Astron 300 Problem Set 5

Due: Wednesday, Oct 13 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.
Reading: Carroll & Ostlie, Chapter 9.2, 10

Problem 1 C&O 9.1

Problem 2 C&O 9.6

For this, you might want to look at C&O Eqn. 8.3 for the velocity of a particle in an ideal gas.

Problem 3 The Eddington Luminosity

Above a certain luminosity, called the Eddington Luminosity, radiation pressure will exceed gravity and matter will be blown away. Below, you will derive this by direct comparison of forces (for an indirect derivation, see C&O, 10.6). We will consider a completely ionized, pure hydrogen gas, in which photons interact with matter (almost) exclusively through electron scattering. We will also assume the gas is optically thin, i.e., all electrons present can interact with photons.

Electron scattering is discussed in C&O 9.2, on page 246. It means that every electron in a plasma (an ionized gas) contributes an equal area σ_T to blocking the photons. For part (b.1) below, you will need to figure out what the equivalent σ for a proton should be.

a. Consider a single electron that scatters a single photon with energy $h\nu$ in a random direction.

   (a.1) On average, what momentum is imparted on the electron? What is its direction?

(a.2) Use your result to show that for a given flux F and cross-section σ, the force on a single electron is $F\sigma/c$ (Here, show may be easier done in a few words rather than equations.).

b. Consider a blob of gas with mass m at distance r from a star with mass M and luminosity L.

(b.1) Write down the equations for the gravitational force and for the radiation force. Do the gravitational force and the radiation force act equally on protons and electrons? If not, what keeps the protons and electrons together?

(b.2) Equate the two forces and show that your result is consistent with C&O, Eq. 10.114 (Hint: the cross section and opacity κ are related by $\sigma n = \kappa\rho$, where n is the number density of relevant particles and ρ the usual density).

Problem 4 Accretion by Stellar Corpses

Dead objects can be made to shine again if one dumps matter on them. We determine how bright they can be maximally, and how far away a good X-ray observatory like XMM-Newton can see them.

a. How much energy is released if we dump 1 kg of matter on a $M = 1.4M_\odot$ neutron star with a radius $R = 10\,\mathrm{km}$? Determine whether this is an efficient way to generate energy by calculating how much of the rest mass one would have to turn into energy to produce the same amount of energy. How does your result compare with the efficiency (energy per rest mass mc^2) of nuclear fusion?

b. Calculate the luminosity (in units of the solar luminosity) one would get for a dumping rate $\dot{M} = 10^{-9}\,M_\odot\,\mathrm{yr}^{-1}$ (this accretion rate is typical for many binary systems). How does this compare with the Eddington luminosity? (Use your result from the previous question, or check C&O, Eq. 10.114, and use an opacity $\kappa = 0.04\mathrm{m}^2\,\mathrm{kg}^{-1}$ for pure hydrogen). For what accretion rate would one reach the Eddington luminosity?

c. Now assume the neutron star accretes at the Eddington rate.

(c.1) What is its effective temperature, T_eff? And what is the typical energy $k_\mathrm{B}T_\mathrm{eff}$ (in eV) of photons? Assuming all photons have this energy, how many photons are emitted?

(c.2) The XMM-Newton observatory has a collecting area of approximately $0.3\,\mathrm{m}^2$ and can detect X-ray sources down to count rates of about $0.01\,\mathrm{photon\,s}^{-1}$. How far out (in pc) could a source accreting at the Eddington luminosity be seen? Compare with the distance to Andromeda (800 kpc) and the Virgo cluster (16 Mpc).