Astron 300 Problem Set 6

Due: Wednesday, Oct 20 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.
Reading: Carroll & Ostlie, Chapter 9.2, 10

Problem 1 Cores of Stars [40 pts]

In this problem, we will examine what are typical densities ρ_c, pressures P_c, and temperatures T_c in cores of stars and how they depend on mass. And we will examine what processes dominate the pressure.

Before you start this part, write down whether you think that for a star more massive (and much more luminous) than the Sun, ρ_c, P_c, and T_c are higher or lower. (Note: this will not be graded; it is simply to find out what your first guess would be.)

a. Look up C&O, App. G, which gives stellar data. Get the masses and radii for main-sequence stars of spectral type B0, A0, F0, the Sun, K0, and M0, and write these in a table. Add columns for ρ_c, P_c, and T_c, and look up the values for the Sun in C&O, Chapter 11.1.

   Estimate values for the other stars using scaling relations that we have discussed (assuming the structure of all main-sequence stars is the same). Do your results match your initial expectation?

b. Use your table to estimate how R actually scales with M (i.e., assuming $R \propto M^a$, what is a? Either fit the data or take the most and least massive star). Given your result, how do ρ_c, P_c, and T_c scale with M?

c. In example 10.2.1, C&O show that for the Sun radiation pressure is not important. Given the scalings you derived, for what mass does radiation pressure (C&O Eqn. 10.19) become important? (If you could not derive the scalings, use the numbers in your table to make a guess).

d. In class, we mentioned that the ideal gas law broke down when particles wavelengths overlap. To estimate whether this happens in the Sun, first calculate the mean particle spacing in the centre of the Sun, $r = n^{-1/3}$ (feel free to assume pure hydrogen composition). Effectively, this implies we “know” particles positions to this accuracy, and this means we cannot “know” their momentum, to a limit given by Heisenbergs uncertainty principle (C&O, Eq. 5.19). Use the uncertainty principle for both an electron and for a proton to estimate the uncertainty in their momentum, and convert that to an uncertainty in their energy. Compare that energy with the thermal energy appropriate for the Suns core. You should find that it is considerably smaller. Given your scalings, for what mass would the energy given by the uncertainty relation become comparable to that inferred from the temperature? For which one do the energies become comparable first, the electron or the proton?

Problem 2 C&O 16.1 [30 pts]

Do not worry about part (e).

Problem 3 C&O 16.4 [30 pts]

For information on Sirius B, look at Sections 16.1 and 16.2