Astron 400 Problem Set 7

Given: Oct 27. Due: Thursday, Nov 3 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Reading: Phillips Chapter 6

Problem 1 Phillips 6.1

Problem 2 Phillips 6.2

Problem 3 Phillips 6.5

Problem 4 Phillips 6.6

Problem 5 GS: Supersonic Skydive

On Oct 14, 2012, Felix Baumgartner jumped from a high-altitude balloon and briefly went supersonic as he fell.

Assume that the atmosphere is well-described by an isothermal gas in hydrostatic equilibrium with $T = 273\,\mathrm{K}$. At $z = 0$, it has $P =$101,000 Pa.

a. Show that both P and ρ decrease like $e^{-z/H}$ with a scale-height H for a height z above the ground. i.e.,

$$P = P_0 e^{-z/H}$$

and:

$$\rho = \rho_0 e^{-z/H}$$

b. Take $H = 7\,\mathrm{km}$ to be the scale height. The sound-speed is defined here as:

$$c_s = \sqrt{\frac{k_B T}{\bar{m}}}$$

so it is constant. The drag force is defined as:

$$\frac{1}{2}AC_d\rho v^2$$

with A the cross-sectional area of the falling body and C_d the drag coefficient (assume $C_d = 1$). Equating this to gravity you get the terminal velocity:

$$v_t = \sqrt{\frac{2mg}{\rho A C_d}}$$

We can define the terminal Mach number $\mathcal{M}_t$, which is the terminal velocity in units of the sound speed:

$$\mathcal{M}_t \equiv \frac{v_t}{c_s}$$

Use the ideal gas law and make other reasonable assumptions to determine the minimum height from which the jump could have occurred to have it be supersonic, i.e., $\mathcal{M}_t > 1$.

c. Now, the jumper needs to start from a higher velocity in order to achieve the terminal velocity. Integrate the equations of motion (gravity and drag forces) to determine the velocity as a function of height starting from $z = (20, 25, 30, 35, 40)$ km. For which of them is the jump actually supersonic ($\mathcal{M} = v(z)/c_s > 1$)? For one of those, plot z on the y-axis against velocity on the x-axis. Show curves for the actual $v(z)$ as well as the terminal velocity $v_t(z)$ and the sound-speed $c_s(z)$. Label the part of the curve where the jumper is supersonic.