Astronomy 299/L&S 295

Lecture I Preliminaries

Course Description: *quantitative* astronomy. Emphasis on the “why” and the “how” rather than just the “what.” There will be some math(!) and physics. There will be no calculus.

If you don’t want that, look at ASTRON 103. If you want a lab, take ASTRON 104. Here we will cover:

- Celestial mechanics
- The nature of light and its interaction with matter
- Telescopes
- The structure and evolution of single stars
- The evolution of binary stars
- The end-products of stellar evolution
- The Solar System
- Extra-solar planets
- Galaxies & quasars
- Expansion of the universe & dark matter
- The big bang

The textbook will be Kutner: *Astronomy: A Physical Perspective*

Evaluation will be:

- Weekly problem sets (50%), with the best 10 of 11 counting.
- Midterm exam (20%)
- Final exam (30%)

I.2 Set the Stage

Astrophysics mostly starts next lecture.

I.3 Physics Synopsis

The level of physics and math that you are expected to be familiar with (but not necessarily know in detail) is:

Newton's Laws : most importantly, $F = ma$

Kinetic Energy : $K = \frac{1}{2}mv^2$

Gravitation : $F = GM_1M_2/r^2$ (on the surface of the Earth, $F = gm$)

Potential Energy : $U = -GM_1M_2/r$ (from gravity; on the surface of the Earth, $U = gmh$)

Centripetal Acceleration : $a = v^2/r$

Ideal Gas Law : $PV = NRT$

Circumference of a Circle : $2\pi r$

Area of Circle : πr^2

Surface Area of a Sphere : $4\pi r^2$

Volume of a Sphere : $\frac{4}{3}\pi r^3$

Radians : $180° = \pi$ radians, $\sin(\pi/2) = 1$, $\sin\pi = 0$, etc.

Small Angles : $\sin x \approx x$ for x very small and measured in radians. Also, $\tan x \approx x$, and $\cos x \approx 1$ (**draw these**)

Scientific notation : $A \times 10^a \cdot B \times 10^b = (AB) \times 10^{a+b}$

I.3.1 Calculus

Not required. However, it is in the book. **Don't Panic!** If you see:

$$\frac{dx}{dt}$$

or

$$\int dx f(x)$$

just read around it. Or ask questions.

I.3.2 Greek

If I use a symbol you don't recognize or can't read, **ask**!

I.4 Precision

We often do not know things very precisely. So we use $\sim$ and $\approx$ and related symbols. $\sim$ is for when we know something to *an order of magnitude*. So we if we know that $x \sim 5$, we know that x is between $5/3$ and $5 * 3$, where 3 is roughly $\sqrt{10}$. This means that the possible range for x is in total a factor of 10. We will also sometimes use $\sim$ to mean *scales as*. For example, if you were to estimate the height of a person as a function of their weight (for a wide range of people), you might expect that as you double the weight, the height changes by $2^{1/3}$. We could write height$\sim$weight$^{1/3}$. There will be a lot of variation, but this is roughly correct.

$\approx$ means more precision. It doesn't necessarily have an exact definition. But generally, if we say $x \approx 5$, that means that 4 is probably OK but 2 is probably not.

Finally, we have $\propto$, which means *proportional to*. This is more precise that the *scales as* use of $\sim$. So while for a person height$\sim$weight$^{1/3}$ is OK, for a sphere (where we know that volume is $4\pi/3r^3$) we could write volume$\propto r^3$: we take this as correct, but leave off the constants ($4\pi/3$ in this case).

I.4.1 Order of Magnitude Problems

There are many problems — in Astronomy, Physics, or Life — where we know only the basics. But we want to estimate something. So we do an "order of magnitude" estimate (also known as a "Fermi problem": see `http://en.wikipedia.org/wiki/Fermi_problem`). Basically, we want to know whether something is 1, 10, or 100, but we do not care whether it is 20 or 30. These make heavy use of the $\sim$ symbol. We will come back to these.

I.4.2 Small Angles

For small angles θ, $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$. We need θ to be in radians. But we also often deal with fractions of a circle. A circle has 360°. We break each degree into 60 minute (or *arcminutes*): $1^\circ = 60'$. And each arcminute into 60 seconds (or *arcseconds*): $1' = 60''$, so $1^\circ = 3600''$. But we also know that 2π radians is 360°, so we can convert between radians and arcsec. This will come up frequently: $1'' = 360 \times 3600/2\pi \approx 1/206265$ radians.

I.5 Example Problem

Consider all of the people on the UWM campus:

- How much do all of the people on the UWM campus weigh?
- How many buses would you need to transport them all?
- If they all jumped in to Lake Michigan, how much would the water level change (numbers: 300 miles long by 118 miles wide, average depth 279 feet, volume 1180 cubic miles)? What about all of the people in the world?

- 30,000 people, 60 kg each, 1,800,000 l or 1800 m^3
- 5×10^{12} m^3
- So answer is 0
- for whole world answer is 1 cm (400,000,000 m^3 total).

Lecture II Celestial Sizes, Distances, and Coordinates

II.1.1 Units

Astronomy emphasizes *natural* units ($\odot$ is for the Sun, $\oplus$ is for the Earth):

- $M_\odot = 2 \times 10^{30}$ kg (solar mass)
- $R_\odot = 7 \times 10^{8}$ m (solar radius)
- $M_\oplus = 6 \times 10^{21}$ kg (earth mass)
- $L_\odot = 4 \times 10^{26}$ W (solar luminosity or power)
- light year $= 10^{16}$ m: the *distance* light travels in one year (moving at $c = 3 \times 10^{8}\,\mathrm{m\,s^{-1}}$)
- parsec = parallax second (we will understand this later) = pc = 3×10^{16} m
- Astronomical Unit = AU = 1.5×10^{11} m (distance between earth and sun)

And then we use usual metric-style prefixes to get things like kpc, Mpc, etc.

II.2 Distances

Kutner 2.6.

How far away/big are things? Use meter stick to draw centimeter, meter, 10 meters.

Chicago : 144 km

circumference of Earth : 40,000 km

distance to Moon : 380,000 km

distance to Sun : 1.5×10^{11} m = 1 AU

solar system (orbit of Neptune): 30 AU in radius

Oort cloud : 50,000 AU

Nearest star (Proxima Centauri): 1.29 pc = 4×10^{16} m

Center of Milky Way : 8.5 kpc = 2.6×10^{20} m

Andromeda Galaxy : 0.6 Mpc = 1.8×10^{22} m and so on. There is *a lot of empty space*!

How do we measure distances to stars? First step: parallax = geometry. $\tan\theta = 1\,\mathrm{AU}/d \rightarrow d = 1\,\mathrm{AU}\tan\theta$. But stars are very far away, so θ is very small. Again, if θ in radians $\tan\theta \approx \theta$ so $d \approx 1\,\mathrm{AU}\theta$ (which puts d in AU too if θ in radians).

But remember, 1 radian is $206265''$. So if θ is in arcsec now, $d = 206265\,\mathrm{AU}(\theta/\mathrm{arcsec})$. 206265 AU has a special name: it is 1 parallax second or 1 parsec (or 1 pc or 3×10^{16} m).

How big is an arcsecond? For a quarter to be $1''$ across (diameter of 25 mm) need to be 5 km away. And we can measure much smaller angles.

II.2.1 Planetary Motion and the Copernican Model

Kutner, Chapter 22.

The geocentric model for the Universe is wrong. The Earth is not the center of the solar system, the galaxy, the universe, etc. Partly this was uncovered through observations of *retrograde motion*: stars appear the same every night, but some objects (often bright ones) move relative to the stars. These are known as *planets* (literally wanderers). Mostly the planets move from West to East. Except when they don't — then they go the other way, which is called retrograde motion. This was a 2000-year old puzzle. **Mars 1994 from Astron 103 week 3**.

In the geocentric view, it was complicated and elaborate (*epicycles*). But the heliocentric view (from Copernicus) has an elegant solution. Taking inner orbits as faster (we'll see why later), we find retrograde motion happens occasionally for the planets that are further out than the Earth.

We can then define two periods:

sidereal period (or sidereal time) is the period relative to the fixed background stars. This is close to (but not exactly the same) as a year.

synodic period is the time between when planets are closest together

and we can relate these by asking how long until the planets line up again. We define the angle of the Earth $\theta_\mathrm{E} = 2\pi t/P_\mathrm{E}$ which goes around from 0 and one revolution happens at $t = P_\mathrm{E}$ (θ in radians). We can do the same for Venus $\theta_\mathrm{V} = 2\pi t/P_\mathrm{V}$. $P_\mathrm{V} < P_\mathrm{E}$, so θ_V goes faster. They line up when $\theta_\mathrm{E} = \theta_\mathrm{V} - 2\pi$: the 2π is since Venus will have gone around one extra time. So we write:

$$\frac{2\pi t}{P_\mathrm{E}} = \frac{2\pi t}{P_\mathrm{V}} - 2\pi$$

and can solve for $1/t = 1/P_\mathrm{V} - 1/P_\mathrm{E}$, and we identify t with the synodic period. For planets outside the Earth's orbit, the sign is opposite.

II.2.2 Motion of the Earth

The Earth rotates around its axis once every 24 hours. So each hour is then $360°/24 = 15°$.

This rotation is what makes the Sun and the stars appear to move over the course of a day. The star to which the Earth's axis appears to point is the North Star (Polaris): it's not a special star, we just point near it. Because of "precession", we point to different stars over the course of about 24,000 years.

This rotation can help you figure out how long you have (for example) until the Sun goes down:

- Your hand is roughly 10° across when you hold your arm out
- Your finger is roughly 1° across

And then, just like all of the planets, the Earth goes around the Sun. Each planet takes its own time. For the Earth, this is 1 year.

II.2.3 Seasons and the Changing Sky

The rotation axis of the Earth is tilted 23.5° to the plane of its orbit. This means that as the Earth orbits around the Sun, the Sun appears to trace a path in the sky. We call this the *ecliptic* (constellations in the ecliptic are the zodiac). The plane of the orbit extending out to infinity is the *ecliptic plane*.

During the nothern summer, the North pole of the Earth points towards the Sun. This is roughly June 21. On the northern winter solstice, what points towards the Sun? Seasons depend on what hemisphere you are in.

solstice : roughly June (northern summer) or December (northern winter), when the "sun stands still." Longest/shortest day. Flips in the southern hemisphere.

equinox : roughly March/September 21. When the night and day are of equal length.

Just like the equator on the Earth, we can extend it into the sky to make the *celestial equator*. This divides the sky into northern & southern halves. We do the same thing with the poles. The Sun crosses the celestial equator twice per year, on the equinoxes.

winter (in north) : Sun is low in the sky, days short

summer (in north) : Sun is high in the sky, days long

spring/fall : in the middle.

II.2.3.1 Weather (Hot vs. Cold)

The Sun puts out energy at a roughly constant rate: $L_\odot$. It is changing the way that we receive this energy that makes seasons. How does this work? Do we move closer to or further from the Sun?

NO!. This wouldn't work since the northern Summer = southern winter. Instead we change the area over which the Sun's energy is spread.

Draw a circle perpendicular to the path of the Sun's light with area $1\,\mathrm{m}^2$. This defines a *flux* which is power per area: $F = L/\mathrm{area}$. The total area over which the Sun's light is spread is the area of a sphere $4\pi R^2$, so at the Earth $F_\odot = L_\odot/4\pi R^2$ with $R = 1$ AU. When the Sun is high overhead (summer), this power gets spread over a patch of the ground with the same area. So each $1\,\mathrm{m}^2$ of the ground gets heated with the full $F_\odot$ of flux. This makes it hot. But in the winter the Sun is low in the sky. So the circle that is perpendicular to the Sun's light gets spread over a wide area on the ground. So we still have $F_\odot$ of flux, but it gets spread over (for example) $2\,\mathrm{m}^2$ of area. This means that each patch of the ground gets heated by less power, which makes it colder.

II.3 How bright are stars?

Kutner 2.1

Historical method: magnitudes (messy and annoying, but has its uses).

lower = brighter, *higher = fainter*. Used to be $m = 1$ is roughly the brightest. But now we have quantified this on a *logarithmic scale*:

- 2.5 mag fainter = 1/10 the brightness
- 5.0 mag fainter = 1/100 the brightness
- 10 mag fainter = 1/10,000 the brightness

We look at things from the Sun ($m = -26.8$, as it appears from the Earth) to $m \approx 30$.

But remember that we are measuring how bright things appear! This would change if they were closer/farther. We are essentially measuring *flux*: energy per time per area. Or (energy per time) per area. (energy per time) is the same as power, and we measure this in Joules per second or Watts. So we measure flux in Watts per area, or Watts per m^2.

If instead we look at total power put out (independent of where you are) that is *Luminosity*, L. L is measured in Joules/s or W. How can we go between these? Take a star of luminosity L, and surround it by a sphere with radius R. The flux is $F = L/4\pi R^2$, since the area of that sphere is $4\pi R^2$. This is the *inverse square law* for fluxes, and it should be familiar (when you go away from a lightbulb, it gets fainter, etc.).

$L_\odot = 4 \times 10^{26}\,\mathrm{W} = 1 L_\odot$. $R = 1$ AU = 1.5×10^{11} m. So the flux on the Earth $F_\odot = 1300\,\mathrm{W\,m^{-2}}$. In comparison, on Neptune at $R = 30$ AU, the flux is 1/900 as much.

So we have magnitudes. How do we tie magnitudes (as a measure of brightness) to real fluxes?

II.3.1 Absolute Magnitudes

=M=the apparent magnitude m of a star if it were 10 pc away.

From the definition of a magnitude, a change of 2.5 mag is a factor of 10 change in F. So we can take two magnitudes m_1 and m_2 and put them with two fluxes F_1 and F_2:

$$\frac{F_2}{F_1} = 10^{-(m_2 - m_1)/2.5}$$

The negative sign comes from having a lower magnitude mean brighter. Or:

$$m_1 - m_2 = -2.5 \log_{10} \frac{F_1}{F_2}$$

To put in absolute magnitudes, we use the inverse square law: $F = L/4\pi d^2$. So $F_1 = L/4\pi d_1^2$ and $F_2 = L/4\pi d_2^2$, and we divide:

$$\frac{F_1}{F_2} = \left(\frac{d_1}{d_2}\right)^{-2}$$

with $d_2 = 10\,\mathrm{pc}$. So $F_1/F_{10\,\mathrm{pc}} = (d_1/10\,\mathrm{pc})^{-2}$. This is the same as:

$$\frac{F_1}{F_{10\,\mathrm{pc}}} = 10^{-(m_1 - M)/2.5}$$

we can work this through to find: $m - M = 5 \log_{10}(d/10\,\mathrm{pc})$ which is known as the *distance modulus*: how much being far away changes the apparent magnitude of something.

So if we know m and M, we can get d. Or if we know m and d, can get M. FOr the Sun: $m_\odot = -26.83$, $d_\odot = 1$ AU = $1/206265\,\mathrm{pc}$. We get: $M_\odot = m_\odot - 5\log_{10}(d/10\,\mathrm{pc}) = +4.74$. This is pretty modest compared to other stars (the Sun is only remarkable by being close).

Knowing what $M_\odot$ is, if we are then given the absolute magnitude of a star we can calculate its flux and actual luminosity. That is because for two objects at the same distance (10 pc in this case) the relation between magnitudes and fluxes can work with luminosities too. So we can say:

$$F = F_\odot 10^{-(M - M_\odot)/2.5}$$

$$L = L_\odot 10^{-(M - M_\odot)/2.5}$$

where $F_\odot$ is now the flux of the Sun as perceived from 10 pc away.

Lecture III Gravity & Celestial Mechanics

And finally some physics. Kutner 5.3, 5.4

Johannes Kepler: used data from Tycho Brahe to determine 3 "laws".

1. Orbits are ellipses with the Sun at a focus. Semi-major axis is a, semi-minor axis is b. The equation of an ellipse says that the distance from the planet to the Sun + the distance from the planet to the other focus is a constant.

2. Planet-sun line traverses equal areas in equal time. (**Kepler's 2nd law from Astron 103 week 3**)

3. $P^2 = a^3$, with P the period of the orbit. For the Sun, this works with P in years and a in AU.

III.2 Elliptical Motion

aphelion = far from star, *perihelion* = close to star. Eccentricity e between 0 (circle) and 1. We find $b^2 = a^2(1 - e^2)$, with perihelion at $a(1 - e)$ and aphelion at $a(1 + e)$. Planets have only slightly eccentric orbits. For the Earth, $e = 0.0167$. So maximum distance from Sun - average is $ae = 0.0167\,\mathrm{AU} \approx 400R_\oplus$. This means a change in the solar flux of about $\pm 3\%$.

III.3 Newton's Laws

1. Inertia

2. $\vec{F} = m\vec{a}$

3. $\vec{F}_{12} = -\vec{F}_{21}$

(here $\vec{F}$ is a force, not a flux). Also have gravitation: $F = GMm/r^2$.

For a circular orbit of something with mass m around something with mass M that is not always the sun, Kepler's third law says: $P^2 \propto r^3$, but the constant of proportionality can change: $P^2 = kr^3$. Let's derive k. $P = 2\pi r/v$, since it has to traverse a distance $2\pi r$. Putting this in gives us

$$\frac{4\pi^2 r^2}{v^2} = kr^3$$

We need a centripetal force $F = mv^2/r$ to keep the planet in orbit: $F = GmM/r^2$. We put these together and re-arrange to get $k = 4\pi^2/GM$.

III.4 Escape: Work and Energy

Potential energy $U = -GMm/r$. It depends just on the start and stop points, not the path.

Kinetic energy $K = 1/2mv^2$.

Total energy $E = K + U = 1/2mv^2 - GMm/r$. Start at r, move away until $v = 0$ at $r = \infty$. $K(\infty) = 0$, $U(\infty) = 0$, so $E(\infty) = 0$. This will always be true (no external source of work), so $K = -U$. We can write $1/2mv^2 = GMm/r$ and get:

$$v = \sqrt{2GM/r} = v_{\rm escape}$$

is the escape speed. For the Earth this is 11 km/s: if you go this fast and point up, you will escape the Earth's gravity.

III.5 Consequences of Gravity

III.5.1 Linear Momentum

If the net force on a system is 0, then momentum is conserved.

III.5.2 Angular Momentum

is conserved also.

III.5.3 Energy

is conserved also.

III.6 2 Body Problem

This is more general than what we did before, and we no longer require $m \ll M$. This applies to binary stars, binary asteroids, black holes, planets, etc. We can write.

Since momentum is conserved, the two bodies will orbit their common center of mass. The COM can move, but it will move at a constant velocity. This is defined such that:

$$m_1 r_1 = m_2 r_2$$

Both stars must go around in the same time such that the line between them always goes through the COM:

$$P = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$

(assuming circular motion). Or $r_1/v_1 = r_2/v_2$. Combining these we get:

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

We also have the force:

$$F = G\frac{m_1 m_2}{(r_1 + r_2)^2}$$

As before, this must be the force needed to keep an object in a circular orbit. So on object 1, $F = m_1 v_1^2 / r_1$:

$$\frac{m_1 v_1^2}{r_1} = G\frac{m_1 m_2}{(r_1 + r_2)^2}$$

Divide both sides by m_1, and use $P = 2\pi r_1 / v_1$:

$$\frac{4\pi^2 r_1}{P^2} = \frac{G m_2}{(r_1 + r_2)^2}$$

But we also define $R = r_1 + r_2 = r_1(1 + r_2/r_1)$. With the ratio of the masses intead of radii:

$$R = r_1(1 + \frac{m_1}{m_2}) = \frac{r_1}{m_1}(m_1 + m_2)$$

Put this in:

$$\frac{4\pi^2 R^3}{G} = (m_1 + m_2)P^2$$

This is Kepler's third law! It's a bit more general than for just a system like the Earth and the Sun. We often write a instead of R.

The angular momentum is constant, and it can be useful. For each object $L_1 = m_1 v_1 r_1$ etc. So overall:

$$L = m_1 v_1 r_1 + m_2 v_2 r_2$$

But we know $m_1 r_1 = m_2 r_2$, so we can write:

$$L = m_1 r_1 (v_1 + v_2) = m_1 r_1 \left(\frac{2\pi r_1}{P} + \frac{2\pi r_2}{P}\right)$$

Substitute from before: $r_1 = m_2 R/(m_1 + m_2)$:

$$L = \frac{2\pi m_1 m_2 R^2}{(m_1 + m_2)P}$$

We substitute for P from Kepler's third law to get:

$$L = m_1 m_2 \sqrt{\frac{GR}{m_1 + m_2}}$$

III.6.1 How to Use This?

A basic result that we will use over and over again is $GM = a^3(2\pi/P)^2$. For circular systems, $v = \sqrt{GM/a}$. Using the center-of-mass we can write $a_1/a_2 = v_1/v_2 = m_2/m_1$, $a_1/a = v_1/v = m_2/M$.

III.6.2 $N > 2$

No general solution. Can be chaotic, not purely periodic, not closed. E.g., Jupiter pertburbs the orbit of the Earth, asteroids, comets, slingshots.

How high does N get? Globular cluster: $N \sim 10^6$ stars. Galaxy cluster: $N \sim 10^3$ galaxies. Universe: $\sim 10^{11}$ galaxies. Need big computers to get approximate solutions.

Lecture IV Doppler Shifts and Waves

Kutner 2.2, 5.2

Light is a wave (and a particle). We talk about waves having wavelength λ and frequency ν: λ is a measure of how far before the wave repeats (in meters or such) and frequency is a measure of how often it repeats (in units of Hz which is 1/s). They are related by the speed of the wave, which for light is c:

$$c = \lambda\nu$$

But, the wavelength that a light is emitted at is not the same as it is absorbed at. It can change if there is relative motion toward or away from the light source. We can this change the Doppler shift, since it shifts the wavelength (or frequency). As long as the velocities are $\ll c$, then:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c}$$

So the shift is $\Delta\lambda$ (Δ usually means change). So if something is moving away from us ($v > 0$), then the wavelength gets longer. Since longer wavelengths are often associated with red, we call this a *red shift*. The opposite, when we go toward a lightsource, is a *blue shift*.

We can also use this in frequency, but the sign is opposite (higher frequency means smaller wavelength, so if the wavelength gets smaller ($\Delta\lambda < 0$) the frequency gets higher ($\Delta\nu > 0$):

$$\frac{\Delta\nu}{\nu_0} = \frac{\nu - \nu_0}{\nu_0} = -\frac{v}{c}$$

Lecture V Extra Solar Planets

Kutner 27.5

This is just how to find them — we will return to talking about the planets themselves later.

How do we find planets around other stars?

- Take pictures? Stars are much brighter, so this is very hard (although it has been done recently in some special cases).
- Main way: Newton

Take a planet in orbit with a star. Mostly the planet moves around the star, but since the mass of the star is finite, it moves a bit too around the center of mass: $v_* m_* = v_p m_p$, so $v_* = v_p(m_p/m_*)$. Even though $m_p \ll m_*$ (so $v_* \ll v_p$) it is still measureable, typically with velocities of a few m/s, via Doppler shifts. We have planets that are similar to Jupiter: $m_p \sim M_{\rm Jupiter} \sim 10^{-3} M_\odot$.

Can also find via eclipses (transits). Here we see the dip in light when a planet goes in front of the star. The amount of light that is lost is $\sim (R_p/R_*)^2$, and since $(R_{\rm Jupiter}/R_\odot) \sim 0.1$, the dip is $\sim 1\%$.

And we can (rarely) see the wobble of the star back and forth during the orbit. Here we use $m_* r_* = m_p r_p$, which gives a wobble of $\ll 1''$.

Lecture VI Tides

Kutner 23.5

Gravity to date has been point masses (or perfect spheres). Not points → tidal forces → not spheres. Tides are from differential forces across an object.

Consider two bits of a bigger thing m_1 and m_2 ($m_1 = m_2$) separated by Δr. $F_1 = GMm_1/r^2$. $F_2 = GMm_2/(r + \Delta r)^2 \approx F_1 + -2GMm/r^3 \Delta r$. It's the extra bit that gives rise to tidal forces, and the $1/r^3$ dependence is pretty general. In the Earth-Moon system, the total force is what keeps the orbit steady. But if you subtract off the forces on the center-of-mass you get the tides, which make a bulge that points at the moon.

Another way to think about it: gravity balances centripetal acceleration $GMm/r^2 = v^2/r = \omega^2 r$ to make a stable orbit. But that is only true at the center of mass: too close and gravity is stronger (bits that are too close get even closer). Too far and gravity is too weak, so bits that are too far get farther. This makes bulges.

Tidal period = $2\times$ forcing periods, from two bulges per rotation (toward and away). Height of tides from Sun $\sim$ that from Moon. With rotation of the Earth, get mostly semi-diurnal (≈ 12.4 h) tides from $P_{\rm rot} = 24$ h, which dominate in Atlantic. But some places hay diurnal tides (complicated interactions between water and gravity).

From the orbit of the moon get spring tides (when lunar lines up with solar) and neap tides (when they cancel out).

From the orbit of the earth get semi-annual tides, since the eccentricity of the Earth is not quite 0.

Tides affect: atmosphere, rock, *ocean*.

$$a_{\rm tide} \sim \frac{2GM_{\rm moon}}{r^3} R_\oplus \sim 2g_\oplus \frac{M_{\rm moon}}{M_\oplus} \left(\frac{R_\oplus}{r}\right)^3$$

But the tidal bulge does not fly away due to this extra acceleration. Instead it makes a bit of extra gravity from the extra mass to cancel it out. The bulge has height h, with extra gravity $g' \sim GM_{\rm bulge}/R_\oplus^2$. We have $M_{\rm bulge} \approx hR_\oplus^2\rho$, which gives $g' \sim GM_\oplus/R_\oplus^2(h/R_\oplus) \sim g_\oplus(h/R_\oplus)$ (we have used $\rho R_\oplus^3 \sim M_\oplus$). Setting g' equal to $a_{\rm tide}$, we find $h/R_\oplus \sim 2(M_{\rm moon}/M_\oplus)(R_\oplus/r)^3$.

The size of this bulge from the moon on the Earth is about $10^{-7}R_\oplus \sim 60$ cm. And about 25 cm from the Sun. The Earth on the moon is about 2 m. This is of the right *order*, although the details are hard. For instance, in the Bay of Fundy, the tidal forcing period is the same as the time it takes for the water to slosh around. So you get a resonance, and the tides are really high (up to 9 m).

VI.2 Tidal Evolution

Water sloshes, loses energy (heat) — or generates electricity!

Note that the Earth spins faster (24h) than the Moon's orbit (28d). Friction drags the tidal bulge ahead of the moon. This leads to a net torque that slows down rotation. Conserving L, the moon moves away ($L \propto \sqrt{a}$). This also makes orbits more circular, as tides are stronger for $e > 0$, and synchronized (like the moon is now, with rotation period of the moon equal to its orbital period).

We can measure: the moon mooves away at $\approx 3\,cm$/yr. And the Earth day slows down at 0.0016 s/century. Note that the length of a day goes up, the length of a month goes up, but the number of days in a month goes down.

For Pluto+Charon, $P_{\rm orb} = P_{\rm Pluto} = P_{\rm Charon} = 6.4$ d, and $e = 0$. So this has already all happened here.

Weird cases exist, like Mars. Its moon Phobos orbits faster than Mars rotates. So tides pull it in! It will hit in about 50 Myr.

VI.3 Roche Limit

When do tides pull things apart? Simple answer:

$$\frac{Gm}{R^2} \sim \frac{2GM_0}{r^3} R$$

where the thing being pulled apart has mass m, size R, and is held together only by gravity. We take $\rho = m/(4\pi/3R^3)$ and $\rho_0 = M_0/(4\pi/3R_0^3)$. Equating the forces we get roughly:

$$r \lesssim 2^{1/3} \left(\frac{\rho_0}{\rho}\right)^{1/3} R_0$$

Could this have given us Saturn's rings? They are inside such a radius (known as the *Roche Limit*, so large moons would have been pulled apart.

VI.4 Tides and Black Holes

Tides are how black holes kill you. It's not that gravity is too strong, it's that the difference in gravity between different parts of you is too strong.

Lecture VII Stars I

Kutner Chapter 9

What is a star?

- Big ball of gas
- Self-gravity $\rightarrow$ need high pressure inside to support against collapse
- high pressure $\rightarrow$ high T inside (basic gas physics)
- high T $\rightarrow$ emit light

A star's life is a long, losing battle with gravity.

Gravity pulls the parts of the stars in. Why do they not fall? Because pressure pushes them out. Which leads to high T, so the star loses energy. This means we need an energy source to keep the star shining. What is that? Gravity? Chemistry? Fission? Fusion?

VII.2 What supports against gravity?

Let us consider:

1. What if there were no support?
2. What provides the support?
3. How much support is necessary?

If the star is spherical, $g_r = GM(r)/r^2$ where $M(r)$ is the amount of mass contained within r. This is independent of $\rho(r)$, and the matter outside r does not matter.

VII.2.1 What if there were no support?

This is relevant for star formation, supernovae. We have things fall inward with radial velocity v_r and acceleration $a_r = -GM(r)/r^2$. The material falls toward the center taking a *free-fall timescale* $t_{\rm ff}$:

$$|v_r| \sim \frac{r_0}{t_{\rm ff}},\ a_r \sim \frac{v_r}{t_{\rm ff}} \sim \frac{r_0}{t_{\rm ff}^2} \sim \frac{GM_0}{r_0^2}$$

From this we can solve $t_{\rm ff} \sim \sqrt{r_0^3/GM_0} \sim \sqrt{1/G\rho_0}$, where $\rho_0 \sim M_0/r_0^3$ is the mean density. We identify this free-fall timescale as the *dynamical time*. If you do out the math in detail (keeping factors of $4\pi/3$ etc.), you find:

$$t_{\rm ff} = \sqrt{\frac{3\pi}{32}}\frac{1}{\sqrt{G\rho_0}}$$

So the time for collapse only depends on the density, not the size.

Object	r	M	ρ	$t_{\rm dyn}$
Earth	6×10^6 m	$10^{-6}\,M_\odot$	5.5 g/cm^3	~ 10 min
Jupiter	7×10^7 m	$10^{-3}\,M_\odot$	1.3 g/cm^3	~ 10 min
Sun	7×10^8 m	$1\,M_\odot$	1.4 g/cm^3	~ 10 min
White Dwarf	7×10^6 m	$1\,M_\odot$	1.4$\times$10^6 g/cm^3	3s
Neutron Star	10^4 m	$1.4\,M_\odot$	7$\times$10^{14} g/cm^3	0.1 ms

VII.2.2 Support comes from gas pressure

Pressure: resists compression. This comes from the kinetic energy of the gas particles. You can think of them each exerting a little force when they bounce off the walls of a box.

Pressure is force/area. In stars, most of the gas is an *ideal gas*, which means that the particles are all independent of each other. In the kinetic theory of ideal gases, the kinetic energy per particle is $3/2k_BT$ (where k_B is Boltzmann's constant). This is an average. It is both an average over time for 1 particle and an average over all particles at one time. From this we can derive the ideal gas law (in a slightly different form to what you might have seen in chemistry):

$$P = nk_BT$$

$n = N/V$ is the number density (units are m^{-3}), the number of particles N per volume V.

Are there non-ideal gases? Yes, we will discuss later. But these are gases where the particles are correlated (the wavefunctions overlap). They can be:

1. Fermi gases, with $P = P(n)$ only (electrons, protons, *degenerate gases*)
2. Bose gases, with $P = P(T)$ only (photons)

But with ideal gases, we have $P = nk_BT$. We can also write the number density n in terms of the mass density ρ (km/m^3). Then we need to figure out how much the average particle weighs: $n = \rho/\mu m_H$. Here m_H is the mass of hydrogen, and μ is the mean molecular weight. If it's only hydrogen, then $\mu = 1$. If it's helium, then $\mu = 4$. What if it is *ionized* hydrogen? Then you have protons and electrons in equal numbers. The protons have mass $\approx m_H$, but the electrons have much less mass. So the average is $m_H/2$, which means $\mu = 0.5$. But writing the ideal gas law this way gives us:

$$P = \frac{\rho k_B T}{\mu m_H}$$

VII.2.3 How much support is needed?

Here we will derive *Hydrostatic equilibium*. This supports fluid against gravity via a pressure *gradient*. We have the gravitational acceleration $mg = (\rho \Delta r\, A)g = P_{\rm bottom}A - P_{\rm top}A$. Cancelling

A, we find $P_{\rm top} = P_{\rm bottom} + \Delta P = P_{\rm bottom} + (\Delta P/\Delta r)\Delta r$, or:

$$\frac{\Delta P}{\Delta r} = -\rho g$$

This is very general, and applies to stars as well as to atmospheres, oceans, etc. What is means is that P increases as you go down/inward.

What can we do with this? In general we need to know $\rho(r)$ and $g(r)$. But we can make some simplifications. We assume that $\rho \approx M/R^3$ and $g = GM(r)/r^2$. Then we get:

$$\frac{\Delta P}{\Delta r} = -\rho(r)\frac{GM(r)}{r^2}.$$

Furthermore, we will take an average over the whole star, the a difference between the inside ($P = P_c$ at $r = 0$) and the outside ($P = 0$ at $r = R$). So we get $\Delta P/\Delta r \approx (0 - P_c)/(R - 0) = -Pc/R$, with P_c the central pressure. Put this in and you get:

$$P_c \approx \frac{GM^2}{R^4}$$

For the Sun this is close: it gives $10^{14}\,\mathrm{N\,m^{-2}}$, which is a bit of an underestimate. For reference, 1 atmosphere is $10^5\,\mathrm{N\,m^{-2}}$. Relations like these are very important for astronomy where the details are hard but we can make general scaling relations between different quantities.

To go further, we can say that the pressure is related to the temperature through the ideal gas law, $P \propto k_B T$. Putting in our approximate density we get $P = (M/R^3)k_B T/\mu m_H$, or $k_B T \approx GM\mu m_H/R$. Again, the details are hard, but the general expression that $T \propto M/R$ is very useful.

Lecture VIII Stellar Energy

Again, a star's life is a long protracted (but losing) battle with gravity. What can balance it? Fundamentally it is pressure, but what keeps the pressure going?

VIII.2 Virial Theorem

Due to Classius (1870). It concerns bound gravitational systems. In essence, the long-term average of kinetic energy is $1/2$ the average of the potential energy. These can each come from different places:

kinetic can be orbital (motion of blobs or stars) or thermal (random motion of particles)

For only orbital energy, that would be something like the Earth-Moon system. Only thermal would be the insides of stars. Or there are situations with a combination like elliptical galaxies and galaxy clusters.

We denote average by $\langle \ldots \rangle$. So the Virial theorem states:

$$\langle K \rangle = -\frac{1}{2}\langle U \rangle$$

This is a bound system, so we have $\langle U \rangle < 0$. We can then look at the total energy $\langle E \rangle = \langle K \rangle + \langle U \rangle$, and if we substitute we find:

$$\langle E \rangle = \frac{1}{2}\langle U \rangle$$

VIII.3 What Powers the Sun and How Long Will It Last?

Kutner 9.1.2, 9.1.3.

We take the Solar luminosity to be 4×10^{26} W, and try to find a way to get that amount of energy out over a long time.

The first estimate was due to Lord Kelvin (1862, in Macmillan's Magazine). This estimate (known now at the Kelvin-Helmholtz time, t_{KH}) was shown to be < 100 Myr. But Darwin said (at the time) that fossils were at least 300 Myr old. So something weird was going on. Kelvin's estimate may have been wrong by a bit, but it couldn't be that bad. So there had to be some unknown energy source.

The lifespan of the Sun could be due to:

1. Chemical energy
2. Gravitational energy
3. Thermal energy (could it have just been a lot hotter in the past?)

4. Fission?

The answers for all of these are no. Kelvin's estimate concerned specifically gravitational. Chemical energy isn't enough, since we know about how much chemical energy a given reaction can release for a given amount of stuff. Same with fission.

VIII.3.1 Gravito-Thermal Collapse, or the Kelvin-Helmholtz Timescale

This ascribes the luminosity to the change in total energy: L is change in $E = K + U$.

If you do this you get a timescale of $t_{\mathrm{KH}} \sim 10^7$ yr, which is $\gg t_{\mathrm{ff}}$:

$$t_{\mathrm{KH}} \sim \frac{E}{L}$$

But $E \sim GM_\odot^2/R_\odot \sim 10^{41}\ \mathrm{J} = 10^{48}\ \mathrm{erg}$ (1 J=10^7 erg).

That is because as collapse occurs, $|U|$ increases so K increases too. That heats up the star, which slows down the collapse.

We can use the Virial theorem to get the central temperature T_c of the Sun. We assume that the center (the hottest/densest bit) dominates K:

$$K \sim \frac{3}{2} k_B T_c \frac{M}{\mu m_H}$$

And $K = -U/2$, with $U \sim -GM_\odot^2/R_\odot$. So we find $T_c \sim GM_\odot m_H/k_B R_\odot \sim 10^7$ K. This is pretty good (the real number is about 1.6×10^7 K).

Lecture IX To Make A Star

1. Support against gravity (P from HSE, E from Virial theorem)
2. Source of energy: nuclear

We can exclude all forms of energy besides nuclear fusion from powering the Sun. How does fusion work?

IX.2 Fusion

Kutner 9.3

What this boils down to is $E = mc^2$: if you can get rid of a bit of mass, you liberate a lot of energy.

atomic unit $u = 1.66054 \times 10^{-27}$ kg (mass of ^{12}C/12)

proton $m_p = 1.6726 \times 10^{-27}$ kg = $1.007u = 938.8$ MeV/c^2

neutron $m_n = 1.6749 \times 10^{-27}$ kg = $1.0087u$

electron $m_e = 9.1 \times 10^{-31}$ kg = $0.0055u$

hydrogen $m_H = 1.0078u = m_p + m_n - \mathrm{electrostatic binding energy}/2$

He nucleus $m_\alpha = 4.002u = 2m_p + 2m_n - \Delta m$, with $\Delta m = 0.03u \sim 0.7\% \times (4m_H) \approx 28\,\mathrm{MeV}/c^2$

So going from 4 protons to 1 He nucleus (α particle) releases 28 MeV. This is the energy released by fusion.

We can think of the binding energy as the energy released when you form something (a nucleus in this case), or as the energy that is required to break something up.

^{1}**H** : $E_b = 0$

^{4}**He** : $E_b = 28$ MeV = 7.08 MeV/nucleon

^{16}**O** : $E_b = 7.97$ MeV/nucleon

^{56}**Fe** : $E_b = 8.798$ MeV/nucleon

^{238}**U** : $E_b = 7.3$ MeV/nucleon

^{56}Fe has the highest binding energy, so it's the most stable. Elements that are lighter or heavier are less stable. This means that reactions would naturally squeeze lighter elements together into Fe (fusion) and break heavier elements apart (fission).

IX.2.1 Basic Nuclear Physics

1. Binding Energy: ${}^A_Z X$, with A the number of nucleons, and Z the number of protons. $E_b = (Zm_p + (A-Z)m_n - m_{\rm nuc})c^2$
2. Strong force: binds nuclei together against Coulomb (electrostatic) repulsion (since protons are positively charged)
3. $A \lesssim 56$: strong force increases faster when A increases than Coulomb forces, so a larger A leads to nuclei that are more bound.
4. $A \gtrsim 56$: the opposite

So fusion builds nuclei up to Fe, while fission breaks them down.

IX.2.1.1 H → Fe

This gives about 9 MeV/nucleon. Going from H to He gets 7 (or about 0.7% of mc^2). Going to O gets about 8 (0.8%). Going to Fe gets about 1% of mc^2 which is the most that fusion can do.

So for each proton you get $\approx 1\% mc^2 \sim 10^{-12}$ J (which means that 1 g of H could supply the annual energy of an american).

Fusion in the Sun: 10^{-12} J $\times M_\odot/m_p \sim 10^{45}$ J $\gg GM_\odot^2/R_\odot$. $t_{\rm nuc} \sim E/L_\odot \sim 10^{11}$ yr, so the Sun could shine for that long.

The actual lifespan is about 10^{10} yr (and it's lived about half of that) for a few reasons:

- $L_\odot$ increases later in life
- Not all H is burned
- It does not get hot enough to burn all the way to Fe

But it is clear that $t_{\rm nuc} \gg t_{\rm KH} \gg t_{\rm dyn}$

Lecture X How to Power the Sun

The basics are $4 \times^1 \mathrm{H} \rightarrow^4 \mathrm{He} + 28\,\mathrm{MeV}$, or releasing $\sim 7\,$MeV/A. But this has some problems.

X.1.2 Problem 1:

coulomb repulsion is strong. In order to have fusion you have to force together multiple hydrogen nuclei. These are protons, and are all positively charged. The strong force can only overcome the repulsion when the protons are *very* close together: $\sim 1\,$fm$= 10^{-15}\,$m (for comparison, an electron orbits at $10^{-11}\,$m).

The *classical* (not quantum) solution to this is that protons get close just because of their motion. They are hot, so they zip around pretty quickly. Sometimes they will approach each other, and this may happen. Can we tell how much?

The Coulomb potential is: $U_C = \frac{1}{4\pi\epsilon_0}\frac{e_1 e_2}{r}$, and energy will be conserved when the protons approach. So if they are travelling fast far away, as they approach the potential barrier they slow down:

$$\frac{1}{2}m_p v_\infty^2 + U_C(\infty) = U_C(1\,\mathrm{fm})$$

taking the limiting case that they have used all of their kinetic energy to get close enough. This gives us a requirement:

$$\frac{1}{2}m_p v_\infty^2 \geq \frac{e^2}{4\pi\epsilon_0(1\,\mathrm{fm})}$$

where we can also relate $\frac{1}{2}m_p v_\infty^2 = \frac{3}{2}k_B T$. So we need

$$T \geq T_{\mathrm{classical}} = \frac{e^2}{6k_B\pi\epsilon_0(1\,\mathrm{fm})} \sim 10^{10}\,\mathrm{K}$$

This is pretty hot, given that we know $T_c \sim 10^7\,$K. So the center of the Sun is not hot enough to sustain nuclear fusion!?

In fact, Arthur Eddington proposed nuclear energy as a power source, but others thought stars were not hot enough. Eddington said: “I am aware that many critics consider the stars are not hot enough. The critics lay themselves open to an obvious retort; we tell them to go and find a hotter place.”

In the end, Eddington was right!

X.1.3 Quantum Mechanics

The problem is that the temperature above was the *classical* result, but quantum mechanics are important here. In particular, we need to consider wave-particle duality and the Heisenberg uncertainty principle: the size of a particle depends on its momentum. Classically you could think of a particle at a certain place with velocity v (and hence momentum mv); but in a quantum sense you need to consider that the particle's position is only known to a de Broglie wavelength $\lambda_B \sim h/p \sim h/mv$, where h is Planck's constant.

When two particles are within λ_B of each other, there is a finite probability that they will "tunnel" to within 1 fm of each other ("tunneling" through the potential barrier):

$$U_c = \frac{e^2}{4\pi\epsilon_0\lambda_B} \leq \frac{1}{2}m_p v_\infty^2 = \frac{1}{2}\frac{p^2}{m_p} \sim \frac{1}{2m_p}\left(\frac{h}{\lambda_B}\right)^2$$

Note that we have replaced 1 fm with λ_B. We then get a constraint on λ_B of:

$$\lambda_B < \frac{4\pi\epsilon_0 h^2}{2e^2 m_p} \sim 10^{-13}\,\mathrm{m} = 100\,\mathrm{fm}$$

which is much bigger than the classical result. So we can be $100\times$ further away and still have fusion. This gives us a much gentler requirement for the temperature as well:

$$\frac{3}{2}k_B T = \frac{1}{2}m_p v_\infty^2 = \frac{1}{2m_p}\left(\frac{h}{\lambda_B}\right)^2$$

Which gives:

$$T \geq T_{\mathrm{quantum}} \sim \frac{m_p e^4}{12\pi^2\epsilon_0^2 k_B h^2} \sim 10^7\,\mathrm{K}$$

which is OK!

So fusion is possible because of *quantum tunneling* at 10^7 K for the Sun. But, if $M < 0.08\,M_\odot$, then it cannot even get this hot and fusion is impossible. Such objects are failed stars known as *brown dwarfs*.

Quantum tunneling being possible does not mean that it always happens. The probability is $\sim e^{-2\pi^2 U_c/k_B T}$. So for 10^7 K, the probability is only 10^{-8} for any two protons, and it goes up to 1 for 10^{10} K. But there are enough protons and the fusion rate increases quickly with temperature that you have *ignition* at 10^7 K.

X.1.4 How Often Do Protons Get Close Enough?

At the center of the Sun the density is $\rho \sim 100\,\mathrm{g\,cm^{-3}}$, so the mean separation $l \sim n^{-1/3}$ is $\sim 10^{-11}$ m which is $\gg \lambda_B$. To have fusion with $4 \times^1\mathrm{H} \rightarrow^4 \mathrm{He}$ we need 4 protons to get very close together, which is hard.

This reaction is not one that needs 4 protons at once, but it is actually a sequence of 2-body reactions. At "low" temperatures it is the *proton-proton* chain (p-p chain), which at "higher temperatures" there is the *carbon-nitrogen-oxygen* (CNO) chain. This does not burn CNO, but uses them as catalysts.

For later fusion reactions to build heavier elements, the higher charges on the nuclei (more protons) need higher temperatures to get close enough. For instance, to fuse He need 10^8 K. Others need close to 10^9 K. These will happen after the central H has been consumed.

X.1.4.1 p-p chain

The main sequence in the Sun. The net reaction is:

$$4^1\mathrm{H} \rightarrow^4 \mathrm{He} + 2e^+ + 2\nu_e + 2\gamma$$

Which is actually:

$$\begin{aligned}
^1\mathrm{H} +^1 \mathrm{H} &\rightarrow {}^2\mathrm{H} + e^+ + \nu_e \\
^1\mathrm{H} +^1 \mathrm{H} &\rightarrow {}^2\mathrm{H} + e^+ + \nu_e \\
^2\mathrm{H} +^1 \mathrm{H} &\rightarrow {}^3\mathrm{He} + \gamma \\
^2\mathrm{H} +^1 \mathrm{H} &\rightarrow {}^3\mathrm{He} + \gamma \\
^3\mathrm{He} +^3 \mathrm{He} &\rightarrow {}^4\mathrm{He} + 2^1\mathrm{H}
\end{aligned}$$

X.1.4.2 CNO chain

For more massive stars:

$$\begin{aligned}
^{12}\mathrm{C} +^1 \mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^+ + \nu_e \\
^{13}\mathrm{C} +^1 \mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
^{14}\mathrm{N} +^1 \mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^+ + \nu_e \\
^{15}\mathrm{N} +^1 \mathrm{H} &\rightarrow {}^{12}\mathrm{C} +^4 \mathrm{He}
\end{aligned}$$

Notice that the CNO all stay the same throughout: anything that is produced is consumed and vice versa.

Lecture XI The Nucleus

We want to explain the binding energy in $m = Zm_p + (A-Z)m_n - E_b/c^2$, where the nucleus has Z protons and $A-Z$ neutrons, for a total number A nucleons.

XI.2 The Liquid Drop Model

$$E_B \approx a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$$

Let's look at each term:

$a_V A$: this is a volume term, since for constant density nucleons the volume will be $\propto A$. This covers the binding due to eht strong force, which is $\propto A$

$a_S A^{2/3}$: this is a surface term. For a volume $\propto A$, the surface area will be $\propto A^{2/3}$. It works as a correction to the volume term since the nucleons near the outside will have fewer other nucleons to interact with.

$a_C \frac{Z(Z-1)}{A^{1/3}}$: this is the Coulomb term, showing the strength of electrostatic repulsion which is $\propto 1/r \sim 1/A^{1/3}$

$a_A \frac{(A-2Z)^2}{A}$: this is an assymetry term, where nuclei with $A \approx 2Z$ are more bound

$\delta(A,Z)$: this is a pairing term

Overall this makes stable nuclei with $Z \approx A/2$, and says that the most bound nuclei are near $A = 60$. It is obviously a simplification, but it can be improved with the addition of a *shell model* (like for electrons) and using empirical data to set the various constants.

Lecture XII Radiation

Astronomy is based (so far) on observing light from objects. This means we see photons (electromagnetic radiation). Which give us information about temperature, density, chemical composition, etc.

We will discuss:

1. Diffusion and random walks
2. Blackbodies and temperature
3. Photospheres and energy transport

XII.2 Photons

Kutner 2.2, 2.4

Photons are light particles, but they also behave as waves. Each photon has energy $E = h\nu$ (with ν the frequency in Hz) or $E = hc\lambda$ (with λ the wavelength). This comes from the definition $c = \lambda\nu$.

We divide up the electromagnetic spectrum into wavelength regions:

γ-rays : $\lambda < 0.01\,\mathrm{nm}$

X-rays : $1\,\mathrm{nm} \rightarrow 10\,\mathrm{nm}$

ultraviolet (UV) : $10\,\mathrm{nm} \rightarrow 400\,\mathrm{nm}$

optical (visual) : $400\,\mathrm{nm} \rightarrow 700\,\mathrm{nm}$

infrared (IR) : $700\,\mathrm{nm} \rightarrow 1\,\mathrm{mm}$

radio : $> 1\,\mathrm{mm}$

XII.3 Diffusion & Random Walks

Heat (energy) is produced at the centers of stars through nuclear reactions. It escapes ultimately as photons. How long does that take? A naive answer is $\sim R_{\odot}/c = 2\,\mathrm{s}$.

But that is wrong (although it is true for neutrinos). It actually takes $\sim 10^7$ yrs. Why? Because a star is a very crowded place, and photons (even though they move fast) cannot move very far before they wack into something else and end up going in another direction. They easily bounce (scatter) off of ions, electrons, and atoms, and even other photons.

Each bounce tends to make the photon lose energy, but more photons are then produced, conserving energy. In the center the photons start out as X-ray photons, but by the time they get to the surface of the star they are optical photons. They get there via a *random walk*.

Assume that a photon will move (on average) a distance $l_{\rm mfp}$ before it hits something and changes direction. That distance is the *mean free path*. It travels a distance d after N collisions. We can determine what $d(N)$ is. Assume each one moves $\vec{l_i}$ for $i = 1 \ldots N$, with $|\vec{l_i}| = l_{\rm mfp}$. So the total distance is the vector sum:

$$\vec{d} = \sum^N \vec{l_i}$$

We want the magnitude of this, $|\vec{d}| = \sqrt{\vec{d} \cdot \vec{d}}$. But

$$\vec{d} \cdot \vec{d} = \sum_i^N \vec{l_i} \cdot \vec{l_i} + \sum_{i \neq j} \vec{l_i} \vec{l_j}$$

The second term there will go to 0 on average, since the directions are different. So $|\vec{d}|^2 = N|\vec{l}| = N l_{\rm mfp}$, or $d = \sqrt{N} l_{\rm mfp}$. This is in fact a general result with applicability to a wide range of areas.

From this we can determine how long does it take for a photon to diffuse out of the star. To go a distance d, it takes:

$$\begin{array}{rcll} & & \frac{d}{c} & l_{\rm mfp} > d \\ N\frac{l_{\rm mfp}}{c} & = & \frac{d^2}{l_{\rm mfp} c} & l_{\rm mfp} < d \end{array}$$

This is also often referred to as a "drunkard's walk".

XII.4 What Happens When A Photon Hits Something?

- Photon A generates an oscillating electromagnetic field
- Matter (ion, electron, atom) is shaken by that field (absorbing the photon)
- But this shaking is itself a fluctuating field, so it makes a new EM field, releasing photon B

There are 3 basic types of interactions:

1. Scattering: A and B have the same energy (or frequency) but different directions. So the matter gains momentum but no energy
2. Absorption: no photon B is emitted. Matter absorbs energy and generally something happens to it
3. Emission: matter emits B (and it can be with or without A)

We take each obstacle as having cross-sectional area σ (in units of m^2). And we have n of them per m^3 (number density n). So what is $l_{\rm mfp}$?

1. A dimensional analysis: $l_{\rm mfp}$ [m] could be $1/\sigma n$, $\sigma^2 n$, $n^{-1/3}$, ...

2. Physical: shoot a bullet into a box of total area A and depth $l_{\rm mfp}$. It has N balloons in it each with area σ. The bullet is likely to hit a balloon if $N\sigma = A$. $N = n \times {\rm volume} = nl_{\rm mfp}A$. Equating $nl_{\rm mfp}A\sigma = A$ gives $l_{\rm mfp} = 1/n\sigma$

3. How big are the balloons (what is σ)?

   electrons $10^{-28}\,{\rm m}^2$

   H atoms $10^{-20}\,{\rm m}^2$

   So for $n \sim 10^{30}\,{\rm m}^{-3}$ (which you get for $\rho = 10^3\,{\rm kg\,m}^{-3}$ like for water), we find $l_{\rm mfp} \sim 10^{-11}$ m to 10^{-2} m, both of which are $\ll R_\odot$. Therefore there are many bounces:

   electrons : $l_{\rm mfp} \sim 10^{-2}$ m, so $t \sim 5000$ yr (10^{22} bounces)

   H atoms : $l_{\rm mfp} \sim 10^{-11}$ m, so $t \sim 5 \times 10^{12}$ yr (10^{40} bounces)

   The actual answer is about 10^7 yrs, taking into account the changing structure and density of the Sun.

XII.5 Temperature of Radiation

Kutner 2.3

Temperature is defined for ideal objects (“blackbodies”) that radiate a *universal* spectrum: the emission depends only on temperature.

To do so, it must absorb all of the light that hits it (hence *black*). But it can appear to have a color when it is hot (like an oven).

Blackbody is a specific shape **sketch**. The peak is at $\lambda = 0.0029/T$ m, which determine the color. The total energy put out per square meter (the flux) is $F = \sigma T^4\,{\rm W\,m}^{-2}$. These are the *Wien displacement law* and *Stefan-Boltzmann* law (σ is sb constant). The important thing is that only T matters.

Wikipedia page, animations.

F is energy per time per area. If the object is a sphere (like a star) it has total area $A = 4\pi R^4$, so total energy per time (luminosity L) is $L = 4\pi R^2\sigma T^4$. This is a very useful expression.

What are “good” blackbodies? Nothing is perfect, but some things are close:

- 3 K cosmic microwave background
- surface or interior of a star
- human skin
- candle
- lava

What are "bad" blackbodies? These don't have a nice smooth distribution, but instead concentrate the light at specific wavelengths:

- neon light
- fluorescents

But even the Sun isn't perfect.

XII.5.1 Planck Function

The detailed function that describes how much light at each wavelength:

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{h\nu/k_BT} - 1}\ \mathrm{J/s/m^2}/\lambda$$

Note that h appears, so this has to be a quantum effect. B_λ is energy per time per area *per wavelength*: if you have a bigger range of wavelengths (i.e., red and green) then you get more energy.

XII.5.1.1 Limits

sketch

$$\frac{hc}{\lambda}\frac{1}{k_BT} \gg 1$$

(long wavelength, low frequency) is the Rayleigh-Jeans limit:

$$B_\lambda(T) \approx \frac{2ck_BT}{\lambda}$$

Note that there is no more h: this limit can be derived classically.

$$\frac{hc}{\lambda}\frac{1}{k_BT} \ll 1$$

(short wavelength, high frequency) is the Wien limit:

$$B_\lambda(T) \propto e^{-hc/\lambda k_BT}/\lambda^5$$

If you add up $B_\lambda(T)$ for all λ, you recover the Stefan-Boltzmann law.

XII.6 Photosphere

What is the surface of a star? What is the temperature of a star?

After all, it's a flaming ball of gas. It's hotter on the inside (Sun is 10^7 K), so why do we see it as cooler on the outside? What defines the temperature that we measure (about 5800 K)?

Photosphere is defined as the "surface", it is where $T(\mathrm{depth}) = T_{\mathrm{effective}}$. We define $T_{\mathrm{effective}}$ such that $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$. This is the layer from which photons escape (stop bouncing or scattering). They end up one mean free path from the surface, and from there they are free!

This is where the star leaves an imprint on the photons that escape. Everything that happens deeper down gets washed away from multiple scatterings.

$$l_{\mathrm{mfp}} \sim \frac{1}{n\sigma} \sim H$$

H is pressure scale heigh - the height at which pressure changes by e: $P \propto e^{-r/H}$. Since we have HSE ($\Delta P/\Delta r = -\rho g$), we can say $\Delta r = H$ and find $P_{\mathrm{phot.}} \sim \rho g H \sim \frac{g\rho}{n\sigma} \sim \frac{g\mu m_H}{\sigma}$. This is about $10^7\,\mathrm{N\,m^{-2}}$ for the Sun.

Lecture XIII Photons & Spectra

Kutner Chapter 3

Spectra: disperse light through "prism", spread it out so we can see each wavelength separately.

Stars generally have *absorption line*: most of the wavelengths are bright, but a few specific wavelengths are dark. To understand this, need Kirchoff's laws:

- Hot background, cold foreground: absorption lines
- Cold background, hot foreground: emission lines

What matters is what is in front. What is in front of the star? It's is that it is hotter on the inside than the outside. So the spectrum of a star is (mostly) a blackbody with some wavelengths absorbed. These wavelengths were identified before we knew what caused them.

Fraunhofer lines: lines in Sun from things like Na, Ca. But there are also lines from H, He that are very important.

Cecilia Payne was one of the first people to identify the spectral lines in the Sun (and other stars). She showed that the elements in the Sun were very different from those on Earth: here we have almost no free H, but that is the majority of what's in the Sun.

XIII.2 Energy Levels for H

proton + electron in Bohr model (approaching proper quantum mechanics, but not quite): "planetary" orbits. Instead of Gravity, Coulomb force:

$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

and we use the Virial theorem again, so $E = K + U = -U/2$. This would have infinite choices for r: anything is OK. The Bohr model says that r can only have particular values that are *quantized*. What is necessary is that, if you take a de Broglie wavelength, the orbit starts and stops in the same part of a wave. You can also write this as:

$$J = m_e v r = n\hbar$$

is the angular momentum. $\hbar = h/2\pi$, so this is quantum mechanical. If you do this, you get discrete energy levels for $n = 1, 2, 3, \ldots$:

$$\frac{-1}{4\pi\epsilon_0}\frac{e^2}{2r} = -\frac{1}{2}m_e v^2 = -\frac{1}{2}\frac{(n\hbar)^2}{m_e r}$$

This can only be true at certain values of r:

$$r = r_n = 4\pi\epsilon_0\frac{\hbar^2 n^2}{m_e e^2} \approx 0.5\,\text{Å}\,n^2$$

The energy levels associated with this are:

$$E_n = \frac{1}{n^2}\frac{-m_e e^4}{2(4\pi\epsilon_0\hbar)^2} = \frac{-13.6\,\mathrm{eV}}{n^2}$$

The constant $13.6\,$eV is the ionization energy of H, known as a Rydberg. Why does this ionize? Start at $n = 1$. How much energy to get to infinitely far away? This would take us to $r = \infty$, so $n = \infty$. The difference in energy levels is how much energy it would take:

$$\Delta E = E_1 - E_\infty$$

But since $E_\infty = 1/\infty = 0$, this is just E_1 or 13.6 eV.
http://astro.unl.edu/classaction/animations/light/hydrogenatom.html

XIII.3 Photon & Matter: Spectral Lines

Spectral lines are associated with transitions between energy levels. See in both absorption and emission.

For example, to excite an atom from $n = 1$ to $n = 2$ takes $\Delta E = hc/\lambda = E_2 - E_1 = (-3.4\,\mathrm{eV} - (-13.6\,\mathrm{eV}) = 10.2\,$eV. THis gives a wavelength of $\lambda = 1216\,$Å. Lymanα. **Sketch Ly, Balmer, Pa, Brackett**. Ly$\alpha = 1216\,$Å, H$\alpha = 6563\,$Å, P$\alpha = 18,700\,$Å, Br$\alpha = 40,500\,$Å.

Emission lines: hot gas on cool background (neon light).

Absorption lines: cool gas in front of hot background.

Some astronomical objects are primarily spectral line emitters. e.g., planetary nebulae and HII regions: clouds of hot gas, where most of the emission is just what we've described.

Lecture XIV Main Sequence

Kutner 3.5, 9

sketch L **vs.** T

temperature from color of star. luminosity from how bright it appears, combined with some knowledge of distance. From this we get *color-magnitude diagram*. Also *Hertzsprung-Russel* diagram. **main sequence** is where they sit for most of the time. Stars do not move along it: they end up at one point determined by their mass.

Stars were originally classified based on spectral lines. We have now been able to re-order that sequence in terms of temperature:

O : O5 is 40,000 K. H ionized, see lines of H, He

B : similar but cooler

A : A0 is 10,000 K, Vega (standard comparison star)

F : start seeing lines of H, “metals”

G : G2 is 5,800 K, Sun

K : start seeing molecules (star is cool, so they can be stable)

M : M0 is 4,000 K

Within each class goes from 0 (hottest) to 9 (coolest).

Taking stars together, we observe:

$$L \propto M^4$$

$$R \propto M^{0.76}$$

Simply put, more massive stars are bigger and (a lot) brighter. They also end up being hotter (via Stefan-Boltzmann law), $T_{\text{eff}} \propto M^{0.6}$.

XIV.2 More massive is bigger and brighter

only mass matters along the main sequence: stars sit there doing almost nothing for most of their lives.

XIV.2.1 Why bigger?

T_c roughly constant (ignition). Virial theorem gives us $T_c \propto M/R$, so then we would have $R \propto M$. Which is close, but actually T_c goes up a bit, so instead we have $R \propto M^{0.76}$.

XIV.2.2 Why brighter?

We can understand $T_c \propto M/R$, and $P_c \propto GM\rho/R$. The other piece we need is to look at how the photons get out of the star. Remember that they have to bounce around a lot. We can write:

$$\frac{T_c}{R} \sim \frac{L\rho}{T_c^3 R^2}$$

which is valid if the amount of time that it takes a photon to get out does not depend on T (true in hot stars). This then gives us: $L \propto M^3$, which is close to what is observed.

XIV.2.3 More massive has shorter life?

lifetime $\propto$ fuel/rate of consuption $\sim M/L$. Since $L \propto M^3$ or M^4, lifetime $\propto M^{-2}$ or M^{-3}.

At the low-mass end, energy is not transported by photons but by bubbles, so this reasoning (and these scalings) break.

XIV.3 A Fundamental Unit for M_*

A star: gravity pushes nucleons together until fusion happens.

Gravitational energy $\sim Gm_H^2/r$ between two atoms. How far apart? 10^{-15} m after fusing, take $r \sim \hbar/m_H c = 2 \times 10^{-16}$ m for scaling.

$$\alpha_G = \frac{\frac{Gm_H^2}{\hbar/m_H c}}{m_H c^2} = \frac{Gm_H^2}{\hbar c} = 5.9 \times 10^{-39}$$

This is the strength of gravity between two nucleons compared to the rest-mass energy of the nucleon. Gravity is very weak! From this, we can derive:

$$M_* = \alpha_G^{-3/2} m_H = 1.85\, M_\odot$$

This is a natural unit for the masses of stars, and it only involves fundamental constants ($G, c, m_H, \hbar$). We can also get:

$$N_* = M_*/m_H = \alpha_G^{-3/2} = 2 \times 10^{57}$$

is the rough number of nucleons (mostly protons) in a star.

XIV.4 What Limits What Could Be A Star?

XIV.4.1 Minimum Mass

No fusion possible (not hot enough): $M < 0.08 M_\odot$ (brown dwarf)

XIV.4.2 Maximum Mass

Star gets very hot inside, and T_c increases as M increases. The pressure due to the blackbody radiation inside increases a lot: $P_{\rm rad} \propto T_c^4$. When this pressure becomes too big, it will dominate over the normal gas pressure ($P \propto \rho k_B T$), and when it does the star becomes unstable: it will blow itself apart. For $M \sim 100\,M_\odot$, unstable, very hard to even form. Even for $\sim 50\,M_\odot$, very violent, short-lived.

Lecture XV Life After the Main Sequence

Remember: stars do not move along the MS. They move onto it when they form, sit there for a long time, and then...

Main sequence is about where we find 80% of the stars. H in the core is burned into He. This is accompanied by a slow increase in L:

- $P = \rho k_B T / \mu m_H$, where μ is mean weight. 0.5 for H, higher for He.
- As H goes to He, μ goes up, so P would go down if T didn't go up.
- As T goes up, fusion region increases, L increases

Then what? End of MS is when H is done in core. After that: giants (bigger and brighter). Depends on mass.

XV.2 Low Mass

$< 8\, M_\odot$

no more energy from core, so contracts, gets hotter (Virial theorem). Layers above core contract too, H burns in "shell" around He core. This makes "red giant": $R \sim 100 R_\odot$, lasts 10^8 yr. As this happens the star is a lot brighter too **draw HR**.

Eventually core contracts, $T_c \sim 10^8$ K, starts burning He via triple-alpha process:

$$^4_2\mathrm{He} + ^4_2\mathrm{He} \rightarrow ^8_4\mathrm{Be}$$

$$^8_4\mathrm{Be} + ^4_2\mathrm{He} \rightarrow ^{12}_6\mathrm{C} + \gamma$$

Unless the second reaction happens, Be will decay (break apart) so needs high density. Higher temperature because He has charge of +2, so Coulomb repulsion is 4 times as much as H.

When enough C, starts burning to O (T higher still).

When He burning starts, star becomes bigger still: Asymptotic Giant Branch **Draw onion diagram, C/O core, He shell, H burning, H envelope**. $\sim 600 R_\odot$ for 10^6 yr, $10^5 L_\odot$.

As this happens the star puffs off a lot of gas: larger R means lower GM/R on the surface, so stuff can escape. Goes from $10^{-14} M_\odot$/yr (Sun) to $10^{-7} M_\odot$/yr (RG) to $10^{-5} M_\odot$/yr (AGB). THis material sticks near the star for a time, forming a *planetary nebula* (nothing to do with planets), but enriches the surrounding area with "metals" formed in star. What's left at the center is C/O *white dwarf* (later).

Moves around on HR diagram, $L = 4\pi R^2 \sigma T^4$.

XV.3 High Mass

Burning keeps going to where Fe is produced. After that, cannot get energy out. Star collapses (*core collapse supernova*). **show onion**. Gravitational potential energy released (10^{46} J), mostly as neutrinos. Some compact remnant (possibly) left behind.

Lecture XVI Clusters: Testing Stellar Evolution

Most stars are not formed alone, but rather in groups (small) or clusters (large).

- Open cluster: young(ish), 10^3 stars, in a size of ∼pc, $\sim 10^3$ known, in the Galactic disk
- Globular cluster: old, 10^5 stars, still ∼pc, ~ 200 around the MW in a spherical cloud (halo). Often very *metal poor*: these stars were formed when the material had not been enriched by previous stellar burning (originally matter was almost all H, He)

In both, the stars are all born together from the same stuff at the same distance. So this removes a lot of the uncertainties that make astronomy hard. They are great testing grounds.

XVI.2 Make HR diagram out of a single cluster

sketch. Can determine distance by finding how bright the stars at $T_\odot$ appear to be.

http://astro.unl.edu/naap/distance/animations/clusterFittingExplorer.html

Can determine age by looking at the highest-mass star that is still on the MS **Sketch**

Look for objects that are not on the MS: *binaries, dwarfs, etc.*

calibrate models of stellar evolution.

Lecture XVII White Dwarfs

Kutner 10.4

leftover remnant from the core of a low-mass star ($< 8M_\odot$).

core gets hotter & denser, as heavier elements need higher T to burn. As each phase of burning ends, collapse a bit, T up, P up. Can this keep going? *NO*

After get to O, P will no longer depend on T, so you cannot keep getting more burning. Why?

XVII.2 Degeneracy

electrons are *Fermions*: 2 cannot be in the same *state*. State = position, momentum, spin (Pauli exclusion principle). From the uncertainty principle:

$$(\Delta x)(\Delta p) \sim \hbar$$

p is momentum, $m_e v$. Density is number per volume, or 1/volume per particle. So $n \sim 1/\Delta x^3$. When the particles get squeezed too close they start to overlap, get to where

$$p = p_F \sim \hbar/\Delta x$$

(Fermi momentum). Which will be $p_F \sim n_e^{1/3}$. We can then use the momentum to get the kinetic energy:

$$E_F = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e(p_F/m_e)^2 = \frac{1}{2}\frac{p_F^2}{m_e}$$

(note that this is only true if $v \ll c$). So $E_F \propto p_F^2 \propto n_e^{2/3}$. This is the total energy per particle. Pressure has the same units as energy per volume (energy density), so we can multiply energy per particle by density to get pressure:

$$P_F \propto n_e E_F \propto n_e^{5/3} \propto \rho^{5/3}$$

There is no T! Unlike ideal gas law ($P = n_e k_B T$), this is *not ideal*. Requires quantum mechanics. Ideal gas law $P = P(\rho, T)$, but here $P = P(\rho)$ only.

XVII.3 Build a Degenerate Star

(This is a white dwarf).

HSE gives:

$$\frac{P}{R} \sim \frac{GM}{R^2}\rho$$

So

$$P \sim \frac{GM}{R}\rho \sim \rho^{5/3}$$

We can then get $M/R \sim \rho^{2/3}$, but $\rho \sim M/R^3$, so we have:

$$M^{1/3} \sim 1/R \; R \sim M^{-1/3}$$

This is weird. Unlike a star ($M \sim R$) or a normal rock ($R \sim M^{+1/3}$, since $\rho \sim$constant) or something, as M increases, it gets smaller!

XVII.3.1 When Does This Matter?

When electron spacing $\Delta x \sim \lambda_B$ (de Broglie wavelength from before). $\lambda \sim h/p \sim h/m_e v$, and $m_e v^2 \sim k_B T$, so

$$\lambda_B \sim 10^{-12}\,\mathrm{m}\left(\frac{10^9\,\mathrm{K}}{T}\right)^{1/2}$$

For $1M_\odot$ and $R = R_\oplus$, get $\Delta x \sim 10^{-12}$ m. Since $T < 10^9$ K (C burning), λ is small enough to be degenerate, and $R_{\mathrm{WD}} \sim R_\oplus$.

So a White Dwarf has the mass of the Sun squeezed into something the size of the Earth.

XVII.3.2 How Bright?

$L = 4\pi R_\oplus^2 \sigma T^4$, so even is T is higher than the Sun, $R_\oplus \ll R_\odot$ and the WD will be very faint.

XVII.3.3 Can A WD Be Any Size?

Can you keep piling mass on, or is there a limit?

$$\rho^{2/3} \sim \frac{M}{R} \sim \frac{M}{M^{-1/3}} \sim M^{4/3}$$

or $\rho \sim M^2$ in the center. So as you make it more massive, the density increases a lot. And as the density increases, so does the pressure and E_F. What happens when $E_F \sim m_e c^2$ (i.e., $v_e \sim c$)? Things get *unstable*. Before,

$$E_F \sim m_e v^2 \sim \frac{p_F^2}{m_e}$$

But now, including *special relativity*, $v \sim c$

$$E_F \sim p_F c \propto n_e^{1/3}$$

$$P \propto n_e E_F \sim n_e^{4/3}$$

Put that into $P \sim GM\rho/R \sim \rho^{4/3}$ and you find

$$\rho^{1/3} \sim \frac{M}{R}$$

But $\rho \sim M/R^3$, so $\rho^{1/3} \sim M^{1/3}/R$:

$$\frac{M^{1/3}}{R} \sim \frac{M}{R}$$

This cannot be true! When $v \sim c$ the velocity cannot increase fast enough to supply enough pressure, and the WD becomes unstable. This happens at the *Chandrasekhar Mass* $1.4M_\odot$: if it gets to this limit, it will collapse.

Lecture XVIII Neutron Stars

XVIII.2 Higher Masses: What Happens?

Kutner 11

Once we get to Fe in the core, we cannot get energy out. The star is still shining (so it's losing energy) but no longer creating it. So the core starts to cool. In order to support the rest of the star, the pressure needs to be the same, so the density goes up and up to keep the pressure constant.

If the mass of the Fe core is less than the Chandrasekhar mass, then electron degeneracy pressure can support the star. But once it gets past there that is not enough. At this point, it is roughly 5×10^9 K, 5000 km in radius.

As it gets to the Chandrasekhar mass, electrons cannot support the star. It starts to collapse. The collapse liberates some gravitational energy, but the Fe *absorbs* that energy, liberating protons. Protons take up extra space, and they are increasingly squeezed by the rest of the star.

Remember β decay (nuclear decay):

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

This happens spontaneously for various nuclei. The inverse can also happen, although it isn't spontaneous:

$$p^+ + e^- \rightarrow n + \nu_e$$

When things get too dense, the inverse reaction is energetically favorable. This makes a bunch of neutrons, removing electron support. Neutrinos also leak out, removing energy.

So the core collapses down in a free-fall timescale of $\sim 1/\sqrt{G\rho} \sim 10\,\mathrm{s}$ or less. This reaction happens at a density of $\sim 10^9\,\mathrm{kg\,m^{-3}}$. Much of the star is blown off in a *supernova explosion*: the gravitational energy of 10^{46} J is released mostly as neutrinos, with a small amount creating a blast wave of material moving at 10,000 km/s.

XVIII.3 Core Collapse

It gets squeezed down to $\sim 10\,\mathrm{km}$. This is a core-collapse SN (there are other kinds), and happens for $M < 25M_\odot$ or so. A lot of the gravitational energy is releasd:

- 10^{46} J total
- 10^{44} J is in the KE of the ejected material
- 10^{43} J is in photons ($10^{10}L_\odot \sim L_{\mathrm{galaxy}}$ for 10 days)

For example, SN1054 which is 2 kpc away was seen during the day quite easily.

99% of the energy (or more) goes out as neutrinos ($10^{19} L_\odot$). About 20 were detected from a SN 50 kpc away.

A lot of the blasted away material goes into the nearby interstellar space, enriching it with metals that were made in the star. THis is how elements heavier than C/O get into the universe.

XVIII.4 What Remains

Kutner 11.2

A neutron star is like a WD, except instead of electron degeneracy, the reaction $p + e \rightarrow n$ makes neutrons, and neutron degeneracy pressure supports it. For $1.4 M_\odot$ the size is about 10 km. This gives $\rho \sim 10^{17-18}\,\mathrm{kg\,m^{-3}}$, compare to a nucleus is 2×10^{17}.

$$\rho_{\rm WD} \sim \frac{m_H}{(h/m_e c)^3} \sim 10^8\,\mathrm{kg\,m^{-3}}$$

$$\rho_{\rm NS} \sim \frac{m_H}{(h/m_p c)^3} \sim 6 \times 10^{17}\,\mathrm{kg\,m^{-3}}$$

We are confining one nucleon (proton or neutron) in a box. For the WD the size of the box is the de Broglie wavelength of the electron. For the NS it's the de Broglie wavelength of the neutron. Since the wavelength is $\sim 1/m$, the neutron's box is much smaller.

Consider a nucleus. The distance between neutrons is $r_0 \sim 10^{-15}$ m. If you take a Solar mass in neutrons, that means $A \sim M_\odot/m_{\rm H} \sim 10^{57}$ neutrons, so the size is $R_{\rm NS} \sim r_0 A^{1/3} \sim 10$ km.

The properties of this object: surface gravity $g \sim 10^{12}\,\mathrm{m\,s^{-2}}$, escape speed $\approx 0.6c$.

The interior is very complicated: still under investigation. Likely superconducting (no electrical resistance) superfluid (no friction).

We have an upper limit to neutron star mass: keep $v_n < c$ gives a limit of $2 - 3 M_\odot$ (details are hard).

XVIII.4.1 Spin

Angular momentum $J = I\omega$, $\omega = 2\pi/P$. Conserved. What made the NS?

$$\frac{R_{\rm core}}{R_{\rm NS}} \sim \frac{m_e}{m_n}\left(\frac{Z}{A}\right)^{5/3} \sim 500$$

going from the core (supported by electron degeneracy) to the NS. Conserving J:

$$I_{\rm core}\omega_{\rm core} = I_{\rm NS}\omega_{\rm NS}$$

and $I \approx MR^2$. So

$$\omega_{\rm NS} \sim \omega_{\rm core}\left(\frac{R_{\rm core}}{R_{\rm NS}}\right)^2$$

since the mass is the same. Or, going to period:

$$P_{\rm NS} \sim 4 \times 10^{-6} P_{\rm core}$$

Much faster! Core can rotate as fast as 30 min, so $P_{\rm NS}$ can be as little as 5 ms.

XVIII.4.2 Magnetic Field

Instead of angular momentum, conserve magnetic flux $\Phi = BR^2$. Same arguments give $B_{\rm NS} \sim 250,000 B_{\rm core}$. What might the initial field be? We measure fields of 10 T in white dwarfs, so the field in a NS could be up to 10^7 T in NS. In fact we see NSs with fields up to $1000\times$ this.

XVIII.4.3 A Limit To Rotation?

How fast can a NS rotate? 2 limits:

- Keep the equator $< c$
- centripetal force $<$ force of gravity

First one:

$$\frac{2\pi R}{P} = v_{\rm equator} < c$$

would give a limit of $2\pi R/c = 0.2$ ms.

The second:

$$\frac{v^2}{R} = \frac{GM}{R^2} = \frac{4\pi^2 R}{P^2}$$

would give a limit $P < \sqrt{4\pi^2 R^3/GM} = 0.4$ ms. (this is actually Kepler's third law, $P^2 \propto R^3$).

XVIII.5 Pulsars

1967 Jocelyn Bell, looking for "twinkling" radio sources. Found something that "blipped" every 1.337 s. It was from the sky: it came every day at the same *sidereal time* (not LGM). It was too fast and too regular to be anything big (white dwarf, binary star, etc.)

More of these were soon found. Some were in supernovae remnants: the clouds of gas flying out at 10,000 km/s from where supernova explosions occured.

Crab Nebula: glowing cloud of gas. Needs $10^5\,L_\odot$ to power it. Found pulsating radio source inside with $P = 33$ ms. But they found that while regular, P was getting longer very slowly, change at $\dot{P} = dP/dt = 4 \times 10^{-13}\,{\rm s\,s^{-1}}$.

What would rotational energy be of neutron star spinning at $\Omega = 2\pi/P$? $(1/2)I\Omega^2$, $I \approx 10^{38}\,{\rm kg\,m^2} = (2/5)MR^2$ is moment of interia. What if the rate of spinning is slowing? Then it is losing kinetic energy at a rate $\sim I\Omega\dot{\Omega}$. If we do this, we find this is 5×10^{31} W is about

$10^5 L_\odot$: just right! This showed that the spin-down of a neutron star is what powers the Crab nebula (Tommy Gold).

Overall, pulsars were found to be rotating neutron stars. We see blips when the "lighthouse" beam crosses the Earth **show animation**. The majority of the energy from the spin-down is invisible: the radio blips are a tiny fraction of the energy.

It is the strong magnetic field that makes this happen.

XVIII.5.1 Spin-Down

(please pardon the calculus)

Light cylinder: where v to go around is c. We take the magnetic field to be a dipole, $B(r) = B_0(r/R)^{-3}$. A changing magnet releases electromagnetic power per unit area S (*Poynting flux*) $\sim cB^2/\mu_0$. We can roughly relate the spin-down energy loss $I\Omega\dot{\Omega}$ to the Poynting flux through the light cylinder:

$$4\pi R_{\rm LC}^2 S_{\rm LC} \approx I\Omega\dot{\Omega}$$

with $\Omega = 2\pi/P$, $\dot{\Omega} = -2\pi\dot{P}/P^2$. $R_{\rm LC} = cP/2\pi$, so $S_{\rm LC} = (c/\mu_0)B_{\rm LC}^2 = (c/\mu_0)B_0^2 R^6/R_{\rm LC}^2$. So we have:

$$4\pi R_{\rm LC}^2 \frac{c}{\mu_0} B_0^2 \frac{R^6}{R_{\rm LC}^6} = 4\pi R^6 \frac{c}{\mu_0} B_0^2 \left(\frac{cP}{2\pi}\right)^{-4} \sim \frac{R^6}{\mu_0}\frac{B_0^2}{c^3}P^{-4} \sim I\frac{\dot{P}}{P^2}$$

This gives:

$$B_0^2 \sim \frac{c^3\mu_0 I}{R^6} P\dot{P}$$

So from the spin period and the rate at which it is slowing down, we can determine what the magnetic field is!

We can then use this (assuming B =constant) to get $P(t)$. We find that the age is $\tau \approx P/2/\dot{P}$, so we also get the age of the system from P and $\dot{P}$. Do this for the Crab pulsar get 1250 years, which is very close to the true age of about 950 years (since people saw the supernova).

P-$\dot{P}$ diagram: HR diagram for pulsars. **draw**. Move through the diagram from upper left to lower right until you die from low voltage (don't actually die, just shut off). This takes 10^{7-8} yrs to get to $P = 10\,$s from a typical starting point of 10 ms. Usually born with 10^8 T, but there is a range.

XVIII.5.2 What happens after death?

Not much, unless in a binary star system. If in a binary: after the pulsar dies (remember, this still happens quickly compared to a main-sequence lifetime), the second star will evolve. It will leave the MS and puff up into a RG. When this happens, the outer bits of that star may get captured by the gravity of the NS **draw**. This is called *Roche-lobe overflow* (remember Roche from the tidal forces?). It leads to *accretion* onto the NS. That dumps angular momentum and mass onto the NS.

In the early 80's, they found pulsar with $P = 1.6\,$ms, $P/2\dot{P} = 200\,$Myr. So it couldn't have been born with that low a period since it is way too old. How could it get there? It was *recycled* into a *millisecond pulsar* **draw**. For these, B is much weaker ($\sim 10^5$ T) although we do not really know why.

XVIII.5.3 Hulse-Taylor Binary

The second star in a binary can also eventually become a NS. The H-T binary is one such system: two NSs in a 8-hr orbit. But, with General Relativity we predict that such a system will lose energy, angular momentum due to *gravitational radiation* (like a moving charge emits EM radiation). The change in the period of the binary was observed from very precise measurements: 1993 Nobel prize in Physics.

Will merge in 300 Myr: explosion and burst of GW.

Lecture XIX Gravitational Redshift & Black Holes

Kutner 8.3, 8.4

a photon needs to climb out of a potential well. When that happens it loses energy. A rocket climbing out of a well would slow down, when it loses energy, but photons must go at c, so change frequency instead.

draw

If we start at R_1 and go to $R_2 = \infty$: starts with $m = E/c^2 = h\nu_1/c^2$, so the total energy of the system is $h\nu_1 - GmM/R_1$ and it ends with the same total energy

$$h\nu_1 - G\frac{h\nu_1 M}{c^2 R_1} = h\nu_2 - \frac{h\nu_2}{c^2\infty}$$

which gives:

$$h(\nu_2 - \nu_1) = h\Delta\nu = \frac{Gh\nu_1 M}{c^2 R_1}$$

or $\Delta\nu/\nu = GM/Rc^2$ (and a shift in wavelength is similar). This is sort of like the Doppler shift, except it comes just from gravity, not velocity.

XIX.1.4 WD

for a white dwarf, $\Delta\lambda/\lambda \approx 74\,(M/M_\odot)^{4/3}\,GM_\odot/R_\odot c^2$. It has been measured. For 40 Eri B (first WD) is it 6×10^{-5}.

XIX.1.5 NS

for a neutron star, this can be 20%! Can be quite large, although yet to be measured (I'm trying...).

XIX.1.6 Earth

1960 at Harvard. Shot γ-ray up a 22.6 m tower. Found a change $\Delta\nu/\nu = -gh/c^2 = -2.5\times 10^{-15}$.

XIX.2 More Mass?

What happens if we take a NS and add mass? $> 3M_\odot$ (or so): like a WD, the particles get to $v \sim c$ and it cannot support itself. Collapses. And nothing can stop it. Gravitational redshift goes up. Escape velocity goes up. $v_{\rm esc} = \sqrt{2GM/R}$ cannot be $> c$: eventually we get to where this is $= c$ at

$$R = R_{\rm Schwarzschild} = \frac{2GM}{c^2}$$

This is a black hole! (note that this derivation is not correct, but it gives the right answer).

At this point, the gravitational redshift becomes:

$$\frac{\Delta\lambda}{\lambda} = \left(1 - \frac{R_{\rm Sch}}{R}\right)^{-1/2} - 1$$

which goes to $1/0 = \infty$ at $R = R_{\rm Sch}$. (and this reduces to the form we already did for $R \gg R_{\rm Sch}$.).

XIX.3 BH

A BH has mass squeezed into $R_{\rm Sch}$: event horizon. Cannot get anything out. For $1M_\odot$, $R_{\rm Sch} = 3\,{\rm km} \ll R_{\rm NS}$.

How do we identify black holes? We look for things moving really fast in a really small volume, and we look for $M > M_{\rm NS}$

draw

Material moves in *accretion disk* around BH, gradually spiralling in. We can deduce that it has a velocity (from redshift/blueshift) such that it would be inside a NS (from Kepler's laws).

Accretion can happen with NS too. But there it can have any orbit. For a BH, stable orbits can only happen for $R > 3R_{\rm Sch}$. Inside that, the material is doomed. Must fall in.

As matter falls in, releases gravitational energy. Gets hot! Makes jet!

draw

Happens for small BHs: X-ray binary, *microquasar*.

Happens for big BH. Most galaxies seem to have one. $M_{\rm BH} = 10^{6-9}M_\odot$, *active galactic nucleus* or AGN. Can outshite rest of galaxy (*quasar*). Brightness depends on how much stuff is falling in. MW: $2 \times 10^6 M_\odot$.

Event horizon does not kill. tidal forces do.

Eventually, GR (Einstein) says that no orbit will stay stable. For massive objects moving quickly (i.e., in short orbits) this will happen in $< t_{\rm Universe}$. Like the Hulse-Taylor binary in 200 Myr.

What will happen? They will merge & (often) explode. Details not know, but for instance NS-NS binary will probably make a gamma-ray burst.

When this happens, the moving masses will also distort space-time, sending out waves. Hopefully we can detect this on Earth with Laser Interferometer Gravitational Observatory: astronomy with gravitational waves, not photons.

Lecture XX Supernovae

Kutner 11.1

explosions, new stars. a few known historically (e.g., Crab in 1054). Here we talk about core collapse (from massive stars).

10^{46} J released, 1% into KE, 0.01%–0.1% into photons. Collapse makes fusion well past Fe.

Lightcurve **draw**: decline set by radioactive decay of Ni (6 days) and Co (77 days) made from shock slamming into Fe.

XX.2 Chemistry

Rest of elements past Fe mostly from SNe. Reactions: r-process (rapid). Heavy element + many neutrons $\rightarrow$ very heavy, unstable nucleus. These then decay to something stable.

Also s-process (slow) in post-MS evolution of stars.

XX.3 Shock Wave

Hot gas out at 10,000 km/s. 10^{44} J at $T = 10^7$ K into ISM (10^4 K, $n \sim 10^6\ \mathrm{m}^{-3}$). Blast wave:

- Starts really fast: free expansion, supersonic
- Until sweeps up mass $\sim$ mass in shell (10's to 100's of years)
- Enters Sedov-Taylor phase (first derived for nuclear explosions): $E =\sim (\rho_{\mathrm{ISM}}/t^2)(r/c)^5$

Lecture XXI Interstellar Medium

Kutner 14

What is between stars? Gas (99%) and dust (1%). Like a star, mostly H, then He.

XXI.2 Dust

Obvious as dark patches in the sky **show**: “holes” w/o stars.

Light is blocked by dust. Work by Trumpler (1930).

Clusters of stars each have main sequence: should line up to get distances. Trumpler: d =distance, D =diameter. Angular diameter $\theta = D/d$:

$$\theta^2 = \frac{D^2}{d^2} \propto \frac{1}{d^2}$$

Apparent brightness is the flux $F = L/4\pi d^2 \propto 1/d^2$.

If L, D are typical values then should see $\theta^2 \propto F$

draw

systematic departuse for distant objects (small F, small θ^2), with F less than expected. Moreover, distant clusters were also redder than expected. Both from dust.

Like a sunset. Red light transmitted, blue light scattered/reflected.

Dust is little balls of C, Si. Makes things dimmer and redder.

XXI.3 Gas

Most of mass of ISM, few % mass of stars.

Gas can be warm or cold, dense or diffuse, atoms or molecules. Average is $n \sim 1\,\mathrm{cm}^{-3} = 10^6\,\mathrm{m}^{-3}$. For comparison, best vacuum on Earth is $10^9\,\mathrm{m}^{-3}$.

component	volume	T	n	state	see via?
molecular clouds	$< 1\%$	10-20	10^{8-12}	H_2	molecules, emit & abs.
CNM	1–5%	70	3×10^7	H	HI 21 cm abs
WNM	10%–20%	10^4	10^6	H	HI emit
WIM	20%–50%	10^4	10^6	H II	Hα
H II regions	$< 1\%$	10^4	10^{8-10}	H II	Hα

The last surrounds hot stars.

Remember: hot light and cold cloud: absorption. Hot cloud: emission.

XXI.3.1 H I 21 cm

diagnostic for gas. Spin flip **draw**. $\nu = 1420$ MHz or $\lambda = 21$ cm. First seen in 1950's.

XXI.3.2 Pressure Equilibrium

Different parts are "roughly" at same $P = nk_BT$ (equilibrium).

Molecular clouds: small and dense, where stars are born

H II regions surround hot stars (OB, WD)

the rest takes up space in between.

measure T from Doppler width of lines. Measure n from brightness of lines.

XXI.3.3 Using H I

Doppler shift gives v_{radial}. In Milky Way, get distance, velocity of gas, can map spiral arms.

XXI.3.4 Physics That Happens

Heat ISM via:

- cosmic rays (protons)
- light from stars
- shocks (SNe)
- stellar winds

Cool via

- emission of lines
- free-free (emission of continuous radiation from hot, ionized gas)

XXI.3.5 H II Regions

H I ($T \sim 10^2$ K) around H II around star.

Star is $> 10^4$ K so enough photons have $h\nu > 13.6$ eV. Can then ionize.

Size of H II: $N_* =$ number of photons per second with $E > 13.6$ eV (beyond Lyman limit).

Assume each photon is absorbed by 1 atom.

But, for each e, p there is a chance that recombine $e + p \rightarrow$H. This balances ionization.

$$\mathcal{R} = \frac{\#\text{recombination}}{\text{volume} \times \text{time}}$$

$\mathcal{R}V = N_* = \mathcal{R}(\frac{4}{3}\pi r^3)$. What is $\mathcal{R}$? Depends on rate that e hits p. $\mathcal{R} \propto n_e n_p \propto n_e^2$. So

$$r = \left(\frac{3N_*}{4\pi\alpha n_e^2}\right)^{1/3}$$

Stromgren sphere. $\alpha = \alpha(T) \sim 3 \times 10^{19}\,\mathrm{m}^3\,\mathrm{s}^{-1}$. $n_e \sim 10^8\,\mathrm{m}^{-3}$, $N_*(\mathrm{O5}) = 3 \times 10^{49}\,\mathrm{s}^{-1}$. Gives r ~few pc.

Can have multiple stars (bigger) or WD (smaller). See via Balmer lines (Hα etc.) in emission, also lines from N, O, He.