# Astron 400 Problem Set 1

Given: Sep 8. Due: Thursday, Sep 15 at the beginning of class

**Homework Policy:** You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

**Reading:** Phillips Chapter 1 (Basic Concepts)

## Problem 1 Phillips Problem 1.1

## Problem 2 Phillips Problem 1.2

## Problem 3 Phillips Problem 1.3

## Problem 4 Radioactive Dating

How old is the Sun? It must be older than the solar system, and the solar system must be older than the oldest rocks on Earth. We will see how radioactive dating can determine those ages.

Radio-active rhenium ($^{187}$Re) decays with a half-life $\tau_{1/2} = 4.6 \times 10^{10}$ yr. The abundance for the decay product osmium ($^{187}$Os) rises accordingly so as to conserve the total number of nuclei of $^{187}$Re and $^{187}$Os. The abundances of these two elements are measured against the abundance of $^{188}$Os, an isotope of Os which is not involved in any decaying process.

a. Show that the following equation is true:

$$\frac{N_{187\mathrm{Os}}(t)}{N_{188\mathrm{Os}}} = (e^{\lambda t} - 1)\frac{N_{\mathrm{Re}}(t)}{N_{188\mathrm{Os}}} + \frac{N_{187\mathrm{Os}}(0)}{N_{188\mathrm{Os}}}$$

where $N_{\mathrm{X}}(y)$ is the number of nuclei of the element X at time $y$ (188Os for $^{188}$Os, etc.), and $\lambda \equiv \ln 2/\tau_{1/2}$.

b. When a rock solidifies (from molten or vaporous forms), all elements are locked in, with initial ratios $N_{187\mathrm{Os}}(0)/N_{188\mathrm{Os}}$ and $N_{187\mathrm{Os}}(0)/N_{\mathrm{Re}}(0)$. Since different isotopes of the same element do not have different chemical behavior, different minerals in the

same piece of rock likely have the same $N_{187\mathrm{Os}}(0)/N_{188\mathrm{Os}}$ value but differ in their $N_{187\mathrm{Os}}(0)/N_{\mathrm{Re}}(0)$ values due to their different chemical compositions. At the present day, one can measure $N_{187\mathrm{Os}}(t)/N_{188\mathrm{Os}}$ and $N_{\mathrm{Re}}(t)/N_{188\mathrm{Os}}$ for different minerals in the same rock. Show that $Y = N_{187\mathrm{Os}}(t)/N_{188\mathrm{Os}}$ should depend linearly on $X = N_{\mathrm{Re}}(t)/N_{188\mathrm{Os}}$, with $Y = aX + b$, and write the values of $a$ and $b$ in terms of the other variables.

c. The table below lists a string of measurements for $X$ and $Y$ (taken from Planetary Sciences, de Pater & Lissauer). Use these to determine what is the initial abundance of $N_{187\mathrm{Os}}(0)/N_{188\mathrm{Os}}$, and how long ago the rock solidified. *(Note: either use a least-square solver or use a graph to determine a and b. The measurements all have similar uncertainty.)*

| $X = N_{\mathrm{Re}}(t)/N_{188\mathrm{Os}}$ | $Y = N_{187\mathrm{Os}}(t)/N_{188\mathrm{Os}}$ |
|---|---|
| 0.668 | 0.148 |
| 0.665 | 0.148 |
| 0.605 | 0.143 |
| 0.483 | 0.133 |
| 0.513 | 0.136 |
| 0.536 | 0.138 |
| 0.415 | 0.128 |
| 0.368 | 0.124 |

## Problem 5 GS: Properties of Matter

Using the relations on p. 59 of *Phillips* and the fundamental constants, reproduce Figure 2.2 in *Phillips*. Your plot should have the various boundaries between the different regions computed from first principles, and include labels like Figure 2.2.

You will have to turn in your source code (only `matlab`, `C`, or `python` without prior permission) and resulting plot via email to `kaplan@uwm.edu`.