Astron 400 Problem Set 3

Given: Sep 22. Due: Thursday, Sep 29 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Reading: Phillips Chapter 3

Problem 1 Phillips 2.4

You might end up with an equation that has no analytic solution. Your options are to solve it numerically or to plot it and estimate the answer from the plot.

Problem 2 Phillips 2.9

Problem 3 Phillips 3.7

Problem 4 Free-Free Absorption

Show that the reaction $\gamma + e^- \rightarrow e^-$ (an electron absorbing a photon) is not allowed because energy and momentum conservation cannot be satisfied simultaneously:

a. Conservation of energy is:

$$E_\gamma + E_e = E_{e'}$$

where e' is the electron on the right-hand side.

b. Conservation of momentum is:

$$\vec{p}_\gamma + \vec{p}_e = \vec{p}_{e'}$$

c. Use these two equations along with the relativistic energy-momentum relation and the mass of a photon to show:

$$\vec{p}_\gamma \cdot \vec{p}_e = E_e E_\gamma$$

d. Argue that this means the reaction cannot happen.

Problem 5 GS: White Dwarf Cooling

Take a white dwarf with $M = 0.4\,M_\odot$. Use:

$$\frac{dT_I}{dt} = -\alpha \left[\frac{T_I}{7\times 10^7\,\mathrm{K}}\right]^{7/2}$$

with

$$\alpha = \frac{2}{3k_B}\frac{12m_\mathrm{H}}{M_\odot}L_\odot$$

and an initial temperature $T_I = 10^8\,\mathrm{K}$ at $t = 0$. Numerically integrate to determine the luminosity and surface (i.e., effective) temperature as a function of time. Now do the same for a $1\,M_\odot$ white dwarf with the same initial temperature. Try to plot both objects on the same set of axes (one axis for L, one for T_eff).

A white dwarf will crystallize when the internal temperature is close to the Debye temperature. You can take that as:

$$T_D \approx 1\times 10^7\,\mathrm{K}\left(\frac{\rho}{10^9\,\mathrm{kg\,m^{-3}}}\right)$$

Assume that the density ρ in that equation is the mean density of the white dwarf. If that is true, at what age and what surface temperature will each white dwarf crystallize? Mark this point on your plots. You might find Eqns. 6.19 and 6.20 helpful.