Astron 400 Problem Set 9

Given: Nov 10. Due: Thursday, Nov 17 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Reading: Phillips Chapter 6

Problem 1   Ages of Pulsars

Assume a pulsar is spinning with period P and period derivative $\dot{P}$. It has moment of interia I and constant magnetic field B. You can relate B to the rest by:

$$B^2 = \frac{3\mu_0 c^3 I P \dot{P}}{8\pi^3 R^6}$$

a. Assume the neutron star is born with some initial period P_0 at $t = 0$. Write a differential equation for $dP(t)/dt$, and solve it to get $P(t)$:

$$P(t) = \sqrt{2\frac{8\pi^3 R^6}{3\mu_0 c^3 I}B^2 t + P_0^2}$$

b. Solve this to then get the time since the pulsar was born.

c. Assume that the pulsar was born with a very short period, $P \gg P_0$. Show then that the time since the pulsar was born is approximately $\tau = P/2\dot{P}$.

Problem 2   Hulse-Taylor Pulsar

The binary neutron star PSR B1913+16 led the 1993 Nobel prize in physics. This is because careful measurement of its orbit showed that the orbit was gradually shrinking: a result of Einstein's theory of General Relativity.

The angular momentum for a binary in a circular orbit is $L = \mu\sqrt{GMa}$, where $\mu = M_1 M_2/(M_1 + M_2)$ is the reduced mass and $M = M_1 + M_2$ is the total mass. The masses stay the same, but a will change. You can assume that both masses are the same $M_0 = 1.4\,M_\odot$.

General Relativity predicts that the orbit will lose angular momentum due to gravitational waves at a rate:

$$\frac{dL}{dt} = \frac{-32}{5} \frac{G^{7/2}}{c^5} \frac{\mu^2 M^{5/2}}{a^{7/2}}$$

a. Take the time derivative of the total angular momentum, $L(t) = \mu\sqrt{GMa(t)}$. Show that for both masses being M_0 and only a changing you get:

$$\frac{dL}{dt} = \sqrt{\frac{GM_0^3}{2}} \frac{1}{2\sqrt{a(t)}} \frac{da}{dt}$$

b. Set this equal to the dL/dt from gravitational waves. This will give you a differential equation for $a(t)$. Assume at $t = 0$ the binary is at $a(0) = a_0$. Solve that to get:

$$a(t) = \frac{(-512tG^3M_0^3 + 5a_0^4 c^5)^{1/4}}{5^{1/4}c^{5/4}}$$

Given this, how long (in years) will it take a pulsar binary to merge (i.e., come to $a = 0$)? What is the maximum a such that they will merge within a Hubble time (the age of the universe, roughly 13 billion years)? What binary period does that correspond to?

These are among the sorts of events that people at UWM hope to detect directly with the Laser Interferometer Gravitational Observatory.

c. Use the result of the previous part and Kepler's laws to determine dP_b/dt, the rate of change of the binary period P_b. Show that it is:

$$\frac{dP_b}{dt} = \frac{-192\pi}{5} \frac{G^{5/3}M_0^{5/3}}{2^{1/3}c^5} \left(\frac{2\pi}{P_b}\right)^{5/3}$$

If the current binary period is $8\,\mathrm{hr}$, what is the value of dP_b/dt? Compare this with the measured value, $(-2.4056 \pm 0.0051) \times 10^{-12}\,\mathrm{s/s}$.

Problem 3 GS: P-$\dot{P}$ Diagrams

Go to the ATNF pulsar catalog:
http://www.atnf.csiro.au/people/pulsar/psrcat/
Download the basic information (period, period derivative, and whether or not there is a binary companion) for all of the pulsars. You should make 3 plots:

a. P-$\dot{P}$.

b. P-B

c. τ-B

where $\tau = P/2\dot{P}$ is the characteristic age (you should do it in units of years) and $B = 3.2 \times 10^{15}\,\mathrm{T}\sqrt{P\dot{P}}$. All plots should have logarithmic axes. For each plot you should plot contours of constant values of the other two parameters (i.e., if you plot P vs. $\dot{P}$, show contours of constant τ and B). Make sure to identify the binary systems with a different symbol.

Make sure you send the plots in to `kaplan@uwm.edu` (not just the code).