Astron 400 Problem Set 2

Given: Sep 15. Due: Thursday, Sep 22 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Reading: Phillips Chapter 2

Problem 1 Phillips 2.2

To "Compare the relative importance..." you should determine the ratio E_{ES}/E_F, where E_{ES} is the electrostatic energy and E_F is the Fermi energy.

Problem 2 Phillips 2.4

You might end up with an equation that has no analytic solution. Your options are to solve it numerically or to plot it and estimate the answer from the plot.

Problem 3 Maximum Rotation of a MS Star

We shall usually neglect the rotation of stars. However, a star rotating fast enough can be significantly distorted from a sphere, and if it rotates too fast it cannot exist at all.

a. Estimate the maximum allowed rotation rates and spin periods for main sequence stars such that gravitational forces are balanced by centripetal forces. How does the maximum possible angular frequency of rotation scale with stellar mass?

b. Suppose you discover a star that is rotating with a period of 1 second. What is the minimum possible mean density of this star?

Problem 4   GS: Empirical Properties of Stars

Using the data below, study various empirical properties of stars.

a. Find the variation of M/R along the main sequence. Assuming that $M/R \propto M^\sigma$ is an adequate fit, estimate σ. [Hint: to fit a relation of this type, you want to fit $y = ax^b$ and solve for a and b. This is a power-law fit. The easiest way to do this is to take the log of both sides, and then it becomes a linear fit.]

b. The mean density is $\bar{\rho}(M) = 3M/4\pi R^3$. The central density $\rho_c(M)$ is roughly proportional to $\bar{\rho}(M)$. Determine the variation of $\rho_c(M)$ with M along the main sequence based on your results from part (a) [Hint: you should not have to fit any more data]. You can normalize this relation by using the fact that for the Sun, $\rho_c \approx 1.5\times10^5\,\mathrm{kg\,m^{-3}}$.

c. Let ε be the energy generated per kg per second in a main sequence star. Assume that $\varepsilon \propto \rho_c(M)(M/R)^\alpha$. Estimate α in a simiilar manner to what you did in part (a).

d. We know that H-burning is responsible for the energy generation in a main sequence star. This involves conversion of four protons in the center of the star into one ^{4}He nucleus, releasing about 27 MeV per ^{4}He nucleus produced. Assuming that a fraction of $f \approx 0.1$ of the entire mass of the star is converted from H into He on the main sequence, estimate the main sequence lifetime as a function of stellar mass using your results from part (c). [Hint: you should not have to fit any more data].

Properties of Main Sequence Stars

$M\,(M_\odot)$	$\log_{10}(L/L_\odot)$	$T\,(\mathrm{K})$
23	5.2	37,000
16	4.6	30,500
10	4.0	24.000
7	3.6	17,700
4.5	2.7	14,000
3.6	2.2	11,800
3.1	1.9	10,500
2.7	1.6	9,500
2.3	1.3	8,500
1.9	1.0	7,900
1.6	0.8	7,350
1.4	0.6	6,800
1.25	0.3	6,300
1.03	0.1	5,900
0.91	−0.1	5,540
0.83	−0.3	5,330
0.77	−0.5	5,090
0.72	−0.6	4,840
0.67	−0.7	4,590
0.62	−0.8	4,350
0.56	−0.9	4,100
0.50	−1.1	3,850