Physics 718 Problem Set 3

Due: March 11, 2019

Problem 1 Supernovae in Binary Systems

Many stars are in binaries. If one of the two is massive enough, at some point its core will implode and its outer layers will be ejected in a supernova explosion. Here, we consider two stars, with masses M_1 and M_2, of which the first explodes, ejecting $M_{\rm env}$.

a. For simplicity, assume the envelope and its associated momentum disappear instantaneously. For what $M_{\rm env}$ (in terms of M_1 and M_2) will the binary unbind (i.e., have the individual stars fly apart)?

b. Brown et al. (2005, *Astroph. J.*, **622**, L33) found a "hypervelocity" star, which is likely a B9 main sequence star (mass of $3M_\odot$) traveling at $850\,\mathrm{km\,s^{-1}}$. They argue it is ejected from the Galactic center. But could it also have been ejected in a binary that became unbound in a supernova explosion? To verify, calculate the maximum velocity the star would have been left with if it was previously in a binary with a companion that exploded. For the companion, try both the lowest possible mass for a star than can go supernova ($\sim 8M_\odot$) and a really massive star ($\sim 50M_\odot$). *Note: for simplicity, you can assume the supernova left no remnant.*

Problem 2 Eccentric Binaries

Consider an initially circular binary system with masses M_1, M_2, and separated a. Star 1 goes supernova and loses ΔM of its mass on the way to becoming a neutron star. Assuming that the mass loss is symmetric, what is the eccentricity of the resulting binary e? What is the minimum that ΔM (and therefore e) could be?

Problem 3 Magnetic Supernova

In class, we discuss the neutrino mechanism for supernova. But the issue is coupling. Here we will consider a different mechanism – magnetic fields.

a. Consider a proto neutron star (PNS) with mass of $1.4\,M_\odot$ and radius of $30\,\mathrm{km}$ with an initial rotation period of $T = 10\,\mathrm{ms}$. What is the rotational KE of this system?

b. Now imagine if there is a dipole magnetic field around the PNS with a magnitude of

$$B = B_0 \left(\frac{r_{\rm PNS}}{r}\right)^3 \tag{1}$$

If the magnetic field rotate rigidly, at what (cylindrical) radius does the speed of the magnetic field equal the speed of light – this is called light cylinder. What is the field strength at this radius?

c. Show that the total Poynting flux leaving through the light cylinder is to order of magnitude.

$$\dot{E} = B_{\rm lc}^2 c r_{\rm lc}^2, \tag{2}$$

where $r_{\rm lc}$ is the light cylinder radius and $B_{\rm lc}$ is the dipolar field strength at that radius. Find an expression in terms of T, B_0, $r_{\rm PNS}$.

d. For what values of B_0 and T for $r_{\rm PNS} = 30\,{\rm km}$, does the integrated energy loss over $10\,{\rm s}$ exceed 10^{51} ergs.

Problem 4 Compactness Problem

In this problem, we will find out why gamma-ray bursts have to be beam. For this problem consider a gamma-ray burst with timescales of $\delta t \sim 100$ and $L_{\rm iso} \sim 10^{53}$ erg/s, and observed gamma rays $dN/dE = f(E/E_0)^{-\alpha}$ (number of photons of energy E as a function of energy) where f is some constant and $\alpha = 2$. Let say that the observed energy of the photons go from $E_{\rm min} = 10\,{\rm keV}$ to $E_{\rm max} = 1\,{\rm GeV}$.

- Calculate the density of photons in the region $R = c\delta t$ assuming it is static. This calculation fixes f. Note that the total luminosity is

$$L = \int \frac{E}{dN} dEdE = L_{\rm iso}$$

- Find the fraction of photons f_e that might produced an electron-positron pair. To make this simple assume the incoming photons are of the same energy. Calculate the optical depth of this region to production of photons.

- You should find that the optical depth is huge, which will give a blackbody spectrum, which is not seen. Now assume that radiation is produced in a moving region (toward the observer) with Lorentz factor Γ. The source region is now larger by $\Gamma^2 c\delta t$ (can you guess why the 2 factors of Γ?). In this case, redo part (a). You should find that photon density is small by a factor of Γ^{-4}

- Redo part (b). You should find that f_e is small by a fraction of $\Gamma^{-2\alpha}$. For what value of Γ does the optical depth fall below unity.

Problem 5   2D Supernova Lightcurves

Your colleague, I.M. Insane, suggests that we have it all wrong with Type Ia supernova and that in fact that they are explosions of cosmic strings. He/She suggests that the explosion releases so much energy that it produces radioactive nickel from conversion of energy to matter. As we have done in class, work out the behavior of light curve of such an explosion by computing the time to peak brightness, early time light curve, and late time light curve. The important point is to get the scaling in relation to time, t. Note that we considered a 3-D spherical explosion in class, whereas we will consider a 2-D cylindrical explosion here. Also note that we will have to parameterize the mass as the mass per unit length, σ, because of the cylindrical nature.