Astron 299/L&S 295 Solution Set 2

Given: Sep 14. Due: Wednesday, Sep 21 at the beginning of class

Problem 1 The Height of the Sun

a. Let's derive a general expression for the altitude of the Sun given its declination δ, which will change through the year, and the latitude λ of the observer. In Figure 1, the altitude is a. Remember too that line OS is parallel to line CS, since the Sun is very far away. The angle b is straightforward: $b = \lambda - \delta$ (this assumes $\lambda > \delta$). So we have triangle OCX, with angles b, 90°, and c. So clearly $c = 90° - b = 90° - (\lambda - \delta)$. But since CS is parallel to OS, then $c = a$, and $a = 90° - (\lambda - \delta)$. So we have our three cases:

Date	Season	δ	a
June 21	Summer	$+23.5°$	$70.5°$
December 21	Winter	$-23.5°$	$23.5°$
March 21/September 21	Equinoxes	0	$47°$

b. This is pretty similar. We just need to use $-\lambda$ and $-\delta$, since we are only interested in altitudes between 0 and 90 degrees. We also need to remember that the seasons are flipped.

Date	Season	δ	a
June 21	Winter	$+23.5°$	$-11.5°$
December 21	Summer	$-23.5°$	$35.5°$

However, we see here that on the Winter solstice the altitude of the Sun is < 0: this means you see no sunlight at all on that day!

c. This is a little more complicated now. Look at Figure 2. The Sun is on the opposite side of the Earth compared to the observer — that's what midnight means. But we can still do similar triangles. We find that angle $x = 180° - \lambda - \delta$, and we can form a triangle with the elevation a (again because the lines are parallel), 90°, and x. So this means that $a = -90° + \lambda + \delta$:

Date	Season	δ	a
June 21	Winter	$-23.5°$	$-35.5°$
December 21	Summer	$+23.5°$	$11.5°$

So we see that in the summer, the Sun never sets (the lowest elevation is still 11.5°). This means 24 hrs of daylight in the summer, and 24 hrs of darkness in the winter.

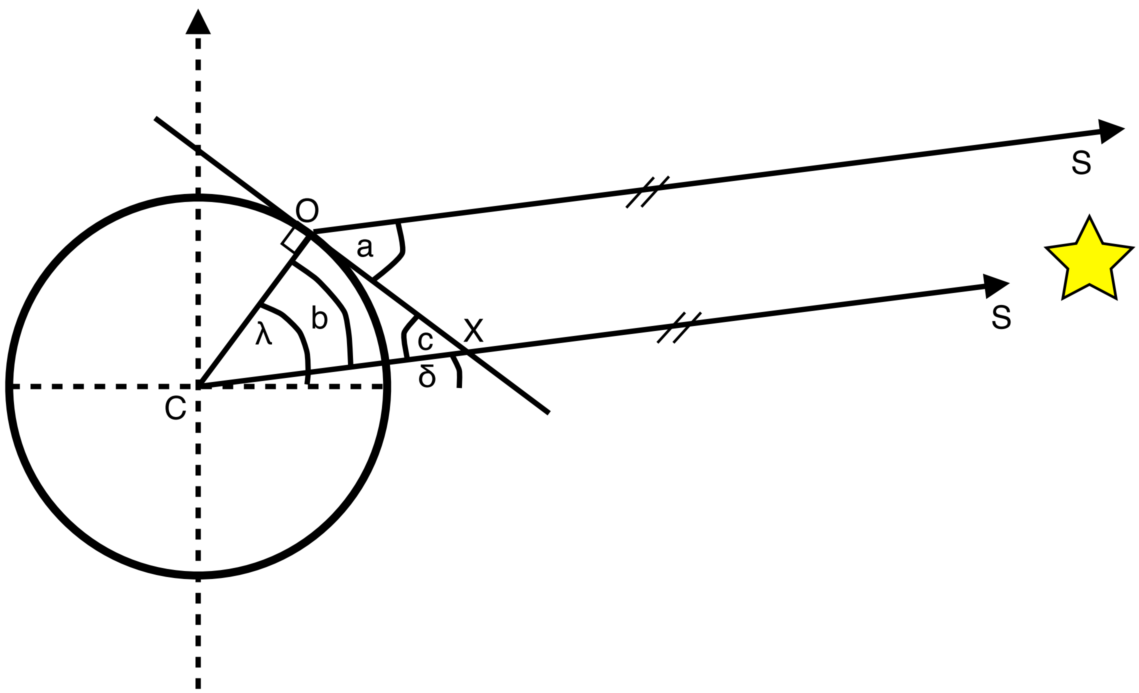

Figure 1: Altitude of the Sun at noon, when it's at declination δ and the observer is at latitude λ

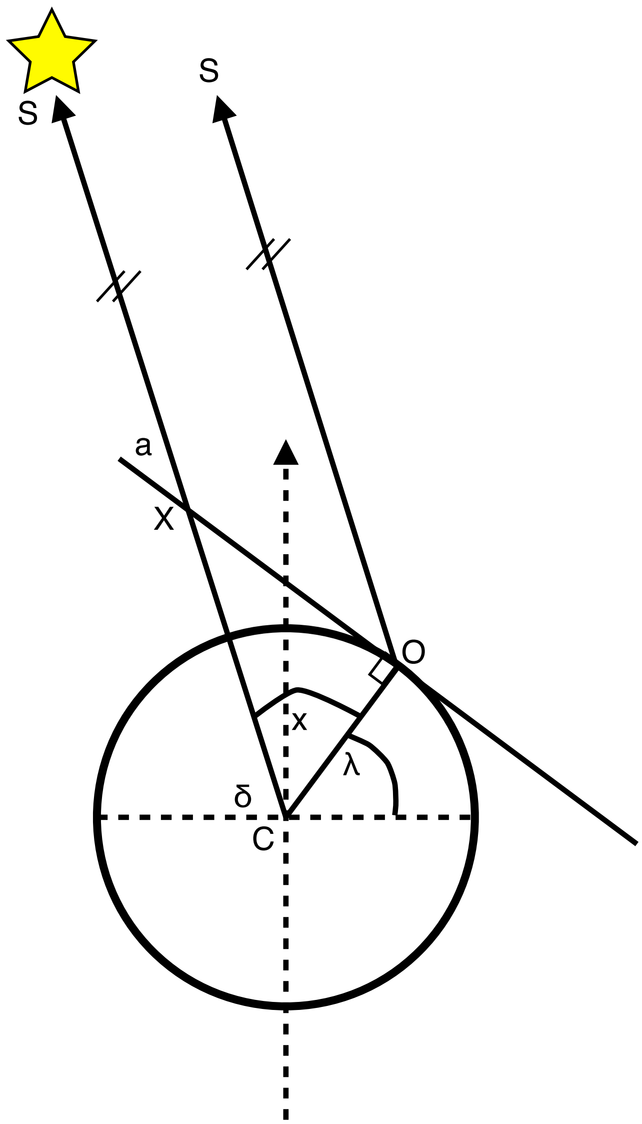

Figure 2: Altitude of the Sun at midnight, when it's at declination δ and the observer is at latitude λ

Problem 2 Synodic Periods

a. This is a lot like the derivation we did in class. The Earth is at an angle $\theta_{\rm E} = (2\pi/P_{\rm E})t$, and Mars is at $\theta_{\rm M} = (2\pi/P_{\rm M})t$. They match at time $t = 0$. When they match again, Earth will have gone around an extra time, so $\theta_{\rm E} = (2\pi/P_{\rm E})t = (2\pi/P_{\rm M})t + 2\pi$, which has the solution $t = S = (1/P_{\rm E} - 1/P_{\rm M})^{-1}$.

b. We use $P_{\rm E} = 365.2\,{\rm d}$. Plugging this in to part (a) and using $S = 583.9$, we solve for $P_{\rm V} = 224.7\,{\rm d}{=}0.615\,{\rm yr}$. For Mars, we use the result we derived in class: $1/S = (1/P_{\rm E} - 1/P_{\rm M})$ the same way, and get $P_{\rm M} = 686.8\,{\rm d}{=}1.88\,{\rm yr}$. Both of these agree with the values in Appendix D.

Problem 3 Kutner 5.13

In circular orbits with total separation r, we know that $m_1 r_1 = m_2 r_2$ and $r_1 + r_2 = r$. What we need are the speeds of the stars, v_1 and v_2 since they will allow us to get the kinetic energy. Since both stars orbit in the same period P, we know $2\pi r_1/P = v_1$ and $2\pi r_2/P = v_2$. Solving for P and equating these, we get:

$$\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$

or (the 2π's cancel)

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

We can get the kinetic energy of each star as $\frac{1}{2}m_1 v_1^2$ etc., so:

$$\frac{K_1}{K_2} = \frac{m_1 v_1^2}{m_2 v_2^2}$$

(the factors of 1/2 cancel). But we know that $v_1/v_2 = m_2/m_1$, so $v_1^2/v_2^2 = (v_1/v_2)^2 = m_2^2/m_1^2$:

$$\frac{K_1}{K_2} = \frac{m_1 m_2^2}{m_2 m_1^2} = \frac{m_2}{m_1}$$

What this says is that if $m_1 > m_2$, then star 2 has more kinetic energy than star 1 despite being less massive. This is because kinetic energy depends on the *square* of the velocity.

Problem 4 Order of Magnitude

[Solutions thanks to Cole Miller.]

a. You may have no idea how much a page of paper weighs. But you know how much a book weighs; a nice 500 page novel might be a kilogram, most of which is the paper. If you adopt 1 kg=500 pages, then its a matter of how much mass you can carry. For most adults it would be between 40 kg and 200 kg, or between roughly 20,000 and 100,000 pages.

b. The amount you drink in a given day depends on the temperature, how much youre moving around, and so on. About two liters per day is probably typical. Lets say you are now 20 years old, which well round to $20 \times 300 = 6{,}000$ days old. Multiplying, you get about 10,000 liters, which is probably accurate to within a factor of three either way.