Astron 300 Problem Set 3

Due: Wednesday, Sep 29 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.
Reading: Carroll & Ostlie, Chapter 19.2, 10.1, 10.2

Problem 1 Bay of Fundy Power [20 pts]

According to Parks Canada, the Bay of Fundy is a 270 km long, generally funnel-shaped embayment with an 80 km wide mouth (treat it like a triangle). Twice daily, water rises and falls by 10m. Suppose we pool our money to form a Bay of Fundy Power Inc., and tap this energy (into electricity). [This problem requires you to do some order of magnitude estimates: concentrate on getting the problem roughly right, and don't worry too much about the exact numbers.]

a. [10 pts]: Show that the maximum power (averaged over a day) the Bay of Fundy could deliver is just over 100 GW. Given current hydro prices, what annual revenue would our Bay of Fundy Inc. have?

b. [10 pts]: Canada has signed the Kyoto protocol, and our Bay of Fundy Inc. may be able to help bring them from a place of 3rd highest emission per capita in the world down to a more modest ranking. Estimate how much CO_2 reduction per year Bay of Fundy Inc. can account for. *(Hint: You can find on the web carbon dioxide emission efficiencies, in units of kg emssion per kWh power, for different fuels).* How does this compare to Canada's total emission?

Problem 2 Supernova Explosions in Binary Stars [15 pts]

Many stars are in binaries. If one of the two is massive enough, at some point its core will implode and its outer layers are ejected in a supernova explosion. Here, we consider two stars, with masses M_1 and M_2, of which the first explodes, ejecting $M_{\rm env}$.

a. [10 pts]: For simplicity, assume the envelope and its associated momentum disappear instantaneously. For what M_{env} (in terms of M_1 and M_2) will the binary unbind (i.e., have the individual stars fly apart)?

b. [5 pts]: Recently, Brown et al. (2005, *Astroph. J.*, **622**, L33) found a "hyper-velocity" star, which is likely a B9 main sequence star (mass of $3M_\odot$) traveling at $850\,\mathrm{km\,s^{-1}}$. They argue it is ejected from the Galactic center. But could it also have been ejected in a binary that became unbound in a supernova explosion? To verify, calculate the maximum velocity the star would have been left with if it was previously in a binary with a companion that exploded. For the companion, try both the lowest possible mass for a star than can go supernova ($\sim 8M_\odot$) and a really massive star ($\sim 50M_\odot$). *Note: for simplicity, you can assume the supernova left no remnant.*

Problem 3 Carroll & Ostlie 10.3 [10 pts]

Problem 4 Dynamical Collapse Time [10 pts]

In class we derived that the collapse time $t_{\text{ff}} \sim 1/\sqrt{G\rho}$. However, I mentioned that you can derive a more exact value. Derive this more exact result for the collapse of the radius of the Sun under the assumption that the Sun is uniformly dense and suddenly becomes pressureless. How long will the collapse take?

Problem 5 Gravitational Contraction [20 pts]

Until Rutherford established that the age of the Earth is approximately 4.5 billion years (roughly), it was thought that the source of energy for the Sun might be gravitational contraction. How fast must the Sun be shrinking in order for its entire luminosity to be generated this way?

a. [10 pts]: Assume that the luminosity L remains constant throughout its lifetime and taking the density of the Sun to be uniform, write down an equation for evolution of the solar radius and determine how long it would take for the Sun to collapse by a factor of 2 in radius. *[Hint: first determine the potential energy from gravity for a sphere of uniform density.]*

b. [5 pts]: How does the central temperature of the Sun change as it shrinks?

c. [5 pts]: In this model, how long ago was the radius of the Sun infinite? Do you find this result objectionable?

Problem 6 Pressure [15 pts]

In C&O, Eqn. 10.8 shows that you can derive the pressure P from the distribution of the momentum that each particle in a gas has:

$$P = \frac{1}{3}\int_0^\infty n_p(p) p v \, dp$$

where $n_p(p)$ is the probability that a gas particle has momentum between p and $p+dp$ (also known as the distribution function). This integrates the contribution of each particle to the pressure $pv/3$ times the distribution function to get the total pressure.

You can also derive:

$$u = \int_0^\infty E_p(p) n_p(p) \, dp$$

where u is the kinetic energy density of the gas (i.e., the total kinetic energy of the gas divided by the volume, in units of $\mathrm{J\,m^{-3}}$), and E_p is the contribution of each particle to the total energy.

a. [10 pts]: Show that $P/u = 2/3$ for a gas of non-relativistic particles.

b. [5 pts]: Show that $P/u = 1/3$ for a gas of relativistic particles (i.e., particles whose velocity is very very close to the speed of light).

Note that these results do not depend on the exact form of n_p.