Astron 400 Final Review

December 8, 2016

Final Exam: 7:30a, Thursday, December 22, 2016, KEN 1130

The final exam will be a closed-book duration. The exam will cover chapters 1–6 in Phillips as well as the additional material in the notes from the end of the semester. It will concentrate on the material from the second half, but anything is fair.

Topics: you should have at least general familiarity with these areas.

- The basics of nucleosynthesis: what are the starting and ending conditions
- Time-scales for the Sun: free-fall, Kelvin-Helmholtz
- Virial theorem, stability and the adiabatic index
- Star formation and Jeans mass/density
- Main sequence, HR diagrams
- Getting energy out of the Sun, random walks, optical depth
- Stellar scalings (how do basic properties of stars depend on quantities like mass and radius), turning derivatives into approximations
- Stellar energy sources, basics of fusion
- Ideal gas law, hydrostatic equilibrium
- Density of states, Fermi-Dirac, Bose-Einstein, Maxwell-Boltzmann distributions
- Quantum concentrations, chemical potential
- Degenerate gases, non-relativistic and relativistic
- Relativistic energy/momentum relation
- Blackbodies, light, radiation pressure

- Saha equation, equilibrium reactions
- Heat transfer, conduction vs. radiation vs. convection
- Critical condition for convection
- White dwarf cooling
- Fusion in stars, barrier penetration (classical)
- Quantum tunneling
- Fusion cross section, Gamow peak
- Hydrogen burning, pp vs. CNO, Solar neutrino problem
- Helium burning, more advanced burning, fusion timescales
- Equations of stellar structure, polytropes
- Minimum and Maximum stellar masses
- Fundamental units for stellar mass
- Degenerate stars, mass/radius/density relations
- Core collapse, loss of electron degeneracy pressure, Chandrasekhar mass, minimum mass
- Neutron stars, neutron star cooling, interior structure
- Pulsars
- Magnetars
- Black hole basics, circular orbits, effective potentials
- Supernova explosions: chemistry and shocks
- Stellar oscillations
- Telescopes, diffraction limit
- Methods for extrasolar planet detection, hot jupiters
- Gravitational microlensing

Formulas:

Virial Theorem $E_{\rm kin} = -\frac{1}{2}E_{\rm pot}$; $E_{\rm tot} = E_{\rm kin} + E_{\rm pot} = \frac{1}{2}E_{\rm pot}$, [where $E_{\rm pot,binary} = -\frac{Gm_1m_2}{a}$ and $E_{\rm pot,star} \approx -\frac{GM^2}{R}$]

Jeans mass $M_J = 3k_BTR/2G\mu m_{\rm H}$; $\rho_J = (3/4\pi M^2)(3k_BT/2G\mu m_{\rm H})^3$

Generalized Ideal Gas number of states $g(p) = g_s(V/h^3)4\pi p^2$

Generalized Ideal Gas occupancy of states $f(\epsilon) = (e^{(\epsilon-\mu)/k_BT} \pm 1)^{-1}$

Generalized Ideal Gas $P = (1/3V)\int_0^\infty dp\, p v_p f(\epsilon) g(p)$

Quantum Concentration $n_{Q,NR} = (2\pi m k_BT/h^2)^{3/2}$, $n_{Q,UR} = 8\pi(k_BT/hc)^3$

Number density $n = \rho/\bar{m}$

Chemical Potential $\mu = mc^2 - k_BT\ln(g_s n_Q/n)$

Fermi momentum $p_F = (3n/8\pi)^{1/3}h$

Fermi pressure $P = K_{NR}n^{5/3}$ or $K_{UR}n^{4/3}$, with $K_{NR} = (h^2/5m)(3/8\pi)^{2/3}$ and $K_{UR} = (hc/4)(3/8\pi)^{1/3}$

Ideal Gas $P = nk_BT = \frac{\rho}{\bar{m}}k_BT$; typical KE per particle is $\frac{3}{2}k_BT$; energy density $u = \frac{3}{2}nk_BT = \frac{3}{2}P$

Saha Equation (example) $n({\rm H}^+)/n({\rm H}) \approx (n_{Q,e}/n_e)e^{-E/k_BT}$

Degenerate Gas $\Delta x \Delta p \sim \hbar$; $E_{\rm F} = \frac{1}{2}\frac{p_{\rm F}^2}{m_e} \propto n_e^{2/3}$; $P \propto n_e E_{\rm F} \propto n_e^{5/3} \propto (\rho/\bar{m})^{5/3} \rightarrow R \propto M^{-1/3}$ [non-relativistic]

Photon Propagation $l_{\rm mfp} = \frac{1}{n\sigma} = \frac{1}{\kappa\rho}$; $t_{\rm randomwalk} = \frac{R}{l_{\rm mfp}}\frac{R}{c}$

Blackbody $L = 4\pi R^2 \sigma T_{\rm eff}^4$, $F = \sigma T_{\rm eff}^4$; $\lambda_{\rm peak} = 0.29\,{\rm cm}/T$, $u = aT^4$, $P = (a/3)T^4$.

Light $c = \lambda\nu$, $E = h\nu = hc/\lambda$, $p = E/c$, energy density $u = aT^4$, pressure $P = (a/3)T^4$

Hydrostatic Equilibrium $\frac{dP}{dr} = -\rho\frac{GM}{r^2} = -g\rho \rightarrow P \propto M^2/R^4$

Stars $T_c \propto M/R$, $\rho_c \propto M/R^3$, $P_c \propto M^2/R^4$

Timescales $\tau_{\rm free-fall} \sim \sqrt{1/G\rho}$; $\tau_{\rm Kelvin-Helmholtz} \sim \frac{GM^2/R}{L}$

Hydrogen Fusion $E = \Delta mc^2 \approx 0.7\%c^2$

Hydrogen Atom $E_n = -13.6\,{\rm eV}/n^2$

Opacity Electron scattering $\kappa = 0.02(1 + X_{\rm H})\,{\rm m}^2\,{\rm kg}^{-1}$, Kramer's law $\kappa \propto \rho T^{-3.5}$

Radiative Heat Flux $dT/dr = (3\rho\kappa/4acT^3)(L/4\pi r^2)$

Convective Heat Flux $dT/dr = (\gamma - 1)/\gamma\,(T/P)dP/dr$

Probability of Barrier Penetration $\approx e^{-\sqrt{E_G/E}}$

Gamow energy $E_G = (\pi\alpha Z_A Z_B)^2 2m_r c^2$ with $m_r = m_A m_B/(m_A + m_B)$

Fusion Rate $R_{AB} \propto n_A n_B (k_B T)^{-3/2} \int dE\, S(E) \exp\left(-E/k_B T - \sqrt{E_G/E}\right)$

Away from resonance $R_{AB} \sim n_A n_B (E_G/4k_B T)^{2/3} e^{-3(E_G/4k_B T)^{1/3}}$

$R_{AB} \propto T^a$ with $a = (E_G/4k_B T)^{1/3}$

Hydrogen burning $4p \rightarrow^4 {\rm He} + 2e^+ + 2\nu_e$, $\epsilon_{pp} \propto X_{\rm H}^2 \rho^2 T^4$

Equilibrium Temperature $T_p = T_s(1 - A)^{1/4}(R_s/2d)^{1/2}$

Fundamental Mass $M_* = \alpha_G^{-3/2} m_{\rm H} = 1.85\,M_\odot$, with $\alpha_G = G m_{\rm H}^2/\hbar c$

Chandrasekhar Mass $M_{\rm Ch} = 3.1 Y_e^2 M_*$

Schwarschild Radius $R_{\rm Sch} = 2GM/c^2$

Inner Most Stable Circular Orbit $R_{\rm ISCO} = 3R_{\rm Sch}$

Diffration limit $\theta = \lambda/D$

Sedov solution $R_s = (E/\rho)^{1/2} t^{2/5}$

Sound speed $c_s = \sqrt{\gamma P/\rho}$