Astron 300 Problem Set 9

Due: Monday Nov 22 in class or by Wed Nov 24 in my mailbox (4th floor of Physics Bldg)

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Reading: Carroll & Ostlie, Chapter 19

Problem 1   Greenhouse Effect and Global Warming

Perhaps the main worry of our time is global warming. Here, we make a overly simple model of the greenhouse effect to get an idea of how numbers scale. In class, we showed that for an air-less Earth, one can derive an equilibrium temperature $T_p = 255\,\text{K}$ (also, Eq. 19.5 of C&O), but this has to be modified when an atmosphere exists. Earth's atmosphere is optically thin (nearly transparent) at visible wavelengths so the solar radiation hits the ground directly. However, the atmosphere is optically thick (opaque) at infrared wavelengths and absorbs the ground's infrared black-body radiation. This heat is lost to space as the atmosphere radiates with a photospheric (top) temperature $T = T_p$ (think why; hint: energy conservation).

a. Imagine the atmosphere as a single opaque layer with a uniform temperature T_p. It is receiving heat from the ground (at temperature T_g) and radiates as much energy towards the ground as it radiates towards space. First ignoring the gradual warming of the atmosphere, use energy conservation to show that $T_g = 2^{1/4}T_p$. Is the current ground temperature ($288\,\text{K}$) colder or hotter than this?

b. A more sophisticated approach is to allow different layers in the atmosphere to have different temperatures, each emitting both upwards and downwards, with a constant net flux passing through. From this, one can derive (C&O, eq. 9.53) that the temperature will follow:

$$T^4 = T_p^4\left[1 + \frac{3}{4}\left(\tau - \frac{2}{3}\right)\right].$$

where τ is the infrared optical depth from the point being considered to the top of the atmosphere. (As discussed in C&O, the atmosphere emits at an effective optical depth $\tau = 2/3$.)

(b.1) Given $T_g = 288\,\mathrm{K}$ and $T_p = 255\,\mathrm{K}$, what is the atmospheric optical depth τ_g to the ground?

(b.2) Also calculate τ_g on Venus, given its no-atmosphere and actual ground temperatures.

(b.3) Supposing, simplistically, that the greenhouse effect scales linearly with CO_2, what is the expected rise in temperature on Earth as CO_2 is doubled from the current abundance?

c. A more accurate prediction requires simulations which consider all greenhouse gases (e.g., water vapour has more effect than CO_2) and includes both positive and negative feedbacks as the earth's temperature rises. Look up the "climate change 2007 synthesis report" from the intergovernmental panel on climate change, and find what they predict for a doubling of CO_2.