Physics 718 Problem Set 1

Due: Feb 8, 2017

Problem 1 Uniformly Stellar

Solve the equilibrium equations for a spherical Newtonian star of uniform density (ρ = constant), to find $m(r)$, $P(r)$ and $\Phi(r)$. Defining the star's surface by the requirement $P = 0$, find the relation between central pressure P_c and radius R. Note that $\Phi = 0$ at $r = \infty$.

Problem 2 Orbital periods and maximum spins

As mentioned in the notes, the free-fall time, time of spherical oscillations, and sound-travel time across a star are all of order $\sqrt{1/G\rho}$. This is also roughly the period of a circular orbit and hence roughly the minimum rotation period of a star.

a. Find the period of a particle in circular orbit just above the surface of a spherical star in terms of G and the average density ρ.

b. Find this period for the Sun and for a typical neutron star.

c. Detailed calculations give the minimum rotation period of a neutron star (the period for which the deformed equator rotates at the speed of a particle in circular orbit) as

$$P_{\min} = \sqrt{\frac{4.4 \times 10^{14} \mathrm{g/cm^3}}{\rho}}\ \mathrm{ms},$$

where ρ is the average density of the nonrotating star. How does this compare to the period of a circular orbit for a star of the density you used? Why does the error have the sign you find?

Problem 3 Bernouilli equation for steady flow

Use the first law of thermodynamics,

$$dE = TdS - PdV$$

to show for a fluid element that conservation of mass $dM = 0$ implies

$$du = -Pd\frac{1}{\rho} + Tds, \text{ and } dh = Tds + \frac{dP}{\rho} \tag{1}$$

Consider a fluid for which all quantities – $P, \rho, \mathbf{v}, \Phi$ – are independent of time: $\partial_t(\text{everything}) = 0$. Using Euler's equation for a fluid, conservation of entropy ($ds = 0$), and the thermodynamic relation for h, show that $\frac{1}{2}v^2 + h + \Phi$ is constant along the flow:

$$\mathbf{v} \cdot \boldsymbol{\nabla} \left(\frac{1}{2}v^2 + h + \Phi \right) = 0. \tag{2}$$

Problem 4 Fluids

Our derivation of conservation of energy for a fluid left out the gravitational field Φ. Show that, when the gravitational field is independent of time, conservation of energy takes the form

$$\partial_t \left[\rho \left(\frac{1}{2}v^2 + u + \Phi \right) \right] + \boldsymbol{\nabla} \cdot \left[\rho\mathbf{v} \left(\frac{1}{2}v^2 + h + \Phi \right) \right] = 0. \tag{3}$$

Problem 5 Stars

- Download *Hipparcos* data from the class website. Note that this is a slightly rearranged version of the data from van Leeuwen (2007, A&A, 474, 653) available at `http://cdsarc.u-strasbg.fr/viz-bin/ReadMe/I/311?format=html&tex=true`.
- Read the file into your favorite plotting/analysis program (`python` preferred but not required). Note that the column names are in the top line. You can read the column descriptions at the website above.
- Restrict the data to points with magnitude $H_p < 8$ and $B - V < 1.75$
- Determine the distance (in pc) from the parallax (`Plx`) column via:

$$d = \frac{1000\,\text{pc}}{\text{Plx}} \tag{4}$$

- Determine the effective temperature from the $B - V$ color via:

$$T_{\text{eff}} = a_0 + a_1(B-V) + a_2(B-V)^2 + a_3(B-V)^3 + a_4(B-V)^4 + a_5(B-V)^5 + a_6(B-V)^6 \tag{5}$$

  where the coefficients are: $a_0 = 9552,\ a_1 = -17443,\ a_2 = 44350,\ a_3 = -68940,\ a_4 = 57338,\ a_5 = -24072,\ a_6 = 4009$ from a fit by Boyajian et al. (2013, ApJ, 771, 40).
- Determine luminosity (in units of $L_\odot$) from the apparent magnitude H_p and the distance d, assuming that the absolute magnitude of the Sun is 4.7.
- Plot a Hertzsprung-Russell diagram of these data. Make sure that temperature increases to the left. Make sure that the luminosities are plotted as a $\log_{10}$ axis. Put a marker where the Sun is on this diagram, and plot lines of constant radii (in units of $R_\odot$).

Problem 6 Stellar Scalings

Assume that for main sequence stars $R \sim M$, and $L \sim M^{3.5}$. Based on these determine how the free-fall timescale and Kelvin Helmholtz timescale scale with mass.