Astron 299/L&S 295 Problem Set 3

Given: Sep 23. Due: Wednesday, Oct 5 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.
Reading: Kutner Chapters 23.5, 9.1–9.4

Problem 1 Order of Magnitude: Bay of Fundy Power [20 pts]

According to Parks Canada, the Bay of Fundy is a 270 km long, generally funnel-shaped embayment with an 80 km wide mouth (treat it like a triangle). Twice daily, water rises and falls by 10m. Suppose we pool our money to form a Bay of Fundy Power Inc., and tap this energy (into electricity). [This problem requires you to do some order of magnitude estimates: concentrate on getting the problem roughly right, and don't worry too much about the exact numbers.]

a. [10 pts]: Show that the maximum power (averaged over a day) the Bay of Fundy could deliver is $\sim 100\,\mathrm{GW}$. Given current hydro prices, what annual revenue would our Bay of Fundy Inc. have?

b. [10 pts]: Canada has signed the Kyoto protocol, and our Bay of Fundy Inc. may be able to help bring them from a place of 3rd highest emission per capita in the world down to a more modest ranking. Estimate how much CO_2 reduction per year Bay of Fundy Inc. can account for. *(Hint: You can find on the web carbon dioxide emission efficiencies, in units of kg emssion per kWh power, for different fuels).* How does this compare to Canada's total emission?

Problem 2 Supernova Explosions in Binary Stars [15 pts]

Many stars are in binaries. If one of the two is massive enough, at some point its core will implode and its outer layers are ejected in a supernova explosion (we'll talk more about this later). Here, we consider two stars, with masses M_1 and M_2, of which the first explodes, ejecting $M_{\rm env}$.

a. [10 pts]: For simplicity, assume the envelope and its associated momentum disappear instantaneously. For what $M_{\rm env}$ (in terms of M_1 and M_2) will the binary unbind (i.e., have the individual stars fly apart)?

   (a) Write the total kinetic energy of the system before explosion. Assume a separation a_0 and velocities v_1 and v_2.

   (b) Write the total potential energy of the system after explosion.

   (c) What is the total energy before explosion? The virial theorem might be useful here to solve for v_1 and v_2.

   (d) Write the total energy after the explosion in terms of the same separation a_0 but new masses. For something to be unbound, the total energy must be ≥ 0. Solve for the new masses and find when this happens.

b. [5 pts]: Recently, Brown et al. (2005, *Astroph. J.*, **622**, L33) found a "hypervelocity" star, which is likely a B9 main sequence star (mass of $3M_\odot$) traveling at $850\,{\rm km\,s^{-1}}$. They argue it is ejected from the Galactic center. But could it also have been ejected in a binary that became unbound in a supernova explosion? To verify, calculate the maximum velocity the star would have been left with if it was previously in a binary with a companion that exploded. For the companion, try both the lowest possible mass for a star than can go supernova ($\sim 8M_\odot$) and a really massive star ($\sim 50M_\odot$). *Note: for simplicity, you can assume the supernova left no remnant.*

   - For the separation of the binary a_0, look at Kutner Appendix E: assume that the initial separation is twice what the radius of the more massive star is, and you can get an idea what that would be by looking at the radii (in terms of the solar radius) for different masses.

Problem 3 Kunter 9.1 [10 pts]

[Look at Equation 8.8 or 8.10 for the "radius" of a black hole]

Problem 4 Kunter 9.2 [10 pts]