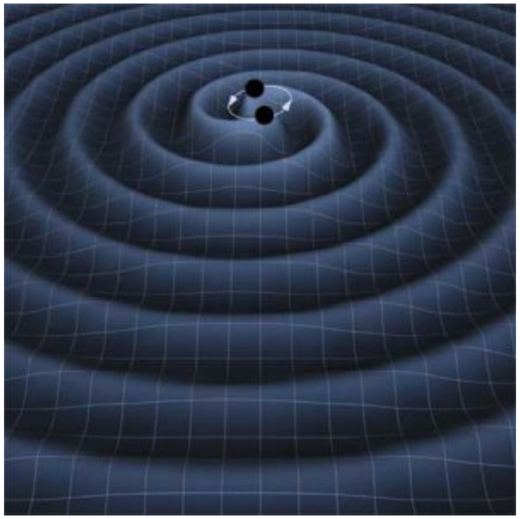
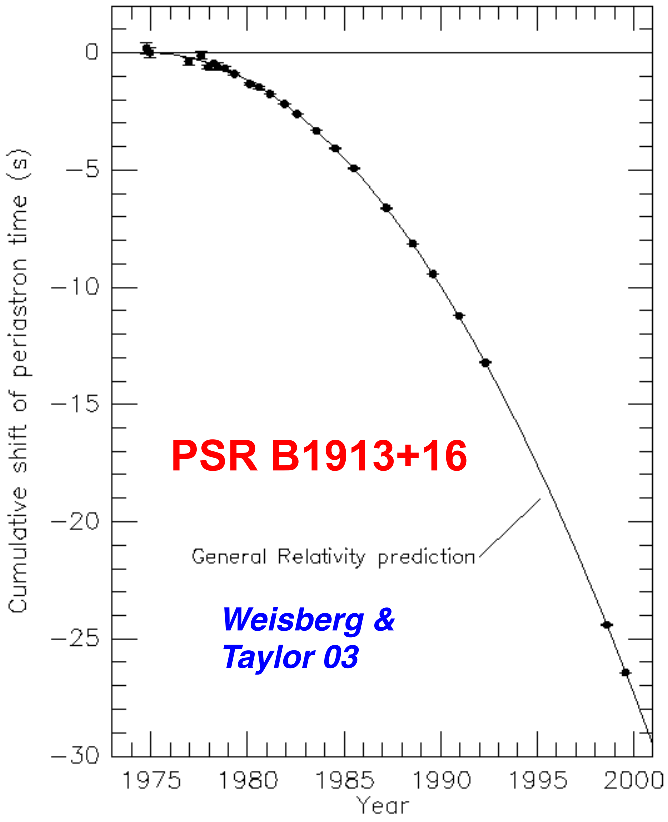
(Ground-based)
Gravitational wave astronomy

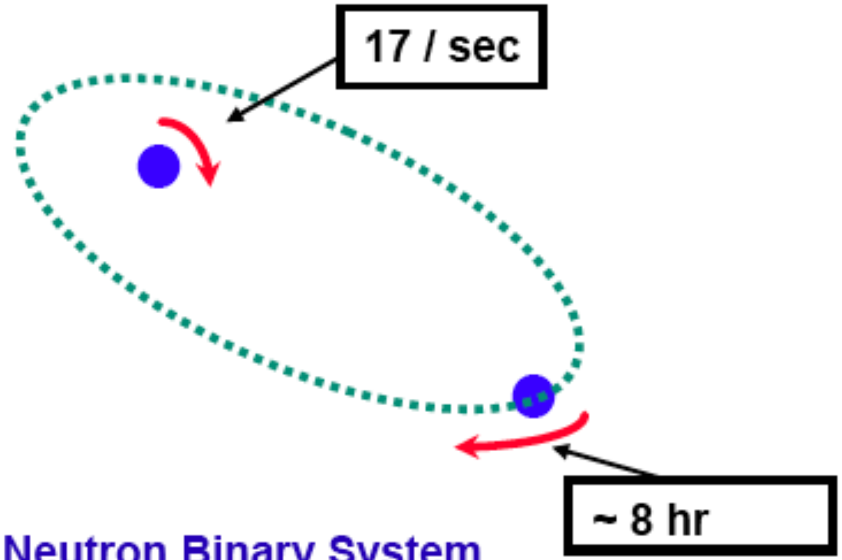


Gravitational waves exist!

- Binary pulsars:



PSR 1913 + 16 -- Timing of pulsars



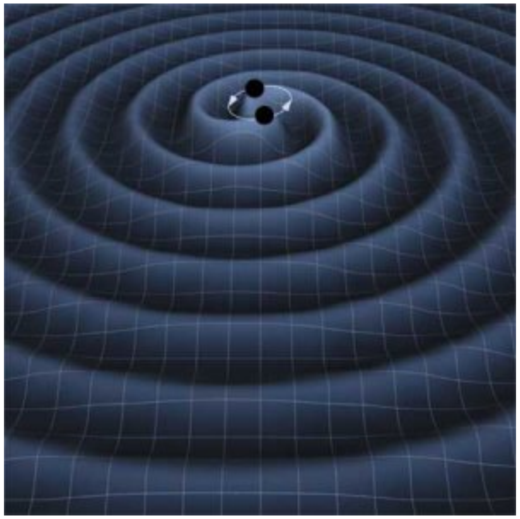
Neutron Binary System

- separated by 10^6 miles
- $m_1 = 1.44 m_\odot$; $m_2 = 1.39 m_\odot$; $\varepsilon = 0.617$

Prediction from general relativity

- spiral in by 3 mm/orbit
- rate of change orbital period

What are GW?

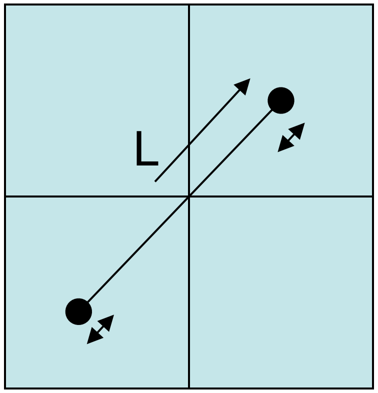


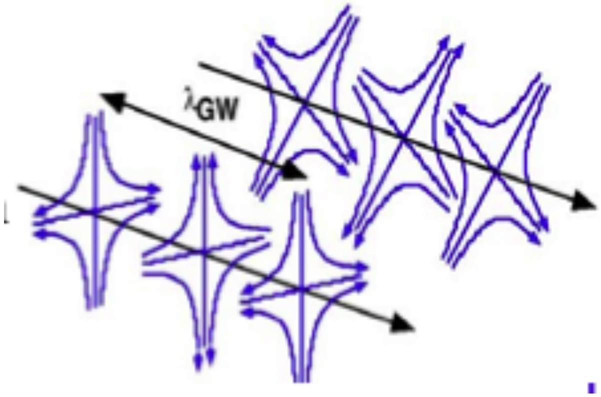
Really:

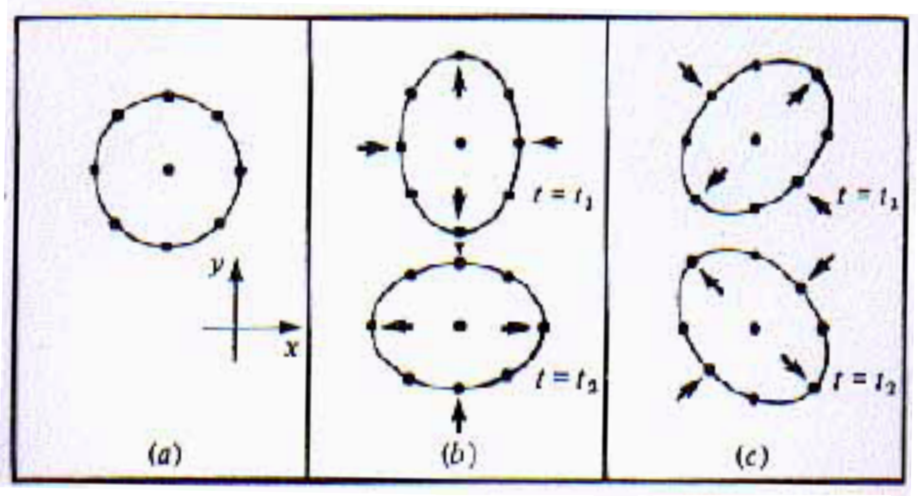
- Required by Einstein's gravity
- "Ripples" on spacetime
- Often highly nonlinear

This talk:

- Linear, spin-2 transverse wave
- Cause "length changes": $h \sim \Delta L / L$
- Like EM:







$h_{xx} = -h_{yy}$ $h_{xy} = -h_{yx}$

What makes GW?

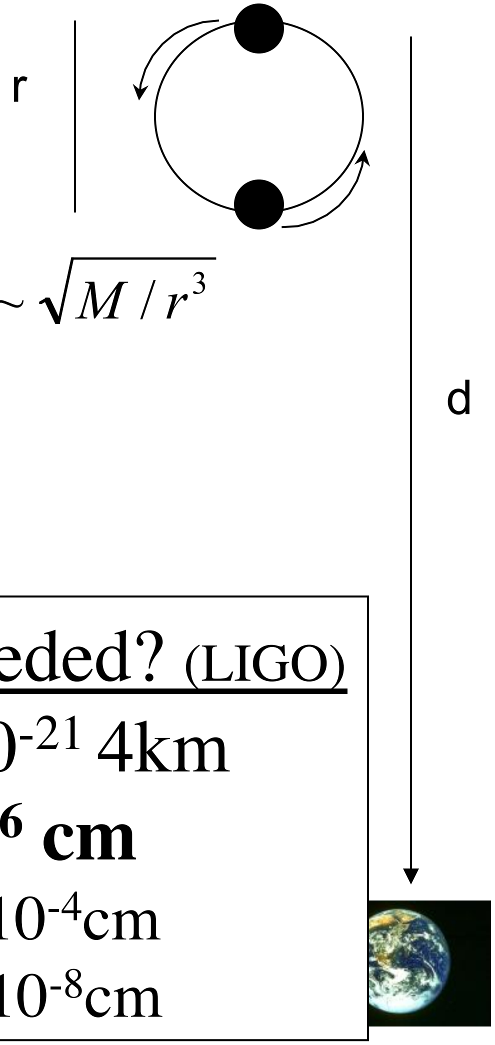
Example: Two black holes (no spin)

Amplitude:

Two black holes

$$f = 2 f_{orb} = 2(\Omega/\pi)$$

$$f = \boxed{10^3\,Hz}(M/8M_o)^{-1}(r/6M)^{-3/2}$$



$$\Omega \sim \sqrt{M/r^3}$$

- **Characteristic relative length changes**

  ~ (kinetic energy)/(distance)

$$h \sim \frac{1}{d}\frac{d^2 I}{dt^2} \sim \frac{Mv^2}{d} \sim \frac{M}{d}(M$$

$$h \sim \boxed{10^{-21}}(M/8M_0)^{5/3}(d/3$$

Sensitivity needed? (LIGO)

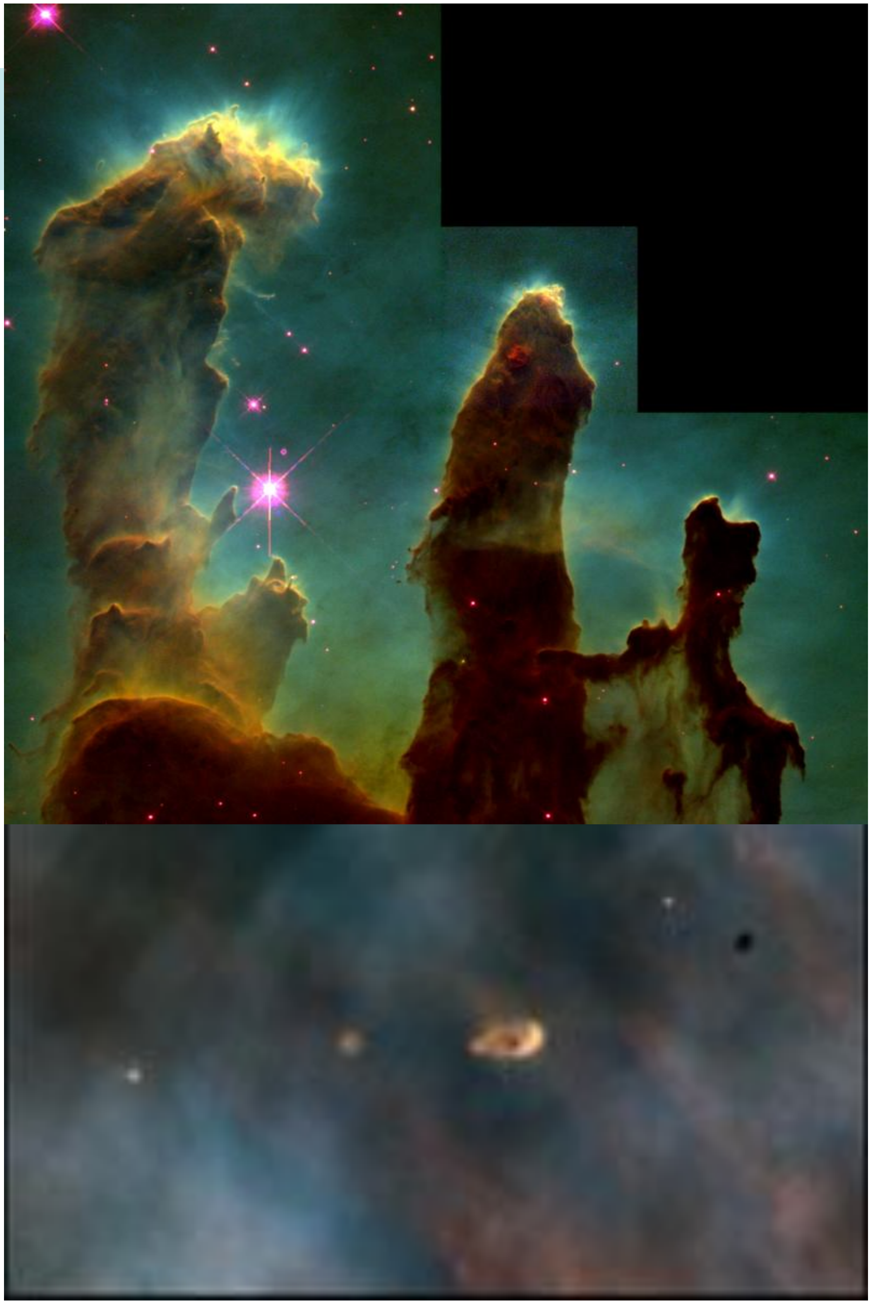
ΔL ~ h L ~ 10^{-21} 4km

~ 4 x 10^{-16} cm

laser light ~ 10^{-4}cm

atom ~ 10^{-8}cm

What makes GW?



EM Waves

Source:

~any accelerating charge

screening limits size…

Strong coupling:

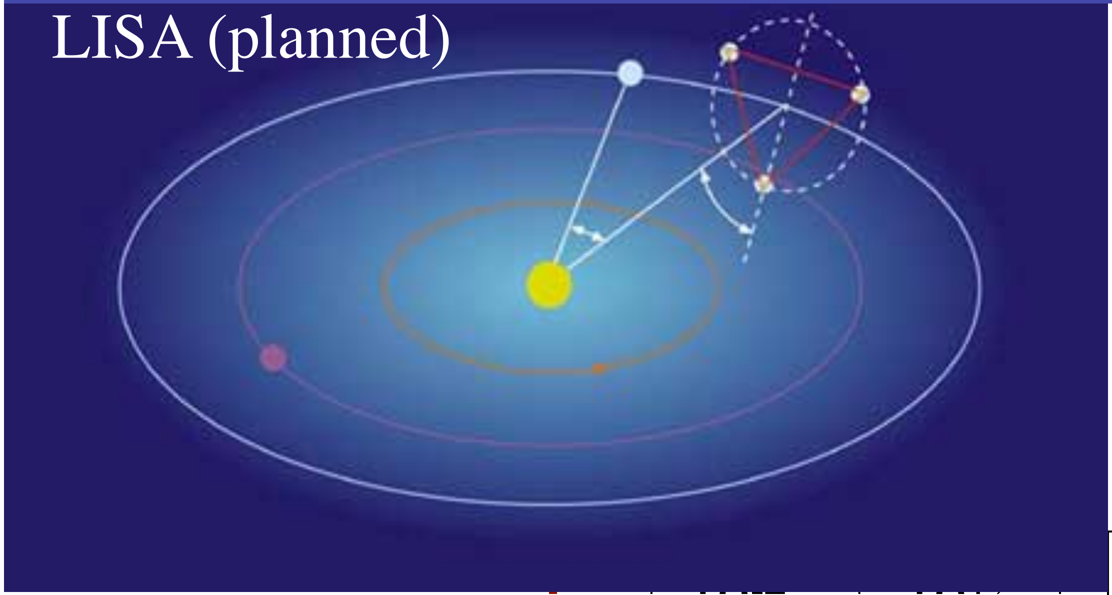
Imaging often practical:

(common sources)

>> wavelength

- Easy to make & detect
- Easy to **obscure**

Detection I: Scale important



Detectors

Pulsar timing
CMB fluctuations

Space-based interferometers
(LISA)

Ground-based interferometers
(LIGO/VIRGO/GEO/TAMA)

Detection II: How sensitive?

Range:

Depends

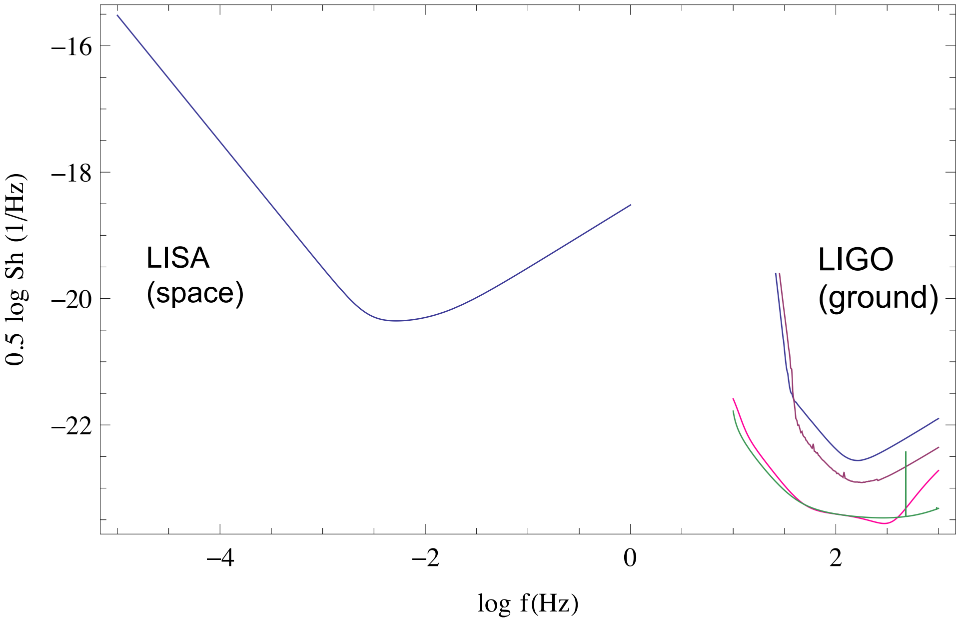
(a) source (how much energy vs frequency) : dE/df

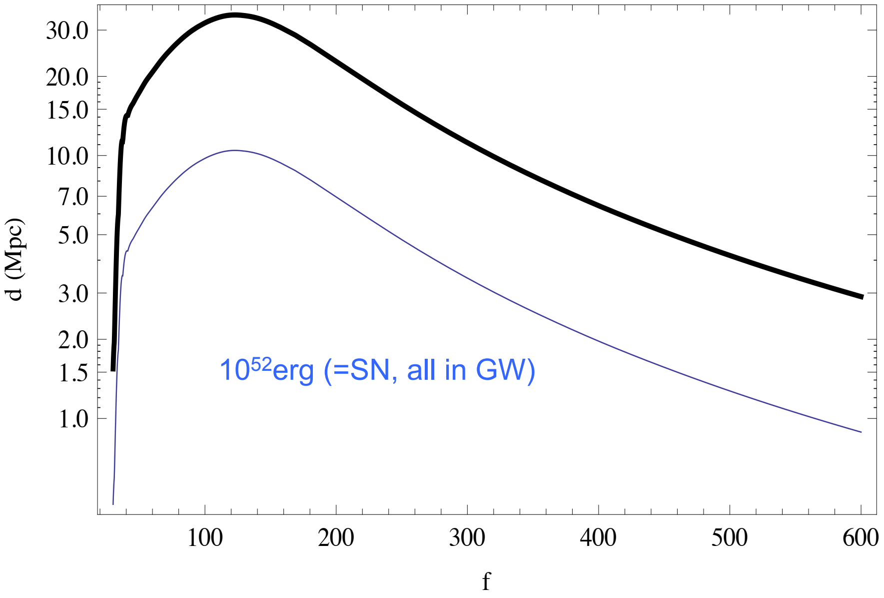
(b) detector (preferred frequencies): $S_h(f)$

compare to
flux threshold

$$D_{burst} \simeq c^{-1}\frac{1}{\rho}\sqrt{\frac{2}{5\pi^2}\frac{\Delta E_{gw}(G/c^5)}{(f^2 S_h(f))}}$$

$$D \simeq \sqrt{L/4\pi F_{crit}}$$

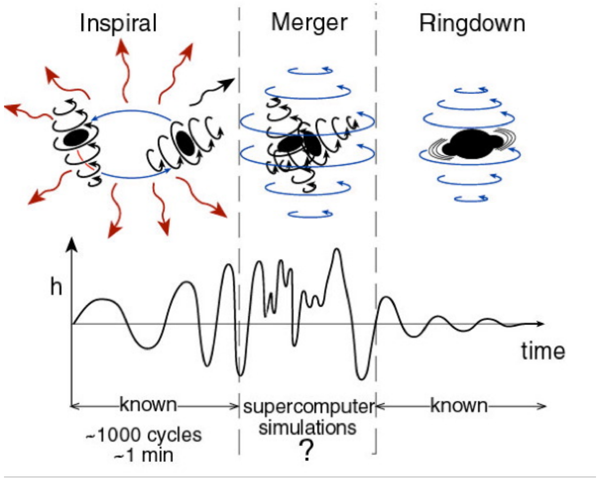




What makes GW?

Example: Two black holes (no spin)

Waveform: 3 epochs



Inspiral:

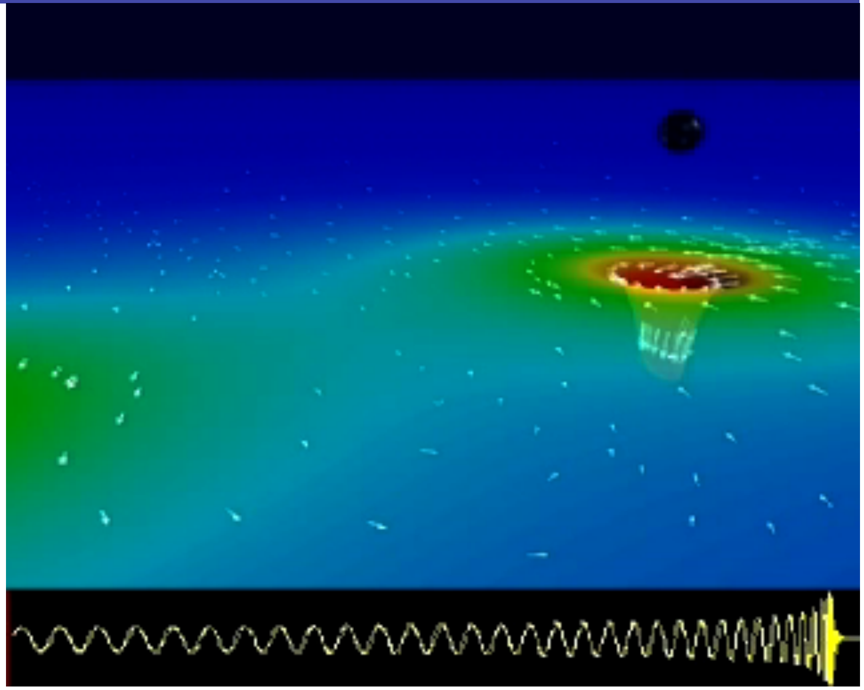
- ~ Quasicircular orbits in potential V(r | L(t))
- Amplitude by changing binding energy

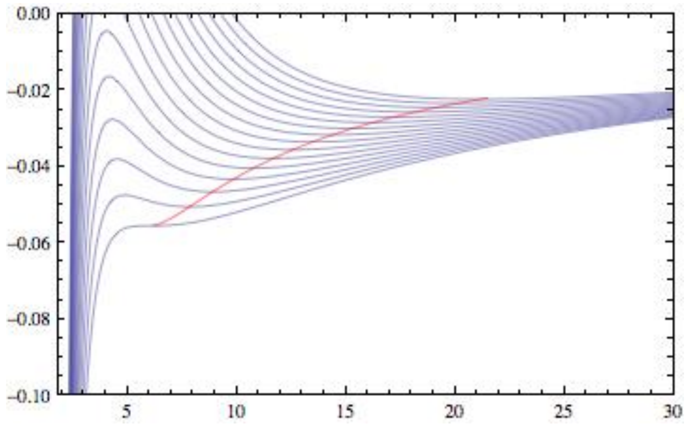
Merger

- Hard

Ringdown:

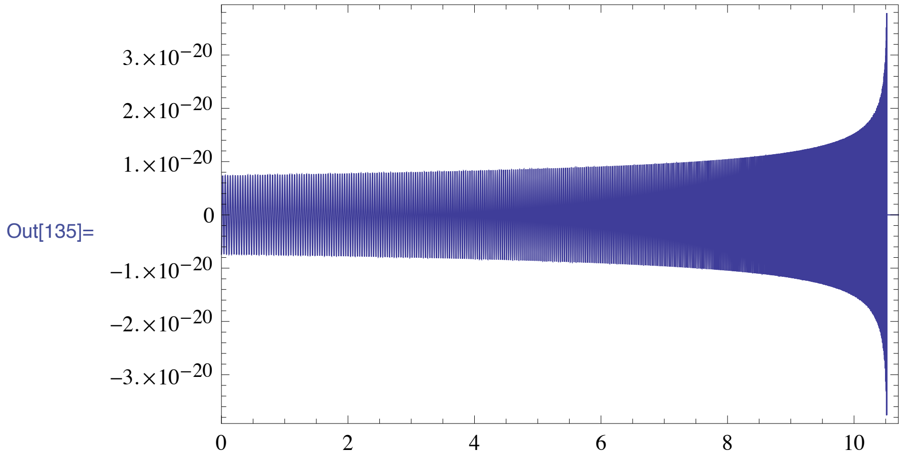
- One perturbed hole “ringing”

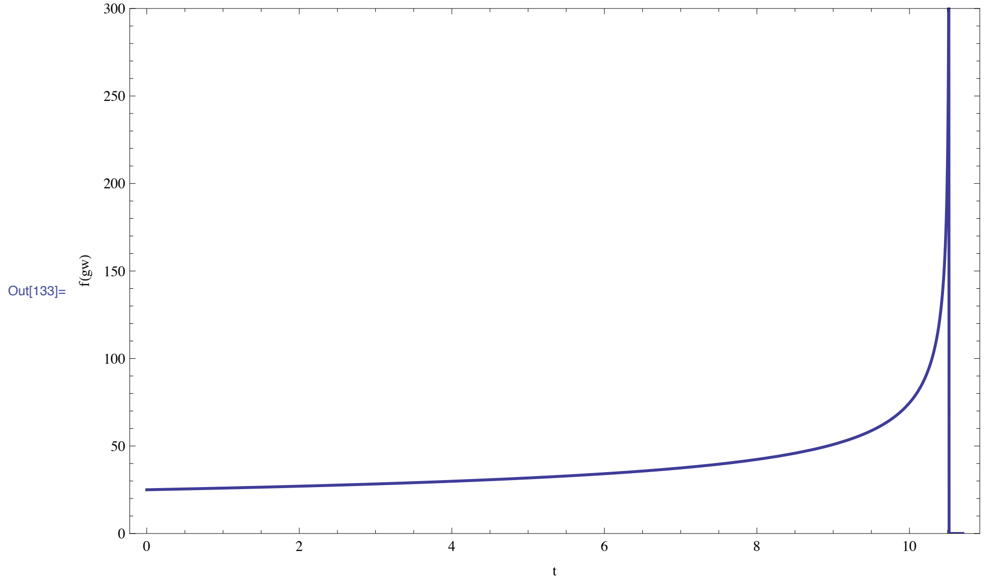




Movie: black-holes.org

Binaries: Chirp





- Frequecy = 2*orbit…
- **Chirp:** Frequency, amplitude increase
  - Set by energy, energy loss rate
- **Identifying source**:

Where on the track are we?

Chirp **rate**, not frequency at any time

Measurables: Inspiral

- Sky location:

$$\delta\theta \simeq \lambda/d$$

Wavelength set by **detector**: 100 Hz
Detector d : 10 ms apart

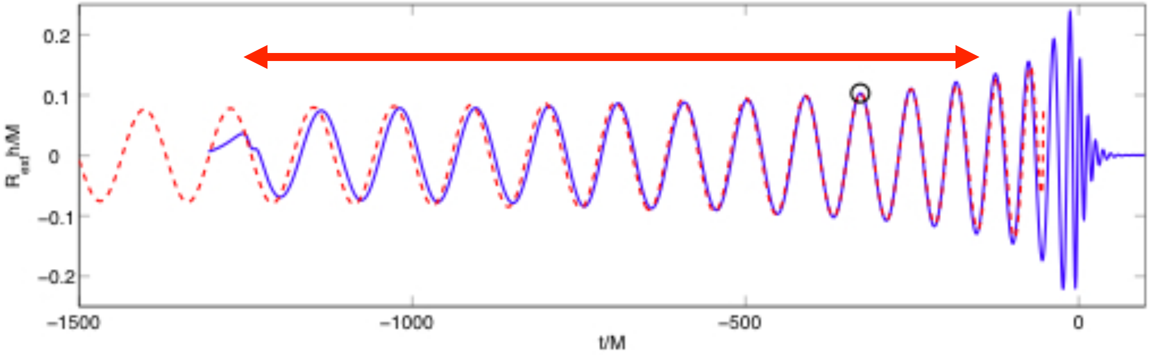
Easier for bright sources (as 1/amplitude)
Easier with complex sources (spin & polarization):
degeneracies

- Mass
  Via chirp rate
  df/dt -> mass
  [mass *ratio* : fine structure]

- Distance

$$SNR \propto \frac{M^{5/6}}{d}$$

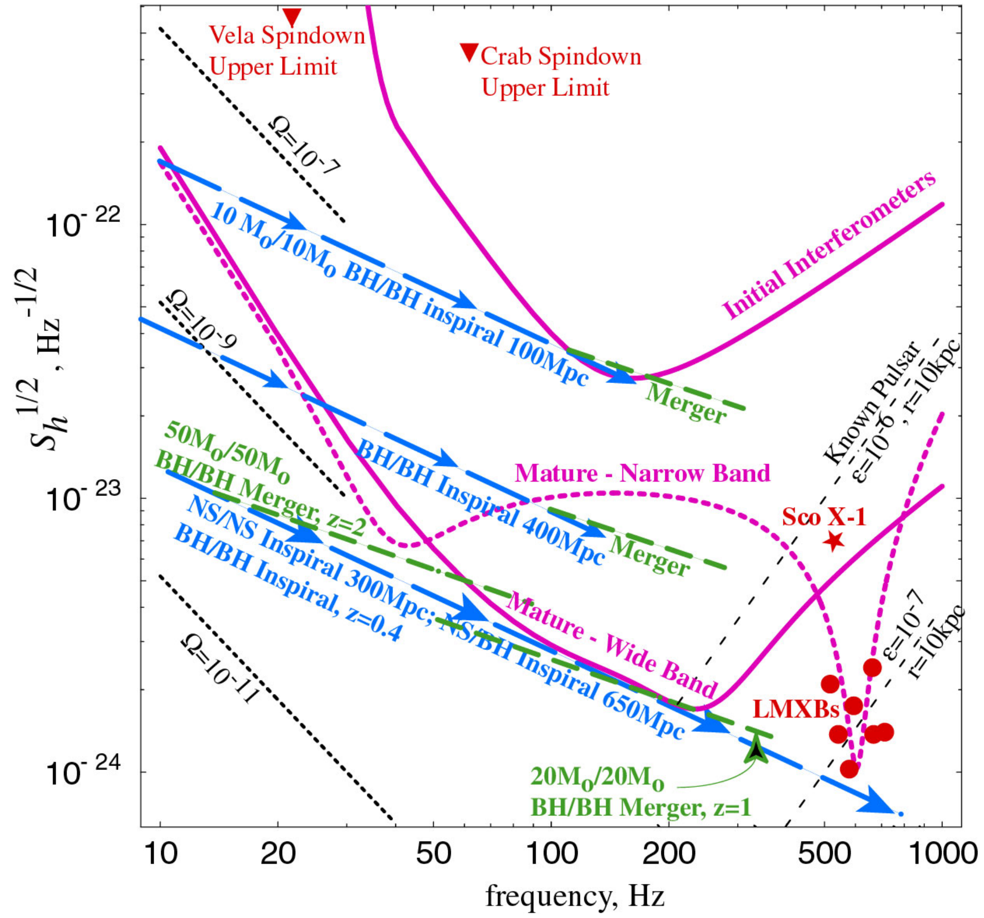
Absolute distance scale

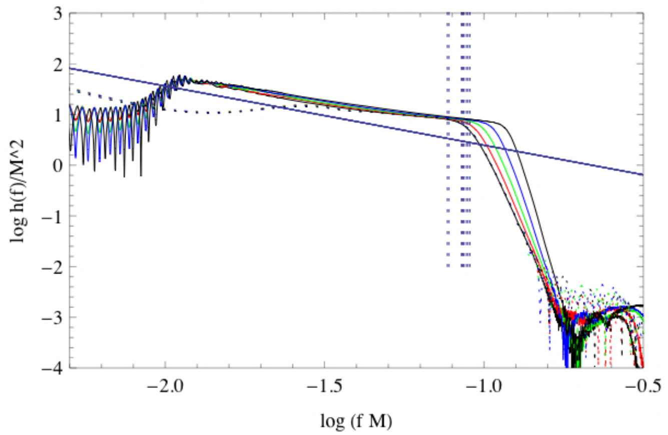


Bonus material: Binary “track” h(f)

“Amplitude” vs f: h(f): $h(f)\text{“}\propto\text{”}\sqrt{dE/df/f^2}$

- Key concept: “Track” h(f (t)) for binaries
- Compare to detector sensitivities



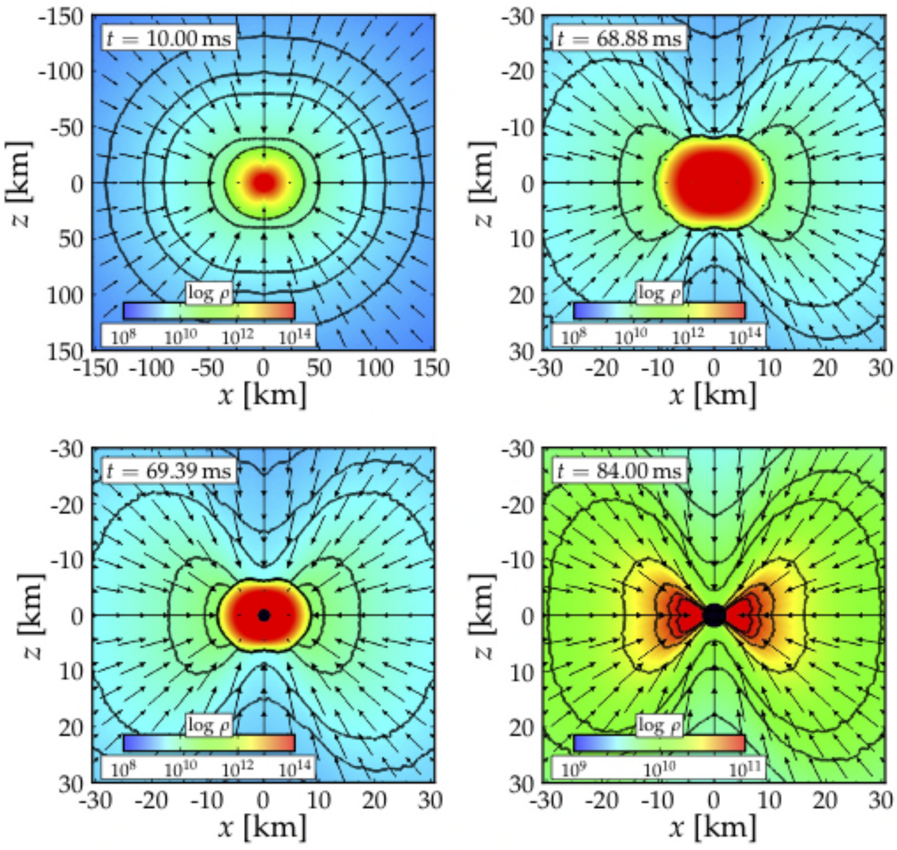


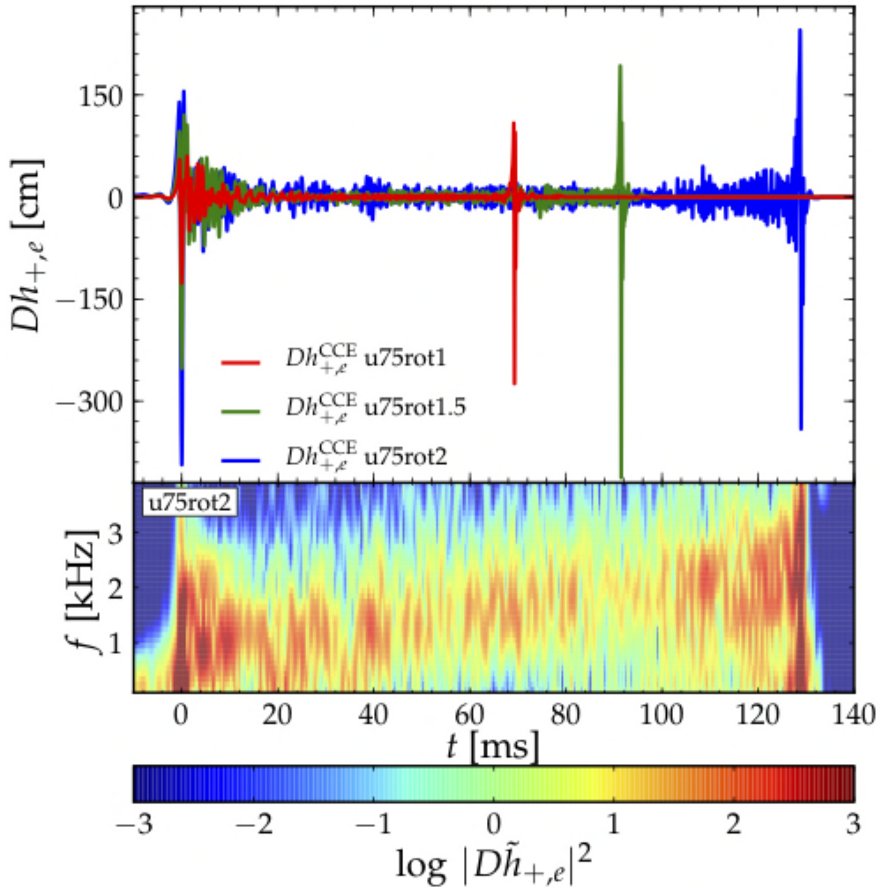
D. Shoemaker, NSF review 2004

Bonus material: Supernova to BH

Complex signal, several epochs [bounce; unstable NS; collapse; ringdown]

Mostly high frequency (collapse time ~ R/c~ms), messy





Ott Dec 2010