Astron 400 Problem Set 6

Given: Oct 20. Due: Thursday, Oct 27 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Reading: Phillips Chapter 1,4

Problem 1 Journey Through the HR Diagram

Here we will explore the evolution of stars using the EZ Web stellar structure code. This is located at the following website:
`http://www.astro.wisc.edu/~townsend/static.php?ref=EZ-Web` Using EZ Web, run a stellar structure model of the Sun by choosing the following settings:

a. `initial mass=1.0` (meaning $1.0\,M_\odot$)

b. `Metallicity=0.02`

c. `Maximum Age=0`

d. `Maximum Number of Steps=0`

e. `Create Detailed Structure Files=no` (leave it unchecked)

Run a second model of a high-mass $15\,M_\odot$ star, with other settings the same. You need to make some figures. You can do this in `python`, `matlab`, `excel`, or whatever you find easy. Make sure to label your axes, and hand in your plots along with the answers to these questions. If you write code in `matlab`, `python`, etc., please hand that in too. See the end of the problem set for some tips.

a. Using the "summary.txt" file, plot an HR diagram (log Luminosity versus surface Temperature) for the evolution of the Sun. Identify the Zero Age Main Sequence (ZAMS) and the red giant branch. Find the point on the track through the HR diagram where the model is the age of the Sun (4.6 Gyr) and mark this point on your figure. Make sure your temperature axis runs in the same sense as in your textbook. Label both the X and Y-axes appropriately.

b. Now, plot the evolution of the $15\,M_\odot$ star in its own HR diagram. Identify the Zero Age Main Sequence (ZAMS). Make sure your temperature axis runs in the same sense as in your textbook. Label both the X and Y-axes appropriately.

c. Answer these questions: Roughly how long will it take the Sun to reach the RGB phase? During its evolution, will the Sun ever be as bright as the ZAMS $15\,M_\odot$ star? How many ZAMS Suns does it take to be as luminous as one ZAMS $15\,M_\odot$ star? How many RGB Suns would it take? State your assumptions.

Problem 2 Structure on the ZAMS

Now we will delve into the detailed structure of a few EZ Web stellar structure models on the Zero Age Main Sequence. Run models with masses $0.3\,M_\odot$, $1.0\,M_\odot$, and $15\,M_\odot$, all at metallicity $Z = 0.02$. With all of these models, choose "`Create Detailed Structure Files=yes`" by clicking on the check box. We'll only need the first timestep, so to make the runs faster and decrease the file sizes, you might set:

```
Maximum Age=1
```

This will cause the model to run for 1 year, generally creating only a few timestep files. You should now have three cases (model runs) in hand before continuing with this problem. Hand in your figures and answers to the questions. If you write code in `matlab`, `python`, etc., please hand that in too.

Power Generation: For each model, which reaction makes the most power? Compare the ϵ values for pp and CNO reactions. How much energy is lost to neutrinos?

Kappa and M: Make a single figure showing the radiative opacity κ in all three models (0.3, 1.0 and $15\,M_\odot$). Do a semilogy() plot (X axis is linear, Y axis is log), and for your X-axis use a scaled radius r/R_* with R_* the stellar radius, so all three stars fall on the same plot. Label the three stars and the axes. What are the qualitative differences between the opacities in the low mass stars (0.3 and $1.0\,M_\odot$) and the $15\,M_\odot$ star? How much lower are the opacities in the $15\,M_\odot$ star than the other two (quantitatively compare at $r/R_* = 0.3$ and $r/R_* = 0.9$)?

Radiation Pressure: Plot $\beta = P_{\rm gas}/P_{\rm total}$ as a function of r/R_* for each model. Is radiation pressure important?

Polytropes: A simple but very useful model for understanding stellar interiors is to take a polytropic relationship between pressure P and density ρ, with

$$P \propto \rho^\gamma$$

and $\gamma = (n+1)/n$, where n is the polytropic index. For the lowest mass model ($0.3\,M_\odot$), do a loglog() plot of P versus ρ. Fit the slope of the resulting line γ and estimate the polytropic index n. Report both numbers. Is this model fit well by a single polytrope? *Extra credit: discuss the physical significance of your answer*

Tips:

- EZ Web outputs a file `summary.txt` that contains 29 columns, with different information about the star at each time-step. See the web site for which column is which. You will need to read in that file and use it. For example, in `matlab` you can just use `D=load('summary.txt')`.
- If you also choose "Detailed Structure Files", you will get additional files like `structure_NNNNN.txt`, with `NNNNN` some timestep along your simulation. You can load this in with the same sort of method you used for the `summary.txt` file.
- As you download your EZ Web runs, you will find that you have several stars worth of data on your hands. Good organization will make this easier to keep track of. Make directories (folders) named with the mass and metallicity of your simulation runs, and put these in the directory where your code is running from. For example, in problem 1, you might call the solar run "1M_0.02" and the high-mass run "15M_0.02".

  Then your directories look like:

  - path/1M_0.02/
  - path/15M_0.02/

[Thanks to Ben Brown]

Problem 3 GS: Lane Emden and Polytropes

Use a numerical method to compute the density profiles $\rho(r)$ for polytropes with $n = 0, 1, 1.5, 3, 5$. Be sure to correctly incorporate the boundary conditions in your integrations. Plot these profiles, and compare them with the anlytic solutions for $n = 0, 1, 5$. Be sure you only plot out to the first zero.