Astron 299/L&S 295 Problem Set 6

Given: Nov 2. Due: Wednesday, Nov 9 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.
Reading: Kutner Chapter 16

Problem 1 Lines of Sight and Olber's Paradox

a. Suppose that in Sherwood Forest, the average radius of a tree is $R = 1$ m and the average number of trees per unit area is $\Sigma = 0.005$ m^{-2}. If Robin Hood shoots an arrow in a random direction, how far, on average, will it travel before it strikes a tree? [Hint: Consider the mean free path in Kutner section 6.2]

b. Suppose you are in an infinitely large, infinitely old universe in which the average density of stars is $n_\star = 10^9$ Mpc^{-3} and the average stellar radius is equal to the Sun's radius, $R_\star = R_\odot = 7 \times 10^8$ m. How far, on average, could you see in any direction before your line of sight struck a star? (Assume standard Euclidean geometry holds true in this universe.) If the stars are clumped into galaxies with a density $n_g = 1$ Mpc^{-3} and average radius $R_g = 2000$ pc, how far, on average, could you see in any direction before your line of sight hit a galaxy?

Problem 2 Mass-to-Light Ratios

a. From information given in the Table below and in Kutner Chapter 16, determine the approximate mass-to-light ratio of the Galaxy interior to a radius of 25 kpc from the center.

Component	Mass	Luminosity	Radius	Shape
Neutral Gas	$5 \times 10^9 M_\odot$	0	25 kpc	disk
Thin Disk	$6 \times 10^{10} M_\odot$	$1.8 \times 10^{10} L_\odot$	25 kpc	disk
Thick Disk	$3 \times 10^9 M_\odot$	$2 \times 10^8 L_\odot$	25 kpc	disk
Bulge	$1 \times 10^{10} M_\odot$	$3 \times 10^9 L_\odot$	4 kpc	sphere
Stellar Halo	$3 \times 10^9 M_\odot$	$1 \times 10^9 L_\odot$	> 100 kpc	sphere
Dark-Matter Halo	$1.9 \times 10^{12} M_\odot$	0	> 230 kpc	sphere

b. Repeat your calculation for a radius of 230 kpc. What can you conclude about the effect that dark matter might have on the average mass-to-light ratio of the universe?

Problem 3 Stars, Supermassive Black Holes and Tidal Forces

In this problem we will calculate whether or not stars swallowed up by the supermassive black hole at the center of the Milky Way are ripped apart before they pass the event horizon of the black hole. The tidal force is the *differential* force across a body due to an external gravitational force; the gravitational force will act more strongly on the side of the body closer to the massive external object. If the tidal force is stronger than the force holding the object together, the object will be ripped apart.

Consider a star of mass $M_\star$ and radius $r_\star$ a distance r from a black hole of mass $M_{\rm BH}$. If the differential tidal force across the star is

$$\Delta F = \frac{G M_{\rm BH} M_\star}{r^3} r_\star \tag{1}$$

and the gravitational force holding the star together is

$$F_{\rm grav} = \frac{G M_\star^2}{r_\star^2}, \tag{2}$$

how close can the star come to the black hole before it is ripped apart? Consider a star with the mass and radius of the Sun, and a black hole of mass $M_{\rm BH} = 3.7 \times 10^6 M_\odot$. What about a star with mass $60 M_\odot$ and radius 13 $R_\odot$? Do these stars cross the Schwarzchild radius of the black hole before they are ripped apart?

If a star is swallowed whole by a black hole it doesn't produce a major outburst of radiation before it crosses the event horizon, but if it's ripped apart first its gas forms a hot and bright accretion disk around the black hole. Would we expect to see an accretion disk around the black hole at the center of the Milky Way? [Note that this process was likely responsible for a new class of explosion seen last spring. See http://www.youtube.com/watch?v=azLDH9ZPbVs

Problem 4 Order of Magnitude: Asteroid 2005 YU55

Consider Asteroid 2005 YU55 (e.g., http://neo.jpl.nasa.gov/news/news171.html). Next week it will pass rather close to the Earth, although it will not hit. But pretend that it will. Take it to be 400 m in diameter.

a. Assuming it hits the Earth (which it will not), what are the chances that it will hit a person? By this you should consider hitting a person directly, not merely injuring a person from the impact.

b. With approximately how much energy would it hit the Earth? How much TNT is this? What would the equivalent rest-mass be, assuming perfect conversion of mass into energy?