Astron 400 Problem Set 11

Given: Dec 1. Due: Tuesday, Dec 13 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

For problems 2–3, make sure you hand in your source code and resulting plots by email, as well as the analytical parts.

Problem 1 Sedov-Taylor Solution

a. On the class homepage, you can find three images of the Trinity nuclear explosion test from 1945. Your task (just like Taylor and Sedov) is to estimate the energy of the explosion from these images. Use the equation:

$$E = K\rho_0 R^5 t^{-2}$$

with $K = 1$. Here ρ_0 is the density of air, E is the explosion energy, R is the radius of the blast-wave, and t is the time since the explosion. Estimate E using each of the three images, and see how well they do (or do not) agree.

b. If one atom of ^{239}Pu releases 207.1 MeV upon fission, what mass of Pu underwent fusion? How much mass was converted to energy?

c. For each image, estimate the mass of air swept up. How does this compare to the mass of the bomb itself (4600 kg)?

d. Look at the image of the Cas A supernova remnant on the class homepage. A scalebar is on the image. Assume that the SNR is 3.4 kpc away and is 340 yr old. Use the same equation to estimate the energy of the supernova explosion for an interstellar medium number density $n_0 = 10^6\ \mathrm{m}^{-3}$

Problem 2 Non-linear Pulsations

In this problem you will carry out non-linear pulsation calculations for the one-zone model. You will use:

$$m\frac{dv}{dt} = -\frac{GMm}{R^2} + 4\pi R^2 P$$

and:

$$v = \frac{dR}{dt}$$

along with an adiabatic equation of state:

$$P_i V_i^\gamma = P_f V_f^\gamma$$

where i and f refer to initial and final, but are any two points along the pulsation cycle. Undergraduate students can use a spreadsheet to do this, although I encourage use of python or similar.

a. Show that:

$$P_i R_i^{3\gamma} = P_f R_f^{3\gamma}$$

b. You will not use derivatives. Instead you will use differences between initial and final values of R and v divided by a time interval Δt. So use $(v_f - v_i)/\Delta t$ for dv/dt and $(R_f - R_i)/\Delta t$ for dR/dt.

If you carefully work through the math, you find that you need to use $R = R_i$ and $P = P_i$ in the equation for $m\,dv/dt$ and $v = v_f$ in $v = dR/dt$. Make these substitutions and show:

$$v_f = v_i + \left(\frac{4\pi R_i^2 P_i}{m} - \frac{GM}{R_i^2}\right)\Delta t$$

and

$$R_f = R_i + v_f \Delta t$$

c. Now you can calculate the oscillation model. Assume $M = 1 \times 10^{31}$ kg for a typical Cepheid, and the surface layers will have $m = 1 \times 10^{26}$ kg. At $t = 0$ take:

$$R_i = 1.7 \times 10^{10}\,\mathrm{m}$$

$$v_i = 0\,\mathrm{m\,s^{-1}}$$

$$P_i = 5.6 \times 10^4\,\mathrm{N\,m^{-2}}$$

Use a time interval $\Delta t = 10^4$ s. Assume $\gamma = 5/3$ for a monatomic gas. First calculate v_f at the end of one time interval Δt, and then calculate R_f and P_f. Then use those values for the initial values for your next time interval. Continue this for 150 time intervals until $t = 1.5 \times 10^6$ s. Plot R vs. t, v vs. t, and P vs. t with time on the horizontal axis.

d. From your plot measure the period Π of the oscillation (in seconds and days) and the equilibrium radius R_0 about which the star is oscillating. Compare this with:

$$\Pi = \frac{2\pi}{\sqrt{\frac{4}{3}\pi G\rho_0(3\gamma - 4)}}$$

and with the period for δ Cephei.

Problem 3 GS: White Dwarf Equation of State

Do Phillips Problem 6.11.

a. You will need to integrate out from the center, and can start with a central density of $10^{10}\,\mathrm{kg\,m^{-3}}$. What do you get for the mass and radius of that white dwarf?

b. Now repeat this for a range of central densities and try to fill out the mass-radius curve for a Carbon/Oxygen white dwarf ($Y_e = 0.5$). How do the mass-radius values compare with the analytic relations such as Phillips 6.23?