Astron 400 Problem Set 10

Given: Nov 22. Due: Thursday, Dec 1 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Problem 1   Dust & Stars

Estimate the temperature of a dust grain $100\,\mathrm{AU}$ from a newly-formed F0 main-sequence star ($T_{\mathrm{Eff}} = 7300\,\mathrm{K}$, $R = 1.4\,R_{\odot}$). Assume that the dust grain is in thermal equilibrium—the amount of energy absorbed by the grain in a given time interval is equal to the amount of energy radiated during the same time interval. Assume the grain is spherically symmetric and absorbs/emits as a blackbody.

Problem 2   Solar Strömgren Sphere

The Sun emits 5×10^{23} photons per second with $h\nu > 13.6\,\mathrm{eV}$. If the density of hydrogen atoms in interplanetary space is $n = 10^{9}\,\mathrm{m}^{-3}$, what is the size of the Sun's Strömgren sphere? Assume a recombination coefficient $\alpha = 2.6 \times 10^{-19}\,\mathrm{m}^{3}\,\mathrm{s}^{-1}$.

Problem 3   Cloudy Collision

Suppose that two cold ($T = 100\,\mathrm{K}$) interstellar clouds of $1\,M_{\odot}$ each collide with a relative velocity $v = 10\,\mathrm{km\,s}^{-1}$, with all the kinetic energy of the collision being converted into heat. What is the temperature of the merged cloud after the collision? You may assume the clouds consist of 100% hydrogen.

Problem 4   Column Density

Column density is the integral of number density along the line of sight. It gives the number of atoms you would find if you had a tube of area $1\,\mathrm{m}^2$ that was infinitely long. For something

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with uniform number density n_H, the column density $N_H = n_H \times L$, where L is the line-of-sight distance through the cloud.

We observe an interstellar cloud, with temperature $T = 80\,\text{K}$ and neutral hydrogen density $n_H = 10^8\,\text{m}^{-3}$, at a distance $d = 100\,\text{pc}$. Suppose that the cloud is spherical and that the column density of neutral hydrogen atoms through its middle is $N_H = 1.5 \times 10^{24}\,\text{m}^{-2}$.

a. What is the diameter of the cloud?

b. How many neutral hydrogen atoms are in the cloud?

c. What is the mass of the cloud (in units of $M_\odot$)?

Problem 5 GS: Non-linear Fitting, with Uncertainties

In problem set 5 you fit a function of the form:

$$y = Ae^{-(x-\mu)^2/2\sigma^2}$$

to data that I provided. Now your task is the same, except that you have to also include uncertainties on your fitted parameters.

Specifically, go to `http://www.gravity.phys.uwm.edu/~kaplan/astron400.html` and download the two data-sets from problem set 5, A and B. The first column is x, and the second is y. Each data-set was created with a particular choice of (A, μ, σ) with some random noise added in. You should fit the model to the data and provide me with your best-fit parameters **and uncertainties** along with a curve comparing your fit to the data.

Moreover, once you have your uncertainties you should compare them with Monte-Carlo results. To do this:

a. Generate a fake data-set with the same properties (number of data-points, noise level) as the one that I provided. Use your best-fit values for (A, μ, σ), and make sure you add some appropriate noise to the data.

b. Fit to that new data-set, getting results (A_i, μ_i, σ_i).

c. Repeat this 1000 times (for $i = 1$ to 1000). Compare your uncertainties on a given parameter with the standard deviation of the ensemble $\sqrt{\text{var}(A_i)}$, $\sqrt{\text{var}(\mu_i)}$, $\sqrt{\text{var}(\sigma_i)}$.