# Astronomy 299/L&S 295

# Lecture I Preliminaries

**Course Description:** *quantitative* astronomy. Emphasis on the "why" and the "how" rather than just the "what." There will be some math(!) and physics. There will be no calculus.

If you don't want that, look at ASTRON 103. If you want a lab, take ASTRON 104. Here we will cover:

- Celestial mechanics
- The nature of light and its interaction with matter
- Telescopes
- The structure and evolution of single stars
- The evolution of binary stars
- The end-products of stellar evolution
- The Solar System
- Extra-solar planets
- Galaxies & quasars
- Expansion of the universe & dark matter
- The big bang

The textbook will be Kutner: *Astronomy: A Physical Perspective*

Evaluation will be:

- Weekly problem sets (50%), with the best 10 of 11 counting.
- Midterm exam (20%)
- Final exam (30%)

### I.2 Set the Stage

Astrophysics mostly starts next lecture.

## I.3 Physics Synopsis

The level of physics and math that you are expected to be familiar with (but not necessarily know in detail) is:

**Newton's Laws** : most importantly, $F = ma$

**Kinetic Energy** : $K = \frac{1}{2}mv^2$

**Gravitation** : $F = GM_1M_2/r^2$ (on the surface of the Earth, $F = gm$)

**Potential Energy** : $U = -GM_1M_2/r$ (from gravity; on the surface of the Earth, $U = gmh$)

**Centripetal Acceleration** : $a = v^2/r$

**Ideal Gas Law** : $PV = NRT$

**Circumference of a Circle** : $2\pi r$

**Area of Circle** : $\pi r^2$

**Surface Area of a Sphere** : $4\pi r^2$

**Volume of a Sphere** : $\frac{4}{3}\pi r^3$

**Radians** : $180° = \pi$ radians, $\sin(\pi/2) = 1$, $\sin\pi = 0$, etc.

**Small Angles** : $\sin x \approx x$ for $x$ very small and measured in radians. Also, $\tan x \approx x$, and $\cos x \approx 1$ (**draw these**)

**Scientific notation** : $A \times 10^a \cdot B \times 10^b = (AB) \times 10^{a+b}$

### I.3.1 Calculus

Not required. However, it is in the book. **Don't Panic!** If you see:

$$\frac{dx}{dt}$$

or

$$\int dx f(x)$$

just read around it. Or ask questions.

### I.3.2 Greek

If I use a symbol you don't recognize or can't read, **ask**!

## I.4 Precision

We often do not know things very precisely. So we use $\sim$ and $\approx$ and related symbols. $\sim$ is for when we know something to *an order of magnitude*. So we if we know that $x \sim 5$, we know that $x$ is between $5/3$ and $5 * 3$, where $3$ is roughly $\sqrt{10}$. This means that the possible range for $x$ is in total a factor of 10. We will also sometimes use $\sim$ to mean *scales as*. For example, if you were to estimate the height of a person as a function of their weight (for a wide range of people), you might expect that as you double the weight, the height changes by $2^{1/3}$. We could write height$\sim$weight$^{1/3}$. There will be a lot of variation, but this is roughly correct.

$\approx$ means more precision. It doesn't necessarily have an exact definition. But generally, if we say $x \approx 5$, that means that $4$ is probably OK but $2$ is probably not.

Finally, we have $\propto$, which means *proportional to*. This is more precise that the *scales as* use of $\sim$. So while for a person height$\sim$weight$^{1/3}$ is OK, for a sphere (where we know that volume is $4\pi/3r^3$) we could write volume$\propto r^3$: we take this as correct, but leave off the constants ($4\pi/3$ in this case).

### I.4.1 Order of Magnitude Problems

There are many problems — in Astronomy, Physics, or Life — where we know only the basics. But we want to estimate something. So we do an "order of magnitude" estimate (also known as a "Fermi problem": see `http://en.wikipedia.org/wiki/Fermi_problem`). Basically, we want to know whether something is 1, 10, or 100, but we do not care whether it is 20 or 30. These make heavy use of the $\sim$ symbol. We will come back to these.

### I.4.2 Small Angles

For small angles $\theta$, $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$. We need $\theta$ to be in radians. But we also often deal with fractions of a circle. A circle has $360°$. We break each degree into 60 minute (or *arcminutes*): $1° = 60'$. And each arcminute into 60 seconds (or *arcseconds*): $1' = 60''$, so $1° = 3600''$. But we also know that $2\pi$ radians is $360°$, so we can convert between radians and arcsec. This will come up frequently: $1'' = 360 \times 3600/2\pi \approx 1/206265$ radians.

## I.5 Example Problem

Consider all of the people on the UWM campus:

- How much do all of the people on the UWM campus weigh?
- How many buses would you need to transport them all?
- If they all jumped in to Lake Michigan, how much would the water level change (numbers: 300 miles long by 118 miles wide, average depth 279 feet, volume 1180 cubic miles)? What about all of the people in the world?

- 30,000 people, 60 kg each, 1,800,000 l or 1800 $m^3$
- $5 \times 10^{12}$ $m^3$
- So answer is 0
- for whole world answer is 1 cm (400,000,000 $m^3$ total).

# Lecture II Celestial Sizes, Distances, and Coordinates

### II.1.1 Units

Astronomy emphasizes *natural* units ($\odot$ is for the Sun, $\oplus$ is for the Earth):

- $M_\odot = 2 \times 10^{30}$ kg (solar mass)
- $R_\odot = 7 \times 10^8$ m (solar radius)
- $M_\oplus = 6 \times 10^{21}$ kg (earth mass)
- $L_\odot = 4 \times 10^{26}$ W (solar luminosity or power)
- light year $= 10^{16}$ m: the *distance* light travels in one year (moving at $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$)
- parsec = parallax second (we will understand this later) = pc = $3 \times 10^{16}$ m
- Astronomical Unit = AU = $1.5 \times 10^{11}$ m (distance between earth and sun)

And then we use usual metric-style prefixes to get things like kpc, Mpc, etc.

### II.2 Distances

Kutner 2.6.

How far away/big are things? Use meter stick to draw centimeter, meter, 10 meters.

**Chicago** : 144 km

**circumference of Earth** : 40,000 km

**distance to Moon** : 380,000 km

**distance to Sun** : $1.5 \times 10^{11}$ m = 1 AU

**solar system** (orbit of Neptune): 30 AU in radius

**Oort cloud** : 50,000 AU

**Nearest star** (Proxima Centauri): 1.29 pc = $4 \times 10^{16}$ m

**Center of Milky Way** : 8.5 kpc = $2.6 \times 10^{20}$ m

**Andromeda Galaxy** : 0.6 Mpc = $1.8 \times 10^{22}$ m and so on. There is *a lot of empty space*!

*How do we measure distances to stars?* First step: parallax = geometry. $\tan\theta = 1\,\mathrm{AU}/d \rightarrow d = 1\,\mathrm{AU}\tan\theta$. But stars are very far away, so $\theta$ is very small. Again, if $\theta$ in radians $\tan\theta \approx \theta$ so $d \approx 1\,\mathrm{AU}\theta$ (which puts $d$ in AU too if $\theta$ in radians).

But remember, 1 radian is $206265''$. So if $\theta$ is in arcsec now, $d = 206265\,\mathrm{AU}(\theta/\mathrm{arcsec})$. 206265 AU has a special name: it is 1 parallax second or 1 parsec (or 1 pc or $3 \times 10^{16}$ m).

How big is an arcsecond? For a quarter to be $1''$ across (diameter of 25 mm) need to be 5 km away. And we can measure much smaller angles.

### II.2.1 Planetary Motion and the Copernican Model

Kutner, Chapter 22.

The geocentric model for the Universe is wrong. The Earth is not the center of the solar system, the galaxy, the universe, etc. Partly this was uncovered through observations of *retrograde motion*: stars appear the same every night, but some objects (often bright ones) move relative to the stars. These are known as *planets* (literally wanderers). Mostly the planets move from West to East. Except when they don't — then they go the other way, which is called retrograde motion. This was a 2000-year old puzzle. **Mars 1994 from Astron 103 week 3**.

In the geocentric view, it was complicated and elaborate (*epicycles*). But the heliocentric view (from Copernicus) has an elegant solution. Taking inner orbits as faster (we'll see why later), we find retrograde motion happens occasionally for the planets that are further out than the Earth.

We can then define two periods:

**sidereal period** (or sidereal time) is the period relative to the fixed background stars. This is close to (but not exactly the same) as a year.

**synodic period** is the time between when planets are closest together

and we can relate these by asking how long until the planets line up again. We define the angle of the Earth $\theta_{\mathrm{E}} = 2\pi t/P_{\mathrm{E}}$ which goes around from 0 and one revolution happens at $t = P_{\mathrm{E}}$ ($\theta$ in radians). We can do the same for Venus $\theta_{\mathrm{V}} = 2\pi t/P_{\mathrm{V}}$. $P_{\mathrm{V}} < P_{\mathrm{E}}$, so $\theta_{\mathrm{V}}$ goes faster. They line up when $\theta_{\mathrm{E}} = \theta_{\mathrm{V}} - 2\pi$: the $2\pi$ is since Venus will have gone around one extra time. So we write:

$$\frac{2\pi t}{P_{\mathrm{E}}} = \frac{2\pi t}{P_{\mathrm{V}}} - 2\pi$$

and can solve for $1/t = 1/P_{\mathrm{V}} - 1/P_{\mathrm{E}}$, and we identify $t$ with the synodic period. For planets outside the Earth's orbit, the sign is opposite.

### II.2.2 Motion of the Earth

The Earth rotates around its axis once every 24 hours. So each hour is then $360°/24 = 15°$.

This rotation is what makes the Sun and the stars appear to move over the course of a day. The star to which the Earth's axis appears to point is the North Star (Polaris): it's not a special star, we just point near it. Because of "precession", we point to different stars over the course of about 24,000 years.

This rotation can help you figure out how long you have (for example) until the Sun goes down:

- Your hand is roughly $10^\circ$ across when you hold your arm out
- Your finger is roughly $1^\circ$ across

And then, just like all of the planets, the Earth goes around the Sun. Each planet takes its own time. For the Earth, this is 1 year.

### II.2.3 Seasons and the Changing Sky

The rotation axis of the Earth is tilted $23.5^\circ$ to the plane of its orbit. This means that as the Earth orbits around the Sun, the Sun appears to trace a path in the sky. We call this the *ecliptic* (constellations in the ecliptic are the zodiac). The plane of the orbit extending out to infinity is the *ecliptic plane*.

During the nothern summer, the North pole of the Earth points towards the Sun. This is roughly June 21. On the northern winter solstice, what points towards the Sun? Seasons depend on what hemisphere you are in.

**solstice** : roughly June (northern summer) or December (northern winter), when the "sun stands still." Longest/shortest day. Flips in the southern hemisphere.

**equinox** : roughly March/September 21. When the night and day are of equal length.

Just like the equator on the Earth, we can extend it into the sky to make the *celestial equator*. This divides the sky into northern & southern halves. We do the same thing with the poles. The Sun crosses the celestial equator twice per year, on the equinoxes.

**winter (in north)** : Sun is low in the sky, days short

**summer (in north)** : Sun is high in the sky, days long

**spring/fall** : in the middle.

#### II.2.3.1 Weather (Hot vs. Cold)

The Sun puts out energy at a roughly constant rate: $L_\odot$. It is changing the way that we receive this energy that makes seasons. How does this work? Do we move closer to or further from the Sun?

*NO!*. This wouldn't work since the northern Summer = southern winter. Instead we change the area over which the Sun's energy is spread.

Draw a circle perpendicular to the path of the Sun's light with area $1\,\mathrm{m}^2$. This defines a *flux* which is power per area: $F = L/\mathrm{area}$. The total area over which the Sun's light is spread is the area of a sphere $4\pi R^2$, so at the Earth $F_\odot = L_\odot/4\pi R^2$ with $R = 1$ AU. When the Sun is high overhead (summer), this power gets spread over a patch of the ground with the same area. So each $1\,\mathrm{m}^2$ of the ground gets heated with the full $F_\odot$ of flux. This makes it hot. But in the winter the Sun is low in the sky. So the circle that is perpendicular to the Sun's light gets spread over a wide area on the ground. So we still have $F_\odot$ of flux, but it gets spread over (for example) $2\,\mathrm{m}^2$ of area. This means that each patch of the ground gets heated by less power, which makes it colder.

## II.3 How bright are stars?

Kutner 2.1

Historical method: magnitudes (messy and annoying, but has its uses).

*lower = brighter*, *higher = fainter*. Used to be $m = 1$ is roughly the brightest. But now we have quantified this on a *logarithmic scale*:

- 2.5 mag fainter = 1/10 the brightness
- 5.0 mag fainter = 1/100 the brightness
- 10 mag fainter = 1/10,000 the brightness

We look at things from the Sun ($m = -26.8$, as it appears from the Earth) to $m \approx 30$.

But remember that we are measuring how bright things appear! This would change if they were closer/farther. We are essentially measuring *flux*: energy per time per area. Or (energy per time) per area. (energy per time) is the same as power, and we measure this in Joules per second or Watts. So we measure flux in Watts per area, or Watts per $m^2$.

If instead we look at total power put out (independent of where you are) that is *Luminosity*, $L$. $L$ is measured in Joules/s or W. How can we go between these? Take a star of luminosity $L$, and surround it by a sphere with radius $R$. The flux is $F = L/4\pi R^2$, since the area of that sphere is $4\pi R^2$. This is the *inverse square law* for fluxes, and it should be familiar (when you go away from a lightbulb, it gets fainter, etc.).

$L_\odot = 4 \times 10^{26}$ W $= 1 L_\odot$. $R = 1$ AU = $1.5 \times 10^{11}$ m. So the flux on the Earth $F_\odot = 1300\,\mathrm{W\,m^{-2}}$. In comparison, on Neptune at $R = 30$ AU, the flux is 1/900 as much.

So we have magnitudes. How do we tie magnitudes (as a measure of brightness) to real fluxes?

### II.3.1 Absolute Magnitudes

=$M$=the apparent magnitude $m$ of a star if it were 10 pc away.

From the definition of a magnitude, a change of 2.5 mag is a factor of 10 change in $F$. So we can take two magnitudes $m_1$ and $m_2$ and put them with two fluxes $F_1$ and $F_2$:

$$\frac{F_2}{F_1} = 10^{-(m_2-m_1)/2.5}$$

The negative sign comes from having a lower magnitude mean brighter. Or:

$$m_1 - m_2 = -2.5\log_{10}\frac{F_1}{F_2}$$

To put in absolute magnitudes, we use the inverse square law: $F = L/4\pi d^2$. So $F_1 = L/4\pi d_1^2$ and $F_2 = L/4\pi d_2^2$, and we divide:

$$\frac{F_1}{F_2} = \left(\frac{d_1}{d_2}\right)^{-2}$$

with $d_2 = 10\,\mathrm{pc}$. So $F_1/F_{10\,\mathrm{pc}} = (d_1/10\,\mathrm{pc})^{-2}$. This is the same as:

$$\frac{F_1}{F_{10\,\mathrm{pc}}} = 10^{-(m_1-M)/2.5}$$

we can work this through to find: $m - M = 5\log_{10}(d/10\,\mathrm{pc})$ which is known as the *distance modulus*: how much being far away changes the apparent magnitude of something.

So if we know $m$ and $M$, we can get $d$. Or if we know $m$ and $d$, can get $M$. FOr the Sun: $m_\odot = -26.83$, $d_\odot = 1$ AU = $1/206265$ pc. We get: $M_\odot = m_\odot - 5\log_{10}(d/10\,\mathrm{pc}) = +4.74$. This is pretty modest compared to other stars (the Sun is only remarkable by being close).

Knowing what $M_\odot$ is, if we are then given the absolute magnitude of a star we can calculate its flux and actual luminosity. That is because for two objects at the same distance (10 pc in this case) the relation between magnitudes and fluxes can work with luminosities too. So we can say:

$$F = F_\odot 10^{-(M-M_\odot)/2.5}$$

$$L = L_\odot 10^{-(M-M_\odot)/2.5}$$

where $F_\odot$ is now the flux of the Sun as perceived from 10 pc away.

# Lecture III Gravity & Celestial Mechanics

And finally some physics. Kutner 5.3, 5.4

Johannes Kepler: used data from Tycho Brahe to determine 3 "laws".

1. Orbits are ellipses with the Sun at a focus. Semi-major axis is $a$, semi-minor axis is $b$. The equation of an ellipse says that the distance from the planet to the Sun + the distance from the planet to the other focus is a constant.
2. Planet-sun line traverses equal areas in equal time. (**Kepler's 2nd law from Astron 103 week 3**)
3. $P^2 = a^3$, with $P$ the period of the orbit. For the Sun, this works with $P$ in years and $a$ in AU.

## III.2 Elliptical Motion

*aphelion* = far from star, *perihelion* = close to star. Eccentricity $e$ between 0 (circle) and 1. We find $b^2 = a^2(1 - e^2)$, with perihelion at $a(1 - e)$ and aphelion at $a(1 + e)$. Planets have only slightly eccentric orbits. For the Earth, $e = 0.0167$. So maximum distance from Sun - average is $ae = 0.0167\,\mathrm{AU} \approx 400R_\oplus$. This means a change in the solar flux of about $\pm 3\%$.

## III.3 Newton's Laws

1. Inertia
2. $\vec{F} = m\vec{a}$
3. $\vec{F}_{12} = -\vec{F}_{21}$

(here $\vec{F}$ is a force, not a flux). Also have gravitation: $F = GMm/r^2$.

For a circular orbit of something with mass $m$ around something with mass $M$ that is not always the sun, Kepler's third law says: $P^2 \propto r^3$, but the constant of proportionality can change: $P^2 = kr^3$. Let's derive $k$. $P = 2\pi r/v$, since it has to traverse a distance $2\pi r$. Putting this in gives us

$$\frac{4\pi^2 r^2}{v^2} = kr^3$$

We need a centripetal force $F = mv^2/r$ to keep the planet in orbit: $F = GmM/r^2$. We put these together and re-arrange to get $k = 4\pi^2/GM$.

## III.4 Escape: Work and Energy

Potential energy $U = -GMm/r$. It depends just on the start and stop points, not the path.

Kinetic energy $K = 1/2mv^2$.

Total energy $E = K + U = 1/2mv^2 - GMm/r$. Start at $r$, move away until $v = 0$ at $r = \infty$. $K(\infty) = 0$, $U(\infty) = 0$, so $E(\infty) = 0$. This will always be true (no external source of work), so $K = -U$. We can write $1/2mv^2 = GMm/r$ and get:

$$v = \sqrt{2GM/r} = v_{\rm escape}$$

is the escape speed. For the Earth this is 11 km/s: if you go this fast and point up, you will escape the Earth's gravity.

## III.5 Consequences of Gravity

### III.5.1 Linear Momentum

If the net force on a system is 0, then momentum is conserved.

### III.5.2 Angular Momentum

is conserved also.

### III.5.3 Energy

is conserved also.

## III.6 2 Body Problem

This is more general than what we did before, and we no longer require $m \ll M$. This applies to binary stars, binary asteroids, black holes, planets, etc. We can write.

Since momentum is conserved, the two bodies will orbit their common center of mass. The COM can move, but it will move at a constant velocity. This is defined such that:

$$m_1 r_1 = m_2 r_2$$

Both stars must go around in the same time such that the line between them always goes through the COM:

$$P = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$

(assuming circular motion). Or $r_1/v_1 = r_2/v_2$. Combining these we get:

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

We also have the force:

$$F = G\frac{m_1 m_2}{(r_1 + r_2)^2}$$

As before, this must be the force needed to keep an object in a circular orbit. So on object 1, $F = m_1 v_1^2 / r_1$:

$$\frac{m_1 v_1^2}{r_1} = G\frac{m_1 m_2}{(r_1 + r_2)^2}$$

Divide both sides by $m_1$, and use $P = 2\pi r_1 / v_1$:

$$\frac{4\pi^2 r_1}{P^2} = \frac{G m_2}{(r_1 + r_2)^2}$$

But we also define $R = r_1 + r_2 = r_1(1 + r_2/r_1)$. With the ratio of the masses intead of radii:

$$R = r_1(1 + \frac{m_1}{m_2}) = \frac{r_1}{m_1}(m_1 + m_2)$$

Put this in:

$$\frac{4\pi^2 R^3}{G} = (m_1 + m_2)P^2$$

This is Kepler's third law! It's a bit more general than for just a system like the Earth and the Sun. We often write $a$ instead of $R$.

The angular momentum is constant, and it can be useful. For each object $L_1 = m_1 v_1 r_1$ etc. So overall:

$$L = m_1 v_1 r_1 + m_2 v_2 r_2$$

But we know $m_1 r_1 = m_2 r_2$, so we can write:

$$L = m_1 r_1 (v_1 + v_2) = m_1 r_1 \left( \frac{2\pi r_1}{P} + \frac{2\pi r_2}{P} \right)$$

Substitute from before: $r_1 = m_2 R/(m_1 + m_2)$:

$$L = \frac{2\pi m_1 m_2 R^2}{(m_1 + m_2)P}$$

We substitute for $P$ from Kepler's third law to get:

$$L = m_1 m_2 \sqrt{\frac{GR}{m_1 + m_2}}$$

### III.6.1 How to Use This?

A basic result that we will use over and over again is $GM = a^3(2\pi/P)^2$. For circular systems, $v = \sqrt{GM/a}$. Using the center-of-mass we can write $a_1/a_2 = v_1/v_2 = m_2/m_1$, $a_1/a = v_1/v = m_2/M$.

### III.6.2 $N > 2$

No general solution. Can be chaotic, not purely periodic, not closed. E.g., Jupiter pertburbs the orbit of the Earth, asteroids, comets, slingshots.

How high does $N$ get? Globular cluster: $N \sim 10^6$ stars. Galaxy cluster: $N \sim 10^3$ galaxies. Universe: $\sim 10^{11}$ galaxies. Need big computers to get approximate solutions.

# Lecture IV Doppler Shifts and Waves

Kutner 2.2, 5.2

Light is a wave (and a particle). We talk about waves having wavelength $\lambda$ and frequency $\nu$: $\lambda$ is a measure of how far before the wave repeats (in meters or such) and frequency is a measure of how often it repeats (in units of Hz which is 1/s). They are related by the speed of the wave, which for light is $c$:

$$c = \lambda\nu$$

But, the wavelength that a light is emitted at is not the same as it is absorbed at. It can change if there is relative motion toward or away from the light source. We can this change the Doppler shift, since it shifts the wavelength (or frequency). As long as the velocities are $\ll c$, then:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c}$$

So the shift is $\Delta\lambda$ ($\Delta$ usually means change). So if something is moving away from us ($v > 0$), then the wavelength gets longer. Since longer wavelengths are often associated with red, we call this a *red shift*. The opposite, when we go toward a lightsource, is a *blue shift*.

We can also use this in frequency, but the sign is opposite (higher frequency means smaller wavelength, so if the wavelength gets smaller ($\Delta\lambda < 0$) the frequency gets higher ($\Delta\nu > 0$):

$$\frac{\Delta\nu}{\nu_0} = \frac{\nu - \nu_0}{\nu_0} = -\frac{v}{c}$$

# Lecture V Extra Solar Planets

Kutner 27.5

This is just how to find them — we will return to talking about the planets themselves later.

How do we find planets around other stars?

- Take pictures? Stars are much brighter, so this is very hard (although it has been done recently in some special cases).
- Main way: Newton

Take a planet in orbit with a star. Mostly the planet moves around the star, but since the mass of the star is finite, it moves a bit too around the center of mass: $v_* m_* = v_p m_p$, so $v_* = v_p(m_p/m_*)$. Even though $m_p \ll m_*$ (so $v_* \ll v_p$) it is still measureable, typically with velocities of a few m/s, via Doppler shifts. We have planets that are similar to Jupiter: $m_p \sim M_{\rm Jupiter} \sim 10^{-3} M_\odot$.

Can also find via eclipses (transits). Here we see the dip in light when a planet goes in front of the star. The amount of light that is lost is $\sim (R_p/R_*)^2$, and since $(R_{\rm Jupiter}/R_\odot) \sim 0.1$, the dip is $\sim 1\%$.

And we can (rarely) see the wobble of the star back and forth during the orbit. Here we use $m_* r_* = m_p r_p$, which gives a wobble of $\ll 1''$.

# Lecture VI Tides

Kutner 23.5

Gravity to date has been point masses (or perfect spheres). Not points → tidal forces → not spheres. Tides are from differential forces across an object.

Consider two bits of a bigger thing $m_1$ and $m_2$ ($m_1 = m_2$) separated by $\Delta r$. $F_1 = GMm_1/r^2$. $F_2 = GMm_2/(r+\Delta r)^2 \approx F_1 + -2GMm/r^3\Delta r$. It's the extra bit that gives rise to tidal forces, and the $1/r^3$ dependence is pretty general. In the Earth-Moon system, the total force is what keeps the orbit steady. But if you subtract off the forces on the center-of-mass you get the tides, which make a bulge that points at the moon.

Another way to think about it: gravity balances centripetal acceleration $GMm/r^2 = v^2/r = \omega^2 r$ to make a stable orbit. But that is only true at the center of mass: too close and gravity is stronger (bits that are too close get even closer). Too far and gravity is too weak, so bits that are too far get farther. This makes bulges.

Tidal period = $2\times$ forcing periods, from two bulges per rotation (toward and away). Height of tides from Sun $\sim$ that from Moon. With rotation of the Earth, get mostly semi-diurnal ($\approx 12.4\,$h) tides from $P_{\rm rot} = 24\,$h, which dominate in Atlantic. But some places hay diurnal tides (complicated interactions between water and gravity).

From the orbit of the moon get spring tides (when lunar lines up with solar) and neap tides (when they cancel out).

From the orbit of the earth get semi-annual tides, since the eccentricity of the Earth is not quite 0.

Tides affect: atmosphere, rock, *ocean*.

$$a_{\rm tide} \sim \frac{2GM_{\rm moon}}{r^3}R_\oplus \sim 2g_\oplus\frac{M_{\rm moon}}{M_\oplus}\left(\frac{R_\oplus}{r}\right)^3$$

But the tidal bulge does not fly away due to this extra acceleration. Instead it makes a bit of extra gravity from the extra mass to cancel it out. The bulge has height $h$, with extra gravity $g' \sim GM_{\rm bulge}/R_\oplus^2$. We have $M_{\rm bulge} \approx hR_\oplus^2\rho$, which gives $g' \sim GM_\oplus/R_\oplus^2(h/R_\oplus) \sim g_\oplus(h/R_\oplus)$ (we have used $\rho R_\oplus^3 \sim M_\oplus$). Setting $g'$ equal to $a_{\rm tide}$, we find $h/R_\oplus \sim 2(M_{\rm moon}/M_\oplus)(R_\oplus/r)^3$.

The size of this bulge from the moon on the Earth is about $10^{-7}R_\oplus \sim 60\,$cm. And about 25 cm from the Sun. The Earth on the moon is about 2 m. This is of the right *order*, although the details are hard. For instance, in the Bay of Fundy, the tidal forcing period is the same as the time it takes for the water to slosh around. So you get a resonance, and the tides are really high (up to 9 m).

### VI.2 Tidal Evolution

Water sloshes, loses energy (heat) — or generates electricity!

Note that the Earth spins faster (24h) than the Moon's orbit (28d). Friction drags the tidal bulge ahead of the moon. This leads to a net torque that slows down rotation. Conserving $L$, the moon moves away ($L \propto \sqrt{a}$). This also makes orbits more circular, as tides are stronger for $e > 0$, and synchronized (like the moon is now, with rotation period of the moon equal to its orbital period).

We can measure: the moon mooves away at $\approx\ 3\,cm$/yr. And the Earth day slows down at $0.0016$ s/century. Note that the length of a day goes up, the length of a month goes up, but the number of days in a month goes down.

For Pluto+Charon, $P_{\rm orb} = P_{\rm Pluto} = P_{\rm Charon} = 6.4$ d, and $e = 0$. So this has already all happened here.

Weird cases exist, like Mars. Its moon Phobos orbits faster than Mars rotates. So tides pull it in! It will hit in about 50 Myr.

### VI.3 Roche Limit

When do tides pull things apart? Simple answer:

$$\frac{Gm}{R^2} \sim \frac{2GM_0}{r^3} R$$

where the thing being pulled apart has mass $m$, size $R$, and is held together only by gravity. We take $\rho = m/(4\pi/3R^3)$ and $\rho_0 = M_0/(4\pi/3R_0^3)$. Equating the forces we get roughly:

$$r \lesssim 2^{1/3} \left(\frac{\rho_0}{\rho}\right)^{1/3} R_0$$

Could this have given us Saturn's rings? They are inside such a radius (known as the *Roche Limit*, so large moons would have been pulled apart.

### VI.4 Tides and Black Holes

Tides are how black holes kill you. It's not that gravity is too strong, it's that the difference in gravity between different parts of you is too strong.

# Lecture VII Stars I

Kutner Chapter 9

What is a star?

- Big ball of gas
- Self-gravity $\rightarrow$ need high pressure inside to support against collapse
- high pressure $\rightarrow$ high $T$ inside (basic gas physics)
- high $T \rightarrow$ emit light

A star's life is a long, losing battle with gravity.

Gravity pulls the parts of the stars in. Why do they not fall? Because pressure pushes them out. Which leads to high $T$, so the star loses energy. This means we need an energy source to keep the star shining. What is that? Gravity? Chemistry? Fission? Fusion?

## VII.2 What supports against gravity?

Let us consider:

1. What if there were no support?
2. What provides the support?
3. How much support is necessary?

If the star is spherical, $g_r = GM(r)/r^2$ where $M(r)$ is the amount of mass contained within $r$. This is independent of $\rho(r)$, and the matter outside $r$ does not matter.

### VII.2.1 What if there were no support?

This is relevant for star formation, supernovae. We have things fall inward with radial velocity $v_r$ and acceleration $a_r = -GM(r)/r^2$. The material falls toward the center taking a *free-fall timescale* $t_{\rm ff}$:

$$|v_r| \sim \frac{r_0}{t_{\rm ff}},\ a_r \sim \frac{v_r}{t_{\rm ff}} \sim \frac{r_0}{t_{\rm ff}^2} \sim \frac{GM_0}{r_0^2}$$

From this we can solve $t_{\rm ff} \sim \sqrt{r_0^3/GM_0} \sim \sqrt{1/G\rho_0}$, where $\rho_0 \sim M_0/r_0^3$ is the mean density. We identify this free-fall timescale as the *dynamical time*. If you do out the math in detail (keeping factors of $4\pi/3$ etc.), you find:

$$t_{\rm ff} = \sqrt{\frac{3\pi}{32}}\frac{1}{\sqrt{G\rho_0}}$$

So the time for collapse only depends on the density, not the size.

| Object | $r$ | $M$ | $\rho$ | $t_{\rm dyn}$ |
|---|---|---|---|---|
| Earth | $6 \times 10^6$ m | $10^{-6}\,M_\odot$ | 5.5 g/cm$^3$ | $\sim 10$ min |
| Jupiter | $7 \times 10^7$ m | $10^{-3}\,M_\odot$ | 1.3 g/cm$^3$ | $\sim 10$ min |
| Sun | $7 \times 10^8$ m | $1\,M_\odot$ | 1.4 g/cm$^3$ | $\sim 10$ min |
| White Dwarf | $7 \times 10^6$ m | $1\,M_\odot$ | 1.4×10$^6$ g/cm$^3$ | 3s |
| Neutron Star | $10^4$ m | $1.4\,M_\odot$ | 7×10$^{14}$ g/cm$^3$ | 0.1 ms |

### VII.2.2 Support comes from gas pressure

Pressure: resists compression. This comes from the kinetic energy of the gas particles. You can think of them each exerting a little force when they bounce off the walls of a box.

Pressure is force/area. In stars, most of the gas is an *ideal gas*, which means that the particles are all independent of each other. In the kinetic theory of ideal gases, the kinetic energy per particle is $3/2k_BT$ (where $k_B$ is Boltzmann's constant). This is an average. It is both an average over time for 1 particle and an average over all particles at one time. From this we can derive the ideal gas law (in a slightly different form to what you might have seen in chemistry):

$$P = nk_BT$$

$n = N/V$ is the number density (units are $m^{-3}$), the number of particles $N$ per volume $V$.

Are there non-ideal gases? Yes, we will discuss later. But these are gases where the particles are correlated (the wavefunctions overlap). They can be:

1. Fermi gases, with $P = P(n)$ only (electrons, protons, *degenerate gases*)
2. Bose gases, with $P = P(T)$ only (photons)

But with ideal gases, we have $P = nk_BT$. We can also write the number density $n$ in terms of the mass density $\rho$ (km/m$^3$). Then we need to figure out how much the average particle weighs: $n = \rho/\mu m_H$. Here $m_H$ is the mass of hydrogen, and $\mu$ is the mean molecular weight. If it's only hydrogen, then $\mu = 1$. If it's helium, then $\mu = 4$. What if it is *ionized* hydrogen? Then you have protons and electrons in equal numbers. The protons have mass $\approx m_H$, but the electrons have much less mass. So the average is $m_H/2$, which means $\mu = 0.5$. But writing the ideal gas law this way gives us:

$$P = \frac{\rho k_B T}{\mu m_H}$$

### VII.2.3 How much support is needed?

Here we will derive *Hydrostatic equilibium*. This supports fluid against gravity via a pressure *gradient*. We have the gravitational acceleration $mg = (\rho\Delta r\, A)g = P_{\rm bottom}A - P_{\rm top}A$. Cancelling

$A$, we find $P_{\rm top} = P_{\rm bottom} + \Delta P = P_{\rm bottom} + (\Delta P/\Delta r)\Delta r$, or:

$$\frac{\Delta P}{\Delta r} = -\rho g$$

This is very general, and applies to stars as well as to atmospheres, oceans, etc. What is means is that $P$ increases as you go down/inward.

What can we do with this? In general we need to know $\rho(r)$ and $g(r)$. But we can make some simplifications. We assume that $\rho \approx M/R^3$ and $g = GM(r)/r^2$. Then we get:

$$\frac{\Delta P}{\Delta r} = -\rho(r)\frac{GM(r)}{r^2}.$$

Furthermore, we will take an average over the whole star, the a difference between the inside ($P = P_c$ at $r = 0$) and the outside ($P = 0$ at $r = R$). So we get $\Delta P/\Delta r \approx (0 - P_c)/(R - 0) = -Pc/R$, with $P_c$ the central pressure. Put this in and you get:

$$P_c \approx \frac{GM^2}{R^4}$$

For the Sun this is close: it gives $10^{14}\,{\rm N\,m^{-2}}$, which is a bit of an underestimate. For reference, 1 atmosphere is $10^5\,{\rm N\,m^{-2}}$. Relations like these are very important for astronomy where the details are hard but we can make general scaling relations between different quantities.

To go further, we can say that the pressure is related to the temperature through the ideal gas law, $P \propto k_B T$. Putting in our approximate density we get $P = (M/R^3) k_B T/\mu m_H$, or $k_B T \approx GM\mu m_H/R$. Again, the details are hard, but the general expression that $T \propto M/R$ is very useful.

# Lecture VIII Stellar Energy

Again, a star's life is a long protracted (but losing) battle with gravity. What can balance it? Fundamentally it is pressure, but what keeps the pressure going?

## VIII.2 Virial Theorem

Due to Classius (1870). It concerns bound gravitational systems. In essence, the long-term average of kinetic energy is $1/2$ the average of the potential energy. These can each come from different places:

**kinetic** can be orbital (motion of blobs or stars) or thermal (random motion of particles)

For only orbital energy, that would be something like the Earth-Moon system. Only thermal would be the insides of stars. Or there are situations with a combination like elliptical galaxies and galaxy clusters.

We denote average by $\langle \ldots \rangle$. So the Virial theorem states:

$$\langle K \rangle = -\frac{1}{2}\langle U \rangle$$

This is a bound system, so we have $\langle U \rangle < 0$. We can then look at the total energy $\langle E \rangle = \langle K \rangle + \langle U \rangle$, and if we substitute we find:

$$\langle E \rangle = \frac{1}{2}\langle U \rangle$$

## VIII.3 What Powers the Sun and How Long Will It Last?

Kutner 9.1.2, 9.1.3.

We take the Solar luminosity to be $4 \times 10^{26}$ W, and try to find a way to get that amount of energy out over a long time.

The first estimate was due to Lord Kelvin (1862, in Macmillan's Magazine). This estimate (known now at the Kelvin-Helmholtz time, $t_{\rm KH}$) was shown to be $< 100$ Myr. But Darwin said (at the time) that fossils were at least 300 Myr old. So something weird was going on. Kelvin's estimate may have been wrong by a bit, but it couldn't be that bad. So there had to be some unknown energy source.

The lifespan of the Sun could be due to:

1. Chemical energy
2. Gravitational energy
3. Thermal energy (could it have just been a lot hotter in the past?)

4. Fission?

The answers for all of these are no. Kelvin's estimate concerned specifically gravitational. Chemical energy isn't enough, since we know about how much chemical energy a given reaction can release for a given amount of stuff. Same with fission.

### VIII.3.1 Gravito-Thermal Collapse, or the Kelvin-Helmholtz Timescale

This ascribes the luminosity to the change in total energy: $L$ is change in $E = K + U$.

If you do this you get a timescale of $t_{\rm KH} \sim 10^7$ yr, which is $\gg t_{\rm ff}$:

$$t_{\rm KH} \sim \frac{E}{L}$$

But $E \sim GM_\odot^2/R_\odot \sim 10^{41}\ {\rm J} = 10^{48}\,{\rm erg}$ (1 J=$10^7$ erg).

That is because as collapse occurs, $|U|$ increases so $K$ increases too. That heats up the star, which slows down the collapse.

We can use the Virial theorem to get the central temperature $T_c$ of the Sun. We assume that the center (the hottest/densest bit) dominates $K$:

$$K \sim \frac{3}{2} k_B T_c \frac{M}{\mu m_H}$$

And $K = -U/2$, with $U \sim -GM_\odot^2/R_\odot$. So we find $T_c \sim GM_\odot m_H/k_B R_\odot \sim 10^7$ K. This is pretty good (the real number is about $1.6 \times 10^7$ K).

# Lecture IX To Make A Star

1. Support against gravity ($P$ from HSE, $E$ from Virial theorem)
2. Source of energy: nuclear

We can exclude all forms of energy besides nuclear fusion from powering the Sun. How does fusion work?

### IX.2 Fusion

Kutner 9.3

What this boils down to is $E = mc^2$: if you can get rid of a bit of mass, you liberate a lot of energy.

**atomic unit** $u = 1.66054 \times 10^{-27}$ kg (mass of $^{12}$C/12)

**proton** $m_p = 1.6726 \times 10^{-27}$ kg = $1.007u = 938.8$ MeV/$c^2$

**neutron** $m_n = 1.6749 \times 10^{-27}$ kg = $1.0087u$

**electron** $m_e = 9.1 \times 10^{-31}$ kg = $0.0055u$

**hydrogen** $m_H = 1.0078u = m_p + m_n - \mathrm{electrostaticbindingenergy}/2$

**He nucleus** $m_\alpha = 4.002u = 2m_p + 2m_n - \Delta m$, with $\Delta m = 0.03u \sim 0.7\% \times (4m_H) \approx 28\,\mathrm{MeV}/c^2$

So going from 4 protons to 1 He nucleus ($\alpha$ particle) releases 28 MeV. This is the energy released by fusion.

We can think of the binding energy as the energy released when you form something (a nucleus in this case), or as the energy that is required to break something up.

$^{1}$**H** : $E_b = 0$

$^{4}$**He** : $E_b = 28$ MeV = 7.08 MeV/nucleon

$^{16}$**O** : $E_b = 7.97$ MeV/nucleon

$^{56}$**Fe** : $E_b = 8.798$ MeV/nucleon

$^{238}$**U** : $E_b = 7.3$ MeV/nucleon

$^{56}$Fe has the highest binding energy, so it's the most stable. Elements that are lighter or heavier are less stable. This means that reactions would naturally squeeze lighter elements together into Fe (fusion) and break heavier elements apart (fission).

### IX.2.1 Basic Nuclear Physics

1. Binding Energy: ${}^A_Z X$, with $A$ the number of nucleons, and $Z$ the number of protons. $E_b = (Zm_p + (A-Z)m_n - m_{\rm nuc})c^2$
2. Strong force: binds nuclei together against Coulomb (electrostatic) repulsion (since protons are positively charged)
3. $A \lesssim 56$: strong force increases faster when $A$ increases than Coulomb forces, so a larger $A$ leads to nuclei that are more bound.
4. $A \gtrsim 56$: the opposite

So fusion builds nuclei up to Fe, while fission breaks them down.

#### IX.2.1.1 H → Fe

This gives about 9 MeV/nucleon. Going from H to He gets 7 (or about 0.7% of $mc^2$). Going to O gets about 8 (0.8%). Going to Fe gets about 1% of $mc^2$ which is the most that fusion can do.

So for each proton you get $\approx 1\% mc^2 \sim 10^{-12}$ J (which means that 1 g of H could supply the annual energy of an american).

Fusion in the Sun: $10^{-12}$ J $\times M_\odot/m_p \sim 10^{45}$ J $\gg GM_\odot^2/R_\odot$. $t_{\rm nuc} \sim E/L_\odot \sim 10^{11}$ yr, so the Sun could shine for that long.

The actual lifespan is about $10^{10}$ yr (and it's lived about half of that) for a few reasons:

- $L_\odot$ increases later in life
- Not all H is burned
- It does not get hot enough to burn all the way to Fe

But it is clear that $t_{\rm nuc} \gg t_{\rm KH} \gg t_{\rm dyn}$

# Lecture X How to Power the Sun

The basics are $4 \times^1 \mathrm{H} \rightarrow^4 \mathrm{He} + 28\,\mathrm{MeV}$, or releasing $\sim 7\,$MeV/$A$. But this has some problems.

### X.1.2 Problem 1:

coulomb repulsion is strong. In order to have fusion you have to force together multiple hydrogen nuclei. These are protons, and are all positively charged. The strong force can only overcome the repulsion when the protons are *very* close together: $\sim 1\,$fm$= 10^{-15}$ m (for comparison, an electron orbits at $10^{-11}$ m).

The *classical* (not quantum) solution to this is that protons get close just because of their motion. They are hot, so they zip around pretty quickly. Sometimes they will approach each other, and this may happen. Can we tell how much?

The Coulomb potential is: $U_C = \frac{1}{4\pi\epsilon_0}\frac{e_1 e_2}{r}$, and energy will be conserved when the protons approach. So if they are travelling fast far away, as they approach the potential barrier they slow down:

$$\frac{1}{2}m_p v_\infty^2 + U_C(\infty) = U_C(1\,\mathrm{fm})$$

taking the limiting case that they have used all of their kinetic energy to get close enough. This gives us a requirement:

$$\frac{1}{2}m_p v_\infty^2 \geq \frac{e^2}{4\pi\epsilon_0(1\,\mathrm{fm})}$$

where we can also relate $\frac{1}{2}m_p v_\infty^2 = \frac{3}{2}k_B T$. So we need

$$T \geq T_{\mathrm{classical}} = \frac{e^2}{6k_B\pi\epsilon_0(1\,\mathrm{fm})} \sim 10^{10}\,\mathrm{K}$$

This is pretty hot, given that we know $T_c \sim 10^7\,$K. So the center of the Sun is not hot enough to sustain nuclear fusion!?

In fact, Arthur Eddington proposed nuclear energy as a power source, but others thought stars were not hot enough. Eddington said: “I am aware that many critics consider the stars are not hot enough. The critics lay themselves open to an obvious retort; we tell them to go and find a hotter place.”

In the end, Eddington was right!

### X.1.3 Quantum Mechanics

The problem is that the temperature above was the *classical* result, but quantum mechanics are important here. In particular, we need to consider wave-particle duality and the Heisenberg uncertainty principle: the size of a particle depends on its momentum. Classically you could think of a particle at a certain place with velocity $v$ (and hence momentum $mv$); but in a quantum sense you need to consider that the particle's position is only known to a de Broglie wavelength $\lambda_B \sim h/p \sim h/mv$, where $h$ is Planck's constant.

When two particles are within $\lambda_B$ of each other, there is a finite probability that they will "tunnel" to within 1 fm of each other ("tunneling" through the potential barrier):

$$U_c = \frac{e^2}{4\pi\epsilon_0\lambda_B} \leq \frac{1}{2}m_p v_\infty^2 = \frac{1}{2}\frac{p^2}{m_p} \sim \frac{1}{2m_p}\left(\frac{h}{\lambda_B}\right)^2$$

Note that we have replaced 1 fm with $\lambda_B$. We then get a constraint on $\lambda_B$ of:

$$\lambda_B < \frac{4\pi\epsilon_0 h^2}{2e^2 m_p} \sim 10^{-13}\,\mathrm{m} = 100\,\mathrm{fm}$$

which is much bigger than the classical result. So we can be $100\times$ further away and still have fusion. This gives us a much gentler requirement for the temperature as well:

$$\frac{3}{2}k_B T = \frac{1}{2}m_p v_\infty^2 = \frac{1}{2m_p}\left(\frac{h}{\lambda_B}\right)^2$$

Which gives:

$$T \geq T_{\mathrm{quantum}} \sim \frac{m_p e^4}{12\pi^2\epsilon_0^2 k_B h^2} \sim 10^7\,\mathrm{K}$$

which is OK!

So fusion is possible because of *quantum tunneling* at $10^7$ K for the Sun. But, if $M < 0.08\,M_\odot$, then it cannot even get this hot and fusion is impossible. Such objects are failed stars known as *brown dwarfs*.

Quantum tunneling being possible does not mean that it always happens. The probability is $\sim e^{-2\pi^2 U_c/k_B T}$. So for $10^7$ K, the probability is only $10^{-8}$ for any two protons, and it goes up to 1 for $10^{10}$ K. But there are enough protons and the fusion rate increases quickly with temperature that you have *ignition* at $10^7$ K.

### X.1.4 How Often Do Protons Get Close Enough?

At the center of the Sun the density is $\rho \sim 100\,\mathrm{g\,cm^{-3}}$, so the mean separation $l \sim n^{-1/3}$ is $\sim 10^{-11}$ m which is $\gg \lambda_B$. To have fusion with $4 \times {}^1\mathrm{H} \rightarrow {}^4\mathrm{He}$ we need 4 protons to get very close together, which is hard.

This reaction is not one that needs 4 protons at once, but it is actually a sequence of 2-body reactions. At "low" temperatures it is the *proton-proton* chain (p-p chain), which at "higher temperatures" there is the *carbon-nitrogen-oxygen* (CNO) chain. This does not burn CNO, but uses them as catalysts.

For later fusion reactions to build heavier elements, the higher charges on the nuclei (more protons) need higher temperatures to get close enough. For instance, to fuse He need $10^8$ K. Others need close to $10^9$ K. These will happen after the central H has been consumed.

#### X.1.4.1 p-p chain

The main sequence in the Sun. The net reaction is:

$$4^1\mathrm{H} \rightarrow^4 \mathrm{He} + 2e^+ + 2\nu_e + 2\gamma$$

Which is actually:

$$\begin{aligned}
{}^1\mathrm{H} +^1 \mathrm{H} &\rightarrow {}^2\mathrm{H} + e^+ + \nu_e \\
{}^1\mathrm{H} +^1 \mathrm{H} &\rightarrow {}^2\mathrm{H} + e^+ + \nu_e \\
{}^2\mathrm{H} +^1 \mathrm{H} &\rightarrow {}^3\mathrm{He} + \gamma \\
{}^2\mathrm{H} +^1 \mathrm{H} &\rightarrow {}^3\mathrm{He} + \gamma \\
{}^3\mathrm{He} +^3 \mathrm{He} &\rightarrow {}^4\mathrm{He} + 2^1\mathrm{H}
\end{aligned}$$

#### X.1.4.2 CNO chain

For more massive stars:

$$\begin{aligned}
{}^{12}\mathrm{C} +^1 \mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^+ + \nu_e \\
{}^{13}\mathrm{C} +^1 \mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} +^1 \mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^+ + \nu_e \\
{}^{15}\mathrm{N} +^1 \mathrm{H} &\rightarrow {}^{12}\mathrm{C} +^4 \mathrm{He}
\end{aligned}$$

Notice that the CNO all stay the same throughout: anything that is produced is consumed and vice versa.

# Lecture XI The Nucleus

We want to explain the binding energy in $m = Zm_p + (A - Z)m_n - E_b/c^2$, where the nucleus has $Z$ protons and $A - Z$ neutrons, for a total number $A$ nucleons.

## XI.2 The Liquid Drop Model

$$E_B \approx a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z)$$

Let's look at each term:

$a_V A$ : this is a volume term, since for constant density nucleons the volume will be $\propto A$. This covers the binding due to eht strong force, which is $\propto A$

$a_S A^{2/3}$ : this is a surface term. For a volume $\propto A$, the surface area will be $\propto A^{2/3}$. It works as a correction to the volume term since the nucleons near the outside will have fewer other nucleons to interact with.

$a_C \frac{Z(Z-1)}{A^{1/3}}$ : this is the Coulomb term, showing the strength of electrostatic repulsion which is $\propto 1/r \sim 1/A^{1/3}$

$a_A \frac{(A-2Z)^2}{A}$ : this is an assymetry term, where nuclei with $A \approx 2Z$ are more bound

$\delta(A, Z)$ : this is a pairing term

Overall this makes stable nuclei with $Z \approx A/2$, and says that the most bound nuclei are near $A = 60$. It is obviously a simplification, but it can be improved with the addition of a *shell model* (like for electrons) and using empirical data to set the various constants.

# Lecture XII Radiation

Astronomy is based (so far) on observing light from objects. This means we see photons (electromagnetic radiation). Which give us information about temperature, density, chemical composition, etc.

We will discuss:

1. Diffusion and random walks
2. Blackbodies and temperature
3. Photospheres and energy transport

### XII.2 Photons

Kutner 2.2, 2.4

Photons are light particles, but they also behave as waves. Each photon has energy $E = h\nu$ (with $\nu$ the frequency in Hz) or $E = hc\lambda$ (with $\lambda$ the wavelength). This comes from the definition $c = \lambda\nu$.

We divide up the electromagnetic spectrum into wavelength regions:

**$\gamma$-rays** : $\lambda < 0.01\,\mathrm{nm}$

**X-rays** : $1\,\mathrm{nm} \rightarrow 10\,\mathrm{nm}$

**ultraviolet (UV)** : $10\,\mathrm{nm} \rightarrow 400\,\mathrm{nm}$

**optical (visual)** : $400\,\mathrm{nm} \rightarrow 700\,\mathrm{nm}$

**infrared (IR)** : $700\,\mathrm{nm} \rightarrow 1\,\mathrm{mm}$

**radio** : $> 1\,\mathrm{mm}$

### XII.3 Diffusion & Random Walks

Heat (energy) is produced at the centers of stars through nuclear reactions. It escapes ultimately as photons. How long does that take? A naive answer is $\sim R_\odot/c = 2\,\mathrm{s}$.

But that is wrong (although it is true for neutrinos). It actually takes $\sim 10^7$ yrs. Why? Because a star is a very crowded place, and photons (even though they move fast) cannot move very far before they wack into something else and end up going in another direction. They easily bounce (scatter) off of ions, electrons, and atoms, and even other photons.

Each bounce tends to make the photon lose energy, but more photons are then produced, conserving energy. In the center the photons start out as X-ray photons, but by the time they get to the surface of the star they are optical photons. They get there via a *random walk*.

Assume that a photon will move (on average) a distance $l_{\rm mfp}$ before it hits something and changes direction. That distance is the *mean free path*. It travels a distance $d$ after $N$ collisions. We can determine what $d(N)$ is. Assume each one moves $\vec{l}_i$ for $i = 1 \ldots N$, with $|\vec{l}_i| = l_{\rm mfp}$. So the total distance is the vector sum:

$$\vec{d} = \sum^{N} \vec{l}_i$$

We want the magnitude of this, $|\vec{d}| = \sqrt{\vec{d}\cdot\vec{d}}$. But

$$\vec{d}\cdot\vec{d} = \sum_i^N \vec{l}_i \cdot \vec{l}_i + \sum_{i \neq j} \vec{l}_i \vec{l}_j$$

The second term there will go to 0 on average, since the directions are different. So $|\vec{d}|^2 = N|\vec{l}| = N l_{\rm mfp}$, or $d = \sqrt{N} l_{\rm mfp}$. This is in fact a general result with applicability to a wide range of areas.

From this we can determine how long does it take for a photon to diffuse out of the star. To go a distance $d$, it takes:

$$\begin{array}{lcl} & \frac{d}{c} & l_{\rm mfp} > d \\ N\frac{l_{\rm mfp}}{c} = & \frac{d^2}{l_{\rm mfp}c} & l_{\rm mfp} < d \end{array}$$

This is also often referred to as a “drunkard’s walk”.

## XII.4 What Happens When A Photon Hits Something?

- Photon A generates an oscillating electromagnetic field
- Matter (ion, electron, atom) is shaken by that field (absorbing the photon)
- But this shaking is itself a fluctuating field, so it makes a new EM field, releasing photon B

There are 3 basic types of interactions:

1. Scattering: A and B have the same energy (or frequency) but different directions. So the matter gains momentum but no energy
2. Absorption: no photon B is emitted. Matter absorbs energy and generally something happens to it
3. Emission: matter emits B (and it can be with or without A)

We take each obstacle as having cross-sectional area $\sigma$ (in units of m$^2$). And we have $n$ of them per $m^3$ (number density $n$). So what is $l_{\rm mfp}$?

1. A dimensional analysis: $l_{\rm mfp}$ [m] could be $1/\sigma n$, $\sigma^2 n$, $n^{-1/3}$, ...

2. Physical: shoot a bullet into a box of total area $A$ and depth $l_{\rm mfp}$. It has $N$ balloons in it each with area $\sigma$. The bullet is likely to hit a balloon if $N\sigma = A$. $N = n \times {\rm volume} = n l_{\rm mfp} A$. Equating $n l_{\rm mfp} A\sigma = A$ gives $l_{\rm mfp} = 1/n\sigma$

3. How big are the balloons (what is $\sigma$)?

   **electrons** $10^{-28}\,{\rm m}^2$

   **H atoms** $10^{-20}\,{\rm m}^2$

   So for $n \sim 10^{30}\,{\rm m}^{-3}$ (which you get for $\rho = 10^3\,{\rm kg\,m}^{-3}$ like for water), we find $l_{\rm mfp} \sim 10^{-11}$ m to $10^{-2}$ m, both of which are $\ll R_\odot$. Therefore there are many bounces:

   **electrons** : $l_{\rm mfp} \sim 10^{-2}$ m, so $t \sim 5000$ yr ($10^{22}$ bounces)

   **H atoms** : $l_{\rm mfp} \sim 10^{-11}$ m, so $t \sim 5 \times 10^{12}$ yr ($10^{40}$ bounces)

   The actual answer is about $10^7$ yrs, taking into account the changing structure and density of the Sun.

## XII.5 Temperature of Radiation

Kutner 2.3

Temperature is defined for ideal objects ("blackbodies") that radiate a *universal* spectrum: the emission depends only on temperature.

To do so, it must absorb all of the light that hits it (hence *black*). But it can appear to have a color when it is hot (like an oven).

Blackbody is a specific shape **sketch**. The peak is at $\lambda = 0.0029/T$ m, which determine the color. The total energy put out per square meter (the flux) is $F = \sigma T^4\,{\rm W\,m}^{-2}$. These are the *Wien displacement law* and *Stefan-Boltzmann* law ($\sigma$ is sb constant). The important thing is that only $T$ matters.

**Wikipedia page, animations**.

$F$ is energy per time per area. If the object is a sphere (like a star) it has total area $A = 4\pi R^4$, so total energy per time (luminosity $L$) is $L = 4\pi R^2 \sigma T^4$. This is a very useful expression.

What are "good" blackbodies? Nothing is perfect, but some things are close:

- 3 K cosmic microwave background
- surface or interior of a star
- human skin
- candle
- lava

What are "bad" blackbodies? These don't have a nice smooth distribution, but instead concentrate the light at specific wavelengths:

- neon light
- fluorescents

But even the Sun isn't perfect.

### XII.5.1 Planck Function

The detailed function that describes how much light at each wavelength:

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{h\nu/k_BT} - 1}\ \mathrm{J/s/m^2/\lambda}$$

Note that $h$ appears, so this has to be a quantum effect. $B_\lambda$ is energy per time per area *per wavelength*: if you have a bigger range of wavelengths (i.e., red and green) then you get more energy.

#### XII.5.1.1 Limits

**sketch**

$$\frac{hc}{\lambda}\frac{1}{k_BT} \gg 1$$

(long wavelength, low frequency) is the Rayleigh-Jeans limit:

$$B_\lambda(T) \approx \frac{2ck_BT}{\lambda}$$

Note that there is no more $h$: this limit can be derived classically.

$$\frac{hc}{\lambda}\frac{1}{k_BT} \ll 1$$

(short wavelength, high frequency) is the Wien limit:

$$B_\lambda(T) \propto e^{-hc/\lambda k_BT}/\lambda^5$$

If you add up $B_\lambda(T)$ for all $\lambda$, you recover the Stefan-Boltzmann law.

## XII.6 Photosphere

What is the surface of a star? What is the temperature of a star?

After all, it's a flaming ball of gas. It's hotter on the inside (Sun is $10^7$ K), so why do we see it as cooler on the outside? What defines the temperature that we measure (about 5800 K)?

Photosphere is defined as the "surface", it is where $T(\mathrm{depth}) = T_{\mathrm{effective}}$. We define $T_{\mathrm{effective}}$ such that $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$. This is the layer from which photons escape (stop bouncing or scattering). They end up one mean free path from the surface, and from there they are free!

This is where the star leaves an imprint on the photons that escape. Everything that happens deeper down gets washed away from multiple scatterings.

$$l_{\mathrm{mfp}} \sim \frac{1}{n\sigma} \sim H$$

$H$ is pressure scale heigh - the height at which pressure changes by $e$: $P \propto e^{-r/H}$. Since we have HSE ($\Delta P/\Delta r = -\rho g$), we can say $\Delta r = H$ and find $P_{\mathrm{phot.}} \sim \rho g H \sim \frac{g\rho}{n\sigma} \sim \frac{g\mu m_H}{\sigma}$. This is about $10^7\,\mathrm{N\,m^{-2}}$ for the Sun.

# Lecture XIII Photons & Spectra

Kutner Chapter 3

Spectra: disperse light through "prism", spread it out so we can see each wavelength separately.

Stars generally have *absorption line*: most of the wavelengths are bright, but a few specific wavelengths are dark. To understand this, need Kirchoff's laws:

- Hot background, cold foreground: absorption lines
- Cold background, hot foreground: emission lines

What matters is what is in front. What is in front of the star? It's is that it is hotter on the inside than the outside. So the spectrum of a star is (mostly) a blackbody with some wavelengths absorbed. These wavelengths were identified before we knew what caused them.

Fraunhofer lines: lines in Sun from things like Na, Ca. But there are also lines from H, He that are very important.

Cecilia Payne was one of the first people to identify the spectral lines in the Sun (and other stars). She showed that the elements in the Sun were very different from those on Earth: here we have almost no free H, but that is the majority of what's in the Sun.

## XIII.2 Energy Levels for H

proton + electron in Bohr model (approaching proper quantum mechanics, but not quite): "planetary" orbits. Instead of Gravity, Coulomb force:

$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

and we use the Virial theorem again, so $E = K + U = -U/2$. This would have infinite choices for $r$: anything is OK. The Bohr model says that $r$ can only have particular values that are *quantized*. What is necessary is that, if you take a de Broglie wavelength, the orbit starts and stops in the same part of a wave. You can also write this as:

$$J = m_e v r = n\hbar$$

is the angular momentum. $\hbar = h/2\pi$, so this is quantum mechanical. If you do this, you get discrete energy levels for $n = 1, 2, 3, \ldots$:

$$\frac{-1}{4\pi\epsilon_0}\frac{e^2}{2r} = -\frac{1}{2}m_e v^2 = -\frac{1}{2}\frac{(n\hbar)^2}{m_e r}$$

This can only be true at certain values of $r$:

$$r = r_n = 4\pi\epsilon_0\frac{\hbar^2 n^2}{m_e e^2} \approx 0.5\,\text{Å}\,n^2$$

The energy levels associated with this are:

$$E_n = \frac{1}{n^2}\frac{-m_e e^4}{2(4\pi\epsilon_0\hbar)^2} = \frac{-13.6\,\mathrm{eV}}{n^2}$$

The constant $13.6\,$eV is the ionization energy of H, known as a Rydberg. Why does this ionize? Start at $n = 1$. How much energy to get to infinitely far away? This would take us to $r = \infty$, so $n = \infty$. The difference in energy levels is how much energy it would take:

$$\Delta E = E_1 - E_\infty$$

But since $E_\infty = 1/\infty = 0$, this is just $E_1$ or 13.6 eV.
**http://astro.unl.edu/classaction/animations/light/hydrogenatom.html**

### XIII.3 Photon & Matter: Spectral Lines

Spectral lines are associated with transitions between energy levels. See in both absorption and emission.

For example, to excite an atom from $n = 1$ to $n = 2$ takes $\Delta E = hc/\lambda = E_2 - E_1 = (-3.4\,\mathrm{eV} - (-13.6\,\mathrm{eV}) = 10.2\,$eV. THis gives a wavelength of $\lambda = 1216\,$Å. Lyman$\alpha$. **Sketch Ly, Balmer, Pa, Brackett**. Ly$\alpha = 1216\,$Å, H$\alpha = 6563\,$Å, P$\alpha = 18,700\,$Å, Br$\alpha = 40,500\,$Å.

Emission lines: hot gas on cool background (neon light).

Absorption lines: cool gas in front of hot background.

Some astronomical objects are primarily spectral line emitters. e.g., planetary nebulae and HII regions: clouds of hot gas, where most of the emission is just what we've described.

# Lecture XIV Main Sequence

Kutner 3.5, 9

**sketch** $L$ **vs.** $T$

temperature from color of star. luminosity from how bright it appears, combined with some knowledge of distance. From this we get *color-magnitude diagram*. Also *Hertzsprung-Russel* diagram. **main sequence** is where they sit for most of the time. Stars do not move along it: they end up at one point determined by their mass.

Stars were originally classified based on spectral lines. We have now been able to re-order that sequence in terms of temperature:

**O** : O5 is 40,000 K. H ionized, see lines of H, He

**B** : similar but cooler

**A** : A0 is 10,000 K, Vega (standard comparison star)

**F** : start seeing lines of H, "metals"

**G** : G2 is 5,800 K, Sun

**K** : start seeing molecules (star is cool, so they can be stable)

**M** : M0 is 4,000 K

Within each class goes from 0 (hottest) to 9 (coolest).

Taking stars together, we observe:

$$L \propto M^4$$

$$R \propto M^{0.76}$$

Simply put, more massive stars are bigger and (a lot) brighter. They also end up being hotter (via Stefan-Boltzmann law), $T_{\text{eff}} \propto M^{0.6}$.

### XIV.2 More massive is bigger and brighter

only mass matters along the main sequence: stars sit there doing almost nothing for most of their lives.

#### XIV.2.1 Why bigger?

$T_c$ roughly constant (ignition). Virial theorem gives us $T_c \propto M/R$, so then we would have $R \propto M$. Which is close, but actually $T_c$ goes up a bit, so instead we have $R \propto M^{0.76}$.

### XIV.2.2 Why brighter?

We can understand $T_c \propto M/R$, and $P_c \propto GM\rho/R$. The other piece we need is to look at how the photons get out of the star. Remember that they have to bounce around a lot. We can write:

$$\frac{T_c}{R} \sim \frac{L\rho}{T_c^3 R^2}$$

which is valid if the amount of time that it takes a photon to get out does not depend on $T$ (true in hot stars). This then gives us: $L \propto M^3$, which is close to what is observed.

### XIV.2.3 More massive has shorter life?

lifetime $\propto$ fuel/rate of consuption $\sim M/L$. Since $L \propto M^3$ or $M^4$, lifetime $\propto M^{-2}$ or $M^{-3}$.

At the low-mass end, energy is not transported by photons but by bubbles, so this reasoning (and these scalings) break.

## XIV.3 A Fundamental Unit for $M_*$

A star: gravity pushes nucleons together until fusion happens.

Gravitational energy $\sim Gm_H^2/r$ between two atoms. How far apart? $10^{-15}$ m after fusing, take $r \sim \hbar/m_H c = 2 \times 10^{-16}$ m for scaling.

$$\alpha_G = \frac{\frac{Gm_H^2}{\hbar/m_H c}}{m_H c^2} = \frac{Gm_H^2}{\hbar c} = 5.9 \times 10^{-39}$$

This is the strength of gravity between two nucleons compared to the rest-mass energy of the nucleon. Gravity is very weak! From this, we can derive:

$$M_* = \alpha_G^{-3/2} m_H = 1.85\, M_\odot$$

This is a natural unit for the masses of stars, and it only involves fundamental constants ($G, c, m_H, \hbar$). We can also get:

$$N_* = M_*/m_H = \alpha_G^{-3/2} = 2 \times 10^{57}$$

is the rough number of nucleons (mostly protons) in a star.

## XIV.4 What Limits What Could Be A Star?

### XIV.4.1 Minimum Mass

No fusion possible (not hot enough): $M < 0.08 M_\odot$ (brown dwarf)

### XIV.4.2 Maximum Mass

Star gets very hot inside, and $T_c$ increases as $M$ increases. The pressure due to the blackbody radiation inside increases a lot: $P_{\rm rad} \propto T_c^4$. When this pressure becomes too big, it will dominate over the normal gas pressure ($P \propto \rho k_B T$), and when it does the star becomes unstable: it will blow itself apart. For $M \sim 100\,M_\odot$, unstable, very hard to even form. Even for $\sim 50\,M_\odot$, very violent, short-lived.

# Lecture XV Life After the Main Sequence

Remember: stars do not move along the MS. They move onto it when they form, sit there for a long time, and then. . .

Main sequence is about where we find 80% of the stars. H in the core is burned into He. This is accompanied by a slow increase in $L$:

- $P = \rho k_B T/\mu m_H$, where $\mu$ is mean weight. 0.5 for H, higher for He.
- As H goes to He, $\mu$ goes up, so $P$ would go down if $T$ didn't go up.
- As $T$ goes up, fusion region increases, $L$ increases

Then what? End of MS is when H is done in core. After that: giants (bigger and brighter). Depends on mass.

### XV.2 Low Mass

$< 8\,M_\odot$

no more energy from core, so contracts, gets hotter (Virial theorem). Layers above core contract too, H burns in "shell" around He core. This makes "red giant": $R \sim 100R_\odot$, lasts $10^8$ yr. As this happens the star is a lot brighter too **draw HR**.

Eventually core contracts, $T_c \sim 10^8$ K, starts burning He via triple-alpha process:

$$^4_2\mathrm{He} + ^4_2\mathrm{He} \rightarrow ^8_4\mathrm{Be}$$

$$^8_4\mathrm{Be} + ^4_2\mathrm{He} \rightarrow ^{12}_6\mathrm{C} + \gamma$$

Unless the second reaction happens, Be will decay (break apart) so needs high density. Higher temperature because He has charge of +2, so Coulomb repulsion is 4 times as much as H.

When enough C, starts burning to O ($T$ higher still).

When He burning starts, star becomes bigger still: Asymptotic Giant Branch **Draw onion diagram, C/O core, He shell, H burning, H envelope**. $\sim 600R_\odot$ for $10^6$ yr, $10^5L_\odot$.

As this happens the star puffs off a lot of gas: larger $R$ means lower $GM/R$ on the surface, so stuff can escape. Goes from $10^{-14}M_\odot$/yr (Sun) to $10^{-7}M_\odot$/yr (RG) to $10^{-5}M_\odot$/yr (AGB). THis material sticks near the star for a time, forming a *planetary nebula* (nothing to do with planets), but enriches the surrounding area with "metals" formed in star. What's left at the center is C/O *white dwarf* (later).

Moves around on HR diagram, $L = 4\pi R^2\sigma T^4$.

### XV.3 High Mass

Burning keeps going to where Fe is produced. After that, cannot get energy out. Star collapses (*core collapse supernova*). **show onion**. Gravitational potential energy released ($10^{46}$ J), mostly as neutrinos. Some compact remnant (possibly) left behind.

# Lecture XVI Clusters: Testing Stellar Evolution

Most stars are not formed alone, but rather in groups (small) or clusters (large).

- Open cluster: young(ish), $10^3$ stars, in a size of ∼pc, $\sim 10^3$ known, in the Galactic disk
- Globular cluster: old, $10^5$ stars, still ∼pc, $\sim 200$ around the MW in a spherical cloud (halo). Often very *metal poor*: these stars were formed when the material had not been enriched by previous stellar burning (originally matter was almost all H, He)

In both, the stars are all born together from the same stuff at the same distance. So this removes a lot of the uncertainties that make astronomy hard. They are great testing grounds.

### XVI.2 Make HR diagram out of a single cluster

**sketch**. Can determine distance by finding how bright the stars at $T_\odot$ appear to be.

**http://astro.unl.edu/naap/distance/animations/clusterFittingExplorer.html**

Can determine age by looking at the highest-mass star that is still on the MS **Sketch**

Look for objects that are not on the MS: *binaries, dwarfs, etc.*

calibrate models of stellar evolution.

# Lecture XVII White Dwarfs

Kutner 10.4

leftover remnant from the core of a low-mass star ($< 8M_\odot$).

core gets hotter & denser, as heavier elements need higher $T$ to burn. As each phase of burning ends, collapse a bit, $T$ up, $P$ up. Can this keep going? *NO*

After get to O, $P$ will no longer depend on $T$, so you cannot keep getting more burning. Why?

### XVII.2 Degeneracy

electrons are *Fermions*: 2 cannot be in the same *state*. State = position, momentum, spin (Pauli exclusion principle). From the uncertainty principle:

$$(\Delta x)(\Delta p) \sim \hbar$$

$p$ is momentum, $m_e v$. Density is number per volume, or 1/volume per particle. So $n \sim 1/\Delta x^3$. When the particles get squeezed too close they start to overlap, get to where

$$p = p_F \sim \hbar/\Delta x$$

(Fermi momentum). Which will be $p_F \sim n_e^{1/3}$. We can then use the momentum to get the kinetic energy:

$$E_F = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e(p_F/m_e)^2 = \frac{1}{2}\frac{p_F^2}{m_e}$$

(note that this is only true if $v \ll c$). So $E_F \propto p_F^2 \propto n_e^{2/3}$. This is the total energy per particle. Pressure has the same units as energy per volume (energy density), so we can multiply energy per particle by density to get pressure:

$$P_F \propto n_e E_F \propto n_e^{5/3} \propto \rho^{5/3}$$

There is no $T$! Unlike ideal gas law ($P = n_e k_B T$), this is *not ideal*. Requires quantum mechanics. Ideal gas law $P = P(\rho, T)$, but here $P = P(\rho)$ only.

### XVII.3 Build a Degenerate Star

(This is a white dwarf).

HSE gives:

$$\frac{P}{R} \sim \frac{GM}{R^2}\rho$$

So

$$P \sim \frac{GM}{R}\rho \sim \rho^{5/3}$$

We can then get $M/R \sim \rho^{2/3}$, but $\rho \sim M/R^3$, so we have:

$$M^{1/3} \sim 1/R \; R \sim M^{-1/3}$$

This is weird. Unlike a star ($M \sim R$) or a normal rock ($R \sim M^{+1/3}$, since $\rho \sim$constant) or something, as $M$ increases, it gets smaller!

### XVII.3.1 When Does This Matter?

When electron spacing $\Delta x \sim \lambda_B$ (de Broglie wavelength from before). $\lambda \sim h/p \sim h/m_e v$, and $m_e v^2 \sim k_B T$, so

$$\lambda_B \sim 10^{-12}\,\mathrm{m}\left(\frac{10^9\,\mathrm{K}}{T}\right)^{1/2}$$

For $1M_\odot$ and $R = R_\oplus$, get $\Delta x \sim 10^{-12}$ m. Since $T < 10^9$ K (C burning), $\lambda$ is small enough to be degenerate, and $R_{\mathrm{WD}} \sim R_\oplus$.

So a White Dwarf has the mass of the Sun squeezed into something the size of the Earth.

### XVII.3.2 How Bright?

$L = 4\pi R_\oplus^2 \sigma T^4$, so even is $T$ is higher than the Sun, $R_\oplus \ll R_\odot$ and the WD will be very faint.

### XVII.3.3 Can A WD Be Any Size?

Can you keep piling mass on, or is there a limit?

$$\rho^{2/3} \sim \frac{M}{R} \sim \frac{M}{M^{-1/3}} \sim M^{4/3}$$

or $\rho \sim M^2$ in the center. So as you make it more massive, the density increases a lot. And as the density increases, so does the pressure and $E_F$. What happens when $E_F \sim m_e c^2$ (i.e., $v_e \sim c$)? Things get *unstable*. Before,

$$E_F \sim m_e v^2 \sim \frac{p_F^2}{m_e}$$

But now, including *special relativity*, $v \sim c$

$$E_F \sim p_F c \propto n_e^{1/3}$$

$$P \propto n_e E_F \sim n_e^{4/3}$$

Put that into $P \sim GM\rho/R \sim \rho^{4/3}$ and you find

$$\rho^{1/3} \sim \frac{M}{R}$$

But $\rho \sim M/R^3$, so $\rho^{1/3} \sim M^{1/3}/R$:

$$\frac{M^{1/3}}{R} \sim \frac{M}{R}$$

This cannot be true! When $v \sim c$ the velocity cannot increase fast enough to supply enough pressure, and the WD becomes unstable. This happens at the *Chandrasekhar Mass* $1.4M_\odot$: if it gets to this limit, it will collapse.

# Lecture XVIII Neutron Stars

## XVIII.2 Higher Masses: What Happens?

Kutner 11

Once we get to Fe in the core, we cannot get energy out. The star is still shining (so it's losing energy) but no longer creating it. So the core starts to cool. In order to support the rest of the star, the pressure needs to be the same, so the density goes up and up to keep the pressure constant.

If the mass of the Fe core is less than the Chandrasekhar mass, then electron degeneracy pressure can support the star. But once it gets past there that is not enough. At this point, it is roughly $5 \times 10^9$ K, 5000 km in radius.

As it gets to the Chandrasekhar mass, electrons cannot support the star. It starts to collapse. The collapse liberates some gravitational energy, but the Fe *absorbs* that energy, liberating protons. Protons take up extra space, and they are increasingly squeezed by the rest of the star.

Remember $\beta$ decay (nuclear decay):

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

This happens spontaneously for various nuclei. The inverse can also happen, although it isn't spontaneous:

$$p^+ + e^- \rightarrow n + \nu_e$$

When things get too dense, the inverse reaction is energetically favorable. This makes a bunch of neutrons, removing electron support. Neutrinos also leak out, removing energy.

So the core collapses down in a free-fall timescale of $\sim 1/\sqrt{G\rho} \sim 10\,$s or less. This reaction happens at a density of $\sim 10^9\,\mathrm{kg\,m^{-3}}$. Much of the star is blown off in a *supernova explosion*: the gravitational energy of $10^{46}$ J is released mostly as neutrinos, with a small amount creating a blast wave of material moving at 10,000 km/s.

## XVIII.3 Core Collapse

It gets squeezed down to $\sim 10\,$km. This is a core-collapse SN (there are other kinds), and happens for $M < 25M_\odot$ or so. A lot of the gravitational energy is releasd:

- $10^{46}$ J total
- $10^{44}$ J is in the KE of the ejected material
- $10^{43}$ J is in photons ($10^{10}L_\odot \sim L_{\rm galaxy}$ for 10 days)

For example, SN1054 which is 2 kpc away was seen during the day quite easily.

99% of the energy (or more) goes out as neutrinos ($10^{19} L_\odot$). About 20 were detected from a SN 50 kpc away.

A lot of the blasted away material goes into the nearby interstellar space, enriching it with metals that were made in the star. THis is how elements heavier than C/O get into the universe.

## XVIII.4 What Remains

Kutner 11.2

A neutron star is like a WD, except instead of electron degeneracy, the reaction $p + e \rightarrow n$ makes neutrons, and neutron degeneracy pressure supports it. For $1.4 M_\odot$ the size is about 10 km. This gives $\rho \sim 10^{17-18}\,\mathrm{kg\,m^{-3}}$, compare to a nucleus is $2 \times 10^{17}$.

$$\rho_{\rm WD} \sim \frac{m_H}{(h/m_e c)^3} \sim 10^8\,\mathrm{kg\,m^{-3}}$$

$$\rho_{\rm NS} \sim \frac{m_H}{(h/m_p c)^3} \sim 6 \times 10^{17}\,\mathrm{kg\,m^{-3}}$$

We are confining one nucleon (proton or neutron) in a box. For the WD the size of the box is the de Broglie wavelength of the electron. For the NS it's the de Broglie wavelength of the neutron. Since the wavelength is $\sim 1/m$, the neutron's box is much smaller.

Consider a nucleus. The distance between neutrons is $r_0 \sim 10^{-15}$ m. If you take a Solar mass in neutrons, that means $A \sim M_\odot / m_{\rm H} \sim 10^{57}$ neutrons, so the size is $R_{\rm NS} \sim r_0 A^{1/3} \sim 10$ km.

The properties of this object: surface gravity $g \sim 10^{12}\,\mathrm{m\,s^{-2}}$, escape speed $\approx 0.6c$.

The interior is very complicated: still under investigation. Likely superconducting (no electrical resistance) superfluid (no friction).

We have an upper limit to neutron star mass: keep $v_n < c$ gives a limit of $2 - 3 M_\odot$ (details are hard).

### XVIII.4.1 Spin

Angular momentum $J = I\omega$, $\omega = 2\pi/P$. Conserved. What made the NS?

$$\frac{R_{\rm core}}{R_{\rm NS}} \sim \frac{m_e}{m_n} \left(\frac{Z}{A}\right)^{5/3} \sim 500$$

going from the core (supported by electron degeneracy) to the NS. Conserving $J$:

$$I_{\rm core}\omega_{\rm core} = I_{\rm NS}\omega_{\rm NS}$$

and $I \approx MR^2$. So

$$\omega_{\rm NS} \sim \omega_{\rm core} \left(\frac{R_{\rm core}}{R_{\rm NS}}\right)^2$$

since the mass is the same. Or, going to period:

$$P_{\rm NS} \sim 4 \times 10^{-6} P_{\rm core}$$

Much faster! Core can rotate as fast as 30 min, so $P_{\rm NS}$ can be as little as 5 ms.

### XVIII.4.2 Magnetic Field

Instead of angular momentum, conserve magnetic flux $\Phi = BR^2$. Same arguments give $B_{\rm NS} \sim 250,000 B_{\rm core}$. What might the initial field be? We measure fields of 10 T in white dwarfs, so the field in a NS could be up to $10^7$ T in NS. In fact we see NSs with fields up to $1000\times$ this.

### XVIII.4.3 A Limit To Rotation?

How fast can a NS rotate? 2 limits:

- Keep the equator $< c$
- centripetal force $<$ force of gravity

First one:

$$\frac{2\pi R}{P} = v_{\rm equator} < c$$

would give a limit of $2\pi R/c = 0.2$ ms.

The second:

$$\frac{v^2}{R} = \frac{GM}{R^2} = \frac{4\pi^2 R}{P^2}$$

would give a limit $P < \sqrt{4\pi^2 R^3/GM} = 0.4$ ms. (this is actually Kepler's third law, $P^2 \propto R^3$).

## XVIII.5 Pulsars

1967 Jocelyn Bell, looking for "twinkling" radio sources. Found something that "blipped" every 1.337 s. It was from the sky: it came every day at the same *sidereal time* (not LGM). It was too fast and too regular to be anything big (white dwarf, binary star, etc.)

More of these were soon found. Some were in supernovae remnants: the clouds of gas flying out at 10,000 km/s from where supernova explosions occured.

Crab Nebula: glowing cloud of gas. Needs $10^5\,L_\odot$ to power it. Found pulsating radio source inside with $P = 33$ ms. But they found that while regular, $P$ was getting longer very slowly, change at $\dot{P} = dP/dt = 4 \times 10^{-13}\,{\rm s\,s^{-1}}$.

What would rotational energy be of neutron star spinning at $\Omega = 2\pi/P$? $(1/2)I\Omega^2$, $I \approx 10^{38}\,{\rm kg\,m^2} = (2/5)MR^2$ is moment of interia. What if the rate of spinning is slowing? Then it is losing kinetic energy at a rate $\sim I\Omega\dot{\Omega}$. If we do this, we find this is $5 \times 10^{31}$ W is about

$10^5 L_\odot$: just right! This showed that the spin-down of a neutron star is what powers the Crab nebula (Tommy Gold).

Overall, pulsars were found to be rotating neutron stars. We see blips when the "lighthouse" beam crosses the Earth **show animation**. The majority of the energy from the spin-down is invisible: the radio blips are a tiny fraction of the energy.

It is the strong magnetic field that makes this happen.

### XVIII.5.1 Spin-Down

(please pardon the calculus)

Light cylinder: where $v$ to go around is $c$. We take the magnetic field to be a dipole, $B(r) = B_0(r/R)^{-3}$. A changing magnet releases electromagnetic power per unit area $S$ (*Poynting flux*) $\sim cB^2/\mu_0$. We can roughly relate the spin-down energy loss $I\Omega\dot{\Omega}$ to the Poynting flux through the light cylinder:

$$4\pi R_{\rm LC}^2 S_{\rm LC} \approx I\Omega\dot{\Omega}$$

with $\Omega = 2\pi/P$, $\dot{\Omega} = -2\pi\dot{P}/P^2$. $R_{\rm LC} = cP/2\pi$, so $S_{\rm LC} = (c/\mu_0)B_{\rm LC}^2 = (c/\mu_0)B_0^2 R^6/R_{\rm LC}^2$. So we have:

$$4\pi R_{\rm LC}^2 \frac{c}{\mu_0} B_0^2 \frac{R^6}{R_{\rm LC}^6} = 4\pi R^6 \frac{c}{\mu_0} B_0^2 \left(\frac{cP}{2\pi}\right)^{-4} \sim \frac{R^6}{\mu_0}\frac{B_0^2}{c^3}P^{-4} \sim I\frac{\dot{P}}{P^2}$$

This gives:

$$B_0^2 \sim \frac{c^3\mu_0 I}{R^6} P\dot{P}$$

So from the spin period and the rate at which it is slowing down, we can determine what the magnetic field is!

We can then use this (assuming $B$ =constant) to get $P(t)$. We find that the age is $\tau \approx P/2/\dot{P}$, so we also get the age of the system from $P$ and $\dot{P}$. Do this for the Crab pulsar get 1250 years, which is very close to the true age of about 950 years (since people saw the supernova).

$P$-$\dot{P}$ diagram: HR diagram for pulsars. **draw**. Move through the diagram from upper left to lower right until you die from low voltage (don't actually die, just shut off). This takes $10^{7-8}$ yrs to get to $P = 10\,$s from a typical starting point of 10 ms. Usually born with $10^8$ T, but there is a range.

### XVIII.5.2 What happens after death?

Not much, unless in a binary star system. If in a binary: after the pulsar dies (remember, this still happens quickly compared to a main-sequence lifetime), the second star will evolve. It will leave the MS and puff up into a RG. When this happens, the outer bits of that star may get captured by the gravity of the NS **draw**. This is called *Roche-lobe overflow* (remember Roche from the tidal forces?). It leads to *accretion* onto the NS. That dumps angular momentum and mass onto the NS.

In the early 80's, they found pulsar with $P = 1.6\,$ms, $P/2\dot{P} = 200\,$Myr. So it couldn't have been born with that low a period since it is way too old. How could it get there? It was *recycled* into a *millisecond pulsar* **draw**. For these, $B$ is much weaker ($\sim 10^5$ T) although we do not really know why.

### XVIII.5.3 Hulse-Taylor Binary

The second star in a binary can also eventually become a NS. The H-T binary is one such system: two NSs in a 8-hr orbit. But, with General Relativity we predict that such a system will lose energy, angular momentum due to *gravitational radiation* (like a moving charge emits EM radiation). The change in the period of the binary was observed from very precise measurements: 1993 Nobel prize in Physics.

Will merge in 300 Myr: explosion and burst of GW.

# Lecture XIX Gravitational Redshift & Black Holes

Kutner 8.3, 8.4

a photon needs to climb out of a potential well. When that happens it loses energy. A rocket climbing out of a well would slow down, when it loses energy, but photons must go at $c$, so change frequency instead.

**draw**

If we start at $R_1$ and go to $R_2 = \infty$: starts with $m = E/c^2 = h\nu_1/c^2$, so the total energy of the system is $h\nu_1 - GmM/R_1$ and it ends with the same total energy

$$h\nu_1 - G\frac{h\nu_1 M}{c^2 R_1} = h\nu_2 - \frac{h\nu_2}{c^2\infty}$$

which gives:

$$h(\nu_2 - \nu_1) = h\Delta\nu = \frac{Gh\nu_1 M}{c^2 R_1}$$

or $\Delta\nu/\nu = GM/Rc^2$ (and a shift in wavelength is similar). This is sort of like the Doppler shift, except it comes just from gravity, not velocity.

### XIX.1.4 WD

for a white dwarf, $\Delta\lambda/\lambda \approx 74\,(M/M_\odot)^{4/3}\,GM_\odot/R_\odot c^2$. It has been measured. For 40 Eri B (first WD) is it $6 \times 10^{-5}$.

### XIX.1.5 NS

for a neutron star, this can be 20%! Can be quite large, although yet to be measured (I'm trying...).

### XIX.1.6 Earth

1960 at Harvard. Shot $\gamma$-ray up a 22.6 m tower. Found a change $\Delta\nu/\nu = -gh/c^2 = -2.5 \times 10^{-15}$.

## XIX.2 More Mass?

What happens if we take a NS and add mass? $> 3M_\odot$ (or so): like a WD, the particles get to $v \sim c$ and it cannot support itself. Collapses. And nothing can stop it. Gravitational redshift goes up. Escape velocity goes up. $v_{\rm esc} = \sqrt{2GM/R}$ cannot be $> c$: eventually we get to where this is $= c$ at

$$R = R_{\rm Schwarschild} = \frac{2GM}{c^2}$$

This is a black hole! (note that this derivation is not correct, but it gives the right answer).

At this point, the gravitational redshift becomes:

$$\frac{\Delta\lambda}{\lambda} = \left(1 - \frac{R_{\rm Sch}}{R}\right)^{-1/2} - 1$$

which goes to $1/0 = \infty$ at $R = R_{\rm Sch}$. (and this reduces to the form we already did for $R \gg R_{\rm Sch}$.).

### XIX.3 BH

A BH has mass squeezed into $R_{\rm Sch}$: event horizon. Cannot get anything out. For $1M_\odot$, $R_{\rm Sch} = 3\,{\rm km} \ll R_{\rm NS}$.

How do we identify black holes? We look for things moving really fast in a really small volume, and we look for $M > M_{\rm NS}$

**draw**

Material moves in *accretion disk* around BH, gradually spiralling in. We can deduce that it has a velocity (from redshift/blueshift) such that it would be inside a NS (from Kepler's laws).

Accretion can happen with NS too. But there it can have any orbit. For a BH, stable orbits can only happen for $R > 3R_{\rm Sch}$. Inside that, the material is doomed. Must fall in.

As matter falls in, releases gravitational energy. Gets hot! Makes jet!

**draw**

Happens for small BHs: X-ray binary, *microquasar*.

Happens for big BH. Most galaxies seem to have one. $M_{\rm BH} = 10^{6-9}M_\odot$, *active galactic nucleus* or AGN. Can outshite rest of galaxy (*quasar*). Brightness depends on how much stuff is falling in. MW: $2 \times 10^6 M_\odot$.

Event horizon does not kill. tidal forces do.

*Eventually*, GR (Einstein) says that no orbit will stay stable. For massive objects moving quickly (i.e., in short orbits) this will happen in $< t_{\rm Universe}$. Like the Hulse-Taylor binary in 200 Myr.

What will happen? They will merge & (often) explode. Details not know, but for instance NS-NS binary will probably make a gamma-ray burst.

When this happens, the moving masses will also distort space-time, sending out waves. Hopefully we can detect this on Earth with Laser Interferometer Gravitational Observatory: astronomy with gravitational waves, not photons.

# Lecture XX Supernovae

Kutner 11.1

explosions, new stars. a few known historically (e.g., Crab in 1054). Here we talk about core collapse (from massive stars).

$10^{46}$ J released, 1% into KE, 0.01%–0.1% into photons. Collapse makes fusion well past Fe.

Lightcurve **draw**: decline set by radioactive decay of Ni (6 days) and Co (77 days) made from shock slamming into Fe.

## XX.2 Chemistry

Rest of elements past Fe mostly from SNe. Reactions: r-process (rapid). Heavy element + many neutrons → very heavy, unstable nucleus. These then decay to something stable.

Also s-process (slow) in post-MS evolution of stars.

## XX.3 Shock Wave

Hot gas out at 10,000 km/s. $10^{44}$ J at $T = 10^7$ K into ISM ($10^4$ K, $n \sim 10^6\,\mathrm{m}^{-3}$). Blast wave:

- Starts really fast: free expansion, supersonic
- Until sweeps up mass $\sim$ mass in shell (10's to 100's of years)
- Enters Sedov-Taylor phase (first derived for nuclear explosions): $E =\sim (\rho_{\rm ISM}/t^2)(r/c)^5$

# Lecture XXI Interstellar Medium

Kutner 14

What is between stars? Gas (99%) and dust (1%). Like a star, mostly H, then He.

### XXI.2 Dust

Obvious as dark patches in the sky **show**: "holes" w/o stars.

Light is blocked by dust. Work by Trumpler (1930).

Clusters of stars each have main sequence: should line up to get distances. Trumpler: $d =$distance, $D =$diameter. Angular diameter $\theta = D/d$:

$$\theta^2 = \frac{D^2}{d^2} \propto \frac{1}{d^2}$$

Apparent brightness is the flux $F = L/4\pi d^2 \propto 1/d^2$.

If $L$, $D$ are typical values then should see $\theta^2 \propto F$

**draw**

systematic departuse for distant objects (small $F$, small $\theta^2$), with $F$ less than expected. Moreover, distant clusters were also redder than expected. Both from dust.

Like a sunset. Red light transmitted, blue light scattered/reflected.

Dust is little balls of C, Si. Makes things dimmer and redder.

### XXI.3 Gas

Most of mass of ISM, few % mass of stars.

Gas can be warm or cold, dense or diffuse, atoms or molecules. Average is $n \sim 1\,\mathrm{cm}^{-3} = 10^6\,\mathrm{m}^{-3}$. For comparison, best vacuum on Earth is $10^9\,\mathrm{m}^{-3}$.

| component | volume | $T$ | $n$ | state | see via? |
|---|---|---|---|---|---|
| molecular clouds | $< 1\%$ | 10-20 | $10^{8-12}$ | $H_2$ | molecules, emit & abs. |
| CNM | 1–5% | 70 | $3 \times 10^7$ | H | HI 21 cm abs |
| WNM | 10%–20% | $10^4$ | $10^6$ | H | HI emit |
| WIM | 20%–50% | $10^4$ | $10^6$ | H II | H$\alpha$ |
| H II regions | $< 1\%$ | $10^4$ | $10^{8-10}$ | H II | H$\alpha$ |

The last surrounds hot stars.

Remember: hot light and cold cloud: absorption. Hot cloud: emission.

### XXI.3.1 H I 21 cm

diagnostic for gas. Spin flip **draw**. $\nu = 1420\,\mathrm{MHz}$ or $\lambda = 21\,\mathrm{cm}$. First seen in 1950's.

### XXI.3.2 Pressure Equilibrium

Different parts are "roughly" at same $P = nk_BT$ (equilibrium).

Molecular clouds: small and dense, where stars are born

H II regions surround hot stars (OB, WD)

the rest takes up space in between.

measure $T$ from Doppler width of lines. Measure $n$ from brightness of lines.

### XXI.3.3 Using H I

Doppler shift gives $v_{\mathrm{radial}}$. In Milky Way, get distance, velocity of gas, can map spiral arms.

### XXI.3.4 Physics That Happens

Heat ISM via:

- cosmic rays (protons)
- light from stars
- shocks (SNe)
- stellar winds

Cool via

- emission of lines
- free-free (emission of continuous radiation from hot, ionized gas)

### XXI.3.5 H II Regions

H I ($T \sim 10^2$ K) around H II around star.

Star is $> 10^4$ K so enough photons have $h\nu > 13.6$ eV. Can then ionize.

Size of H II: $N_* =$ number of photons per second with $E > 13.6$ eV (beyond Lyman limit).

Assume each photon is absorbed by 1 atom.

But, for each $e$, $p$ there is a chance that recombine $e+p \rightarrow$H. This balances ionization.

$$\mathcal{R} = \frac{\#\text{recombination}}{\text{volume} \times \text{time}}$$

$\mathcal{R}V = N_* = \mathcal{R}(\frac{4}{3}\pi r^3)$. What is $\mathcal{R}$? Depends on rate that $e$ hits $p$. $\mathcal{R} \propto n_e n_p \propto n_e^2$. So

$$r = \left(\frac{3N_*}{4\pi\alpha n_e^2}\right)^{1/3}$$

Stromgren sphere. $\alpha = \alpha(T) \sim 3 \times 10^{19}\,\mathrm{m}^3\,\mathrm{s}^{-1}$. $n_e \sim 10^8\,\mathrm{m}^{-3}$, $N_*(\mathrm{O5}) = 3 \times 10^{49}\,\mathrm{s}^{-1}$. Gives $r \sim$few pc.

Can have multiple stars (bigger) or WD (smaller). See via Balmer lines (H$\alpha$ etc.) in emission, also lines from N, O, He.

# Lecture XXII Star Formation

See other notes

# Lecture XXIII Solar System

Kutner 22–26

Solar system: terrestrial (like Earth) planets are Mercury, Venus, Earth, Mars; rocky, with very thin atmospheres. Gas giants Jupiter, Saturn, Uranus, Neptune; have cores, but outer layers are mostly gas. And then there are other objects.

Study planets now, get clues to how they formed.

see other notes.

# Lecture XXIV  Terrestrial Planets & Atmospheres

Kutner 23, 24.

### XXIV.2 Atmospheres

The Earth's atmosphere is 78% $N_2$, 21% $O_2$, and 1% others (Ar, $CO_2$). How did it get that way? Did it happen naturaly during the formation of the Earth?

No: at 1 AU, conditions during the formation of the Earth were too hot for those elements to condense:

- Mass to low to attact the gas on its own (unlike gas giant)
- Hot early on, so gases would escape

Answer: gradual accumulation + *life*.

First atmosphere:

- H and He
- Much escapes

Second:

- $CO_2$/$NH_3$ which outgassed from volcanos
- $H_2O$, some from comets (icy balls)
- Pressure $\sim 100$ bar (Earth now is $\sim 1$ bar)
- $< 100°$ C

Third:

- $CO_2$ absorbed
- $H_2O$ liquid
- $O_2$ produced
- 1 bar
- 15° C

Sink of $CO_2$: rock (with water, it dissolves), water, life
sources: volcanos (+humans)
Sink of $H_2O$: tectonic plates subduct, carry water into the mantle
sources: volcanic outgassing, comets

It is a sensitive balance, where too much $CO_2$ can drive off water

Venus: similar to Earth at some point in the past. Why different now?

- Closer to Sun, so warmer, less water?
- Too much $CO_2$ to start?

Non-linear system, so even if starting points were very similar small differences can be amplified (in unexpected ways)

$O_2$: not natural, unstable. Produced over billions of years by life: $\mathrm{CO_2 + H_2O \rightarrow O_2 + sugar}$.

**Show greenhouse**, energy balance.

### XXIV.3 Interiors of Planets

Differentiated (in layers)

- Dense metallic core (solid interior, liquid outer)
- Lighter rocky mantle
- Lighter rocky crust

Heat sources:

- Accretion (things falling)
- Differentiation (settling into layers, releases potential energy)
- Radioactive (U, Th)
- Tidal

Heat flow:

- Conduction in core
- Convection in mantle (lava lamp w/ $\sim 100$ Myr, rock creep)
- Conduction through surface

Relative sizes of the components change a lot between the different planets.

Exteriors shaped by:

- Impacts (rare now, more common before, so # can be used to "date" surface)
- Volcanos (Earth, Venus now; Mars before)
- Tectonics (Earth now)
- Erosion by winds, liquids (Earth, Mars, ?)

Continental Drift/Plate Tectonics **show**
Evidence:

- Similar fossils on different continents
- Continents fit together
- Spreading of mid-ocean ridges

All explained by motions of plates on plastic mantle.

Venus shows evidence of tectonics but not plates: is this because less water makes the rock more plastic, so it will not be in pieces?

## XXIV.4 $CO_2$ Cycle

Where is all of the $CO_2$ on Earth compared to Venus?

Sink: dissolves in water, forms carbonate rocks, rain erodes rock and the pieces fall into the ocean.

Source: volcanos

So the sink depends on the temperature, but the source does not.

Earth:

- If it were too cool
  - Ocean dissolves less $CO_2$
  - Fewer carbonates
  - more $CO_2$
  - more greenhouse effect, so it would be warmer
- If it were too warm
  - dissolve more $CO_2$

- more carbonates
- less $CO_2$ in atmosphere
- less greenhouse, so it would be cooler

this acts as a thermostat to regulate the temperature (within some range).

### XXIV.4.1 Snowball Earth

(not entirely solved problem)

Try to explain recurrent freezings of (much of) Earth. These were triggered by some even (volanoc, impact):

- drives temperature down
- extra ice, so shinier (*albedo* goes up)
- temperature goes down
- this makes more ice, $T$ goes down further
- eventually, $\sim$ total ice (evidence for glaciers at equator)
- once this happens, oceans are covered, so $CO_2$ sinks disappear, builds up
- greenhouse, rapid warming, melt, albedo down

Most recent episode $\sim 650$ Myr ago? Questions regarding snowball vs. slushball, details, extend, frequency.

# Lecture XXV Formation of Solar System

Kutner 27

Protoplanetary disk: disk around proto-Sun 4.6 Gyr ago (**show**).

Common around other stars (first 10–100 Myr of star's life)

Formation: details still unknown, but basic concepts:

- Minimum mass solar nebula
- planetismals
- frost line

ways we test the theory:

- giant planets vs. rocky
- isotope dating

## XXV.2 Minimum Mass Solar Nebula

Sun collapsed out of gas cloud. Angular momentum made much of it form disk. Had same elements as the Sun: 70% H, 28% He, CNO etc. 1% (also seen in meteorites). Early on this was substantial, but since then it has either been formed into planets or been blown away.

Current total of all objects (except Sun): 1.5 $M_J$=0.0015 $M_\odot$. Was a lot more in the past. How can we reconstruct what might have been? Can we set a *lower limit*?

Put back all light elements (H, He) to make planets have the same abundances as the Sun (only adding):

- Mercury: $\times 350$
- Earth: $\times 235$
- Jupiter: $\times 5$
- Saturn: $\times 8$
- Uranus: $\times 15$

Add this all together → MMSN $= 10 M_J = 0.01 M_\odot$.

- This would have been densest near the Sun

- at least 85% was lost before forming planets
- Similar in mass wrt Sun as many observed disks

Problem: this disk cannot collapse on its own. Any bulge that forms will be quickly torn apart by tidal forces from the Sun (tidal force $\gg$ self-gravity)

### XXV.2.1 Cannot Collapse

Ball with mass $M_{\rm gas}$, size $d$, $a$ from Sun:

$$F_{\rm tide} \sim \frac{GM_\odot M_{\rm gas}}{a^2}\left(\frac{d}{a}\right)$$

And self-gravity:

$$F_{\rm self} = \frac{GM_{\rm gas}^2}{d^2} = G\rho d M_{\rm gas}$$

($\rho \approx M_{\rm gas}/d^3$). To make something collapse, need $F_{\rm self} > F_{\rm tide}$

$$G\rho d M_{\rm gas} > \frac{GM_{\rm gas}M_\odot}{a^3}d$$

which needs $\rho > M_\odot/a^3 \sim 2\times 10^{-4}\,{\rm kg\,m^{-3}}(1\,{\rm AU}/a)^3$. This is known as the *Toomre criterion*. Compare with:

$$\rho_{\rm MMSN} \sim 10^{-5}\,{\rm kg\,m^{-3}}(1\,{\rm AU}/a)^{5/2}$$

which is too low by at least a factor of several–20. So how did planets form?

### XXV.2.2 Planetismal Hypothesis

dust

(grains are small & sticky, can stick together with a force that is $>$ gravity)

- Disk cools, forms dust, $m \sim 10^{-15}$ kg, size $\sim \mu$m
- These stick together to become pebbles (∼g, cm)
- These collide to form *planetismal* ($\sim 10^{12}$ kg, km)

The remnants of this process might form the asteroid belt (in part).

- A few of these dominate everything nearby, *planetary embryos* ($\sim 10^{19}$ kg, 100 km)
- These collide, *planetary cores* ($\sim 10^{21}$ kg, 1000 km, proto-Earth
- The cores accrete gas, make gaseous planets ($\sim 10^{24}$ kg, proto-Jupiter).

### XXV.2.3 Hill Sphere

What does *dominate* mean? Define Hill Radius $R_H$: where $P_{\rm orb}$ around embryo = $P_{\rm orb}$ around Sun. Get:

$$R_H = a \left( \frac{M}{M_\odot} \right)^{1/3}$$

which is $1.4 R_\oplus$ for 10 km comet at 5 AU. Stuff within that region will be bound to the embryo.

### XXV.2.4 Frost Line

(or Snow line)

**sketch disk, with frost line**

Where is equilibrium temperature $T < 150$ K or so: make ices. water, ammonia, methane solid rather than gas (vacuum lowers melting point, no liquid phase)

Outside frost line, planets can grab a lot of H in the form of ices (mix with C, H, N). These will be accreted onto the rocky cores.

Inside, planets largely rocky, made of *refractory* elements (high boiling point).

Outside, volatiles. So the mass of the planet jumps a lot since you can get many of the light elements that dominate the MMSN. Masses go up by $> 5$. With even more mass, can get gasses too. Somewhere $\sim 2-3$ AU.

This explains:

- giants have much gas, ice
- terristrial do not

From this we expect a rocky core inside the giant planets.

A related concept is the *Habitable Zone*. It is where liquid water can exist (needs pressure $> 0$). Here it is 0.7–3 AU (Venus to Mars). But this is rough, since greenhouse effect and the history of the planet can change the answer.

## XXV.3 Date Of Formation

Look at pristine materials: asteroids, comets, moon, Mars. We want things that have not melted (and driven off any elements) since they formed.

We see that e.g., meteorites that land on Earth have abundances much like the Sun. From these we measure ratios of isotopes with natural decay, like $^{238}{\rm U} \rightarrow ^{206}{\rm Pb}$ by way of Th, etc. Takes $t_{1/2} = 4.47$ Gyr. From this, get:

- Age $= 4567.2 \pm 0.6$ Myr

- From other isotopes, expect was a supernova a few MYr before formation ($^{60}\mathrm{Fe}$, 1.5 Myr). Was star formatino triggered by SN shock?
- Earth formed within 10 Myr
- Moon $\sim 30$ Myr after the solar system
- oldest fossils, 3.5 Gyr

### XXV.4 Earth's Moon

size is uniquely large compared to all other planets, 1% of Earth mass.

We can tell that areas of the surface are older or younger based on numbers of craters

Moon rocks: little water or other volatiles. Low in Fe (core $< 200$ km?), but similar isotopes relative to Earth.

Possibilities?

- Capture (was floating around system)
- Accretion disk (might account for Jupiter's moon, like mini solar nebula)
- Impact (explains lack of Fe, volatiles)

Need $M$ ~Mars and $v \sim 10$ km/s to melt Earth. This gives

$$\frac{1}{2}mv^2 \approx k_B T \frac{M_\oplus}{\mu m_H}$$

Some mass was lost, but the rest became the Moon.

# Lecture XXVI Extrasolar Planets

Kutner 27

Planets outside the solar system, first found in 1995 (except for pulsar system in 1992)

### XXVI.2 How To Find?

**show app?** Can we see directly? How bright compared to star (contrast)?

Reflected light:

$$\frac{\pi R_p^2}{4\pi a^2} \approx 10^{-8}\left(\frac{a}{1\,\mathrm{AU}}\right)^{-2}$$

Thermal light (planets are warm, like Jupiter):

$$\frac{\pi R_p^2}{\pi R_S^2}\left(\frac{T_p}{T_S}\right)^4 \approx 10^{-6}$$

Angular separation: $0.1''(a/1\,\mathrm{AU})$ at 10 pc away. This is very hard, but has been done in a few occasions.

Main method: radial velocities (relative motion of star tugged by planet)

$$\Delta v = 30\,\mathrm{m/s}\left(\frac{a}{1\,\mathrm{AU}}\right)^{-1/2}\left(\frac{M_p}{M_J}\right)$$

**sketch**. Can see multiple planets. But inclination unknown.

Astrometry: see motion of star itself, not the velocity. Very hard.

$$\Delta\theta \sim 0.1\,\mathrm{mas}\left(\frac{a}{\mathrm{AU}}\right)\left(\frac{M_p}{M_J}\right)\left(\frac{d}{10\,\mathrm{pc}}\right)^{-1}$$

This depends on $d$, unlike the previous two methods.

A lot of progress lately on *transits*: see dip in light when planet goes in front of star (Kepler!). Depth

$$\frac{\pi R_p^2}{\pi R_S^2} \sim 0.01$$

or less, but only if orbit is edge-on. Chance of transit (**sketch**):

$$\sim \frac{R_s}{a} \sim 0.05\left(\frac{0.1\,\mathrm{AU}}{a}\right)$$

Works best for big, nearby planets.

All search methods have biases: certain systems show up more easily. Hard to understand what the real underlying population is.

These are from protoplanetary disks. We see disks around 5% of stars younger than 10 Myr. We see planets around roughly 5% of stars with $M \sim M_J$.

Occasionally we see systems with both disk and planet: the planet clears a space around it (like Saturn's rings) and accretes material, growing.

## XXVI.3 Properties

Look at exoplanets.eu

### XXVI.3.1 Mass

large range, high-M easier to detect, see up to $10M_J$. $N \propto 1/M$ roughly.

### XXVI.3.2 Period

short end: very close, orbits down to a few days (makes it easier to detect). We call these *hot jupiters*

long end: limited by how long a search goes on and sensitivity of searches.

### XXVI.3.3 Eccentricity

at low $a$, most have $e = 0$ (tides make things circular)

at high $a$, $e$ can be anything.

### XXVI.3.4 Hot Jupiters

$$T_p = T_S \sqrt{\frac{R_S}{2a}} (1 - A)^{1/4} \approx 1100\,\mathrm{K} \left( \frac{0.1\,\mathrm{AU}}{a} \right)^{1/2}$$

Compare to 120 K for Jupiter. How can they be so hot? Wouldn't they boil away? Could they form there?

See large planets with orbits all the way down to 0.01 AU. Mercury is at 0.4 AU.

### XXVI.3.5 Metallicity

Find more around stars with high metallicity. More metals = more grains and ices = bigger cores?

- $< 1\%$ when metals are $< 30\%$ of Sun
- $> 20\%$ when metals are $3\times$ Sun

**sketch**

### XXVI.3.6 Radii

Depth, length of transit can get radius of planet too (otherwise just mass) Many are large than Jupiter **show**. Large range, with a fractof of $\sim 4$ in $\rho$.

Are they bloated because they are hot? This is hard. Did they just never shrink when they formed?

### XXVI.3.7 Detecting Atmospheres

Transit: planet blocks star light, blocks all wavelengths

If there is an atmosphere around the star, certain wavelengths will get extra absorbed right near when the main part of the planet transits

### XXVI.3.8 Migration

Expect that we form giants far out (past frost line). But we see many close in (hot Jupiters). Why?

Even Jupiter might have formed $\sim 0.5$ AU further out. Slowly spirals in (*migration*) from interactions between disk and planet. Hard to get details right:

- need it to happen fast enough that close is common
- need it to happen slow enough that they stay there

# Lecture XXVII Telescopes

Kutner 4

Why do we use telescopes? How big a telescope do you need?

photons come at some rate: source has a flux density $F_\lambda$ in J/s/m$^2$/Å. To get rate of photons:

$$F_\lambda \times \frac{\text{telescope area}}{\text{photon energy}} \times \text{filter width} \times \text{efficiency}$$

gives $N$ (photons/s). Number detected is $N\times$time:

$$n = F_\lambda \frac{A}{hc/\lambda} \Delta\lambda \Delta t \eta$$

e.g., magnitude 20 (faint for eyes, not for telescopes) star: $F_\lambda = 3.6 \times 10^{-20} \mathrm{W\,m^{-2}\,\AA^{-1}}$ (by eye can see mag$=6$, so this is $10^{-(20-6)/2.5} = 2.5 \times 10^{-6}$ times as bright). Use:

- $\lambda = 500\,\mathrm{nm}$, so $hc\lambda = 4 \times 10^{-19}$ J
- diameter $D = 10\,\mathrm{m}$, so area $A = 78\,\mathrm{m}^2$
- efficiency $\eta = 20\%$
- width $\Delta\lambda = 1000\,\mathrm{\AA}$

gives $1.4 \times 10^3$ photons/s. How long do we need to observe for? How many seconds are enough?

### XXVII.2 Poisson Statistics

for counting

expect $r$ events/s, wait $t$ seconds. So we *expect* $rt = n$ events (cars, raindrops, people, etc.). How many actually come? Probability that we see $m$ when expect $n$:

$$\frac{e^{-n} n^m}{m!}$$

- $P(0) = e^{-n}$
- $P(1) = ne^{-n}$
- $P(2) = n^2 e^{-n}/2$

**sketch**.

*Expected* number is $m = n$, but we often get more or less. What is important here is width: we don't always see exactly as many as we expect, but we want to know how close we will come on average. In general, 68% of the time we see somewhere in $n \pm \sqrt{n}$ (this is 1-$\sigma$, central limit theorem, $n \gg 1$). 95% of the time we see $n \pm 2\sqrt{n}$. 99.7% of the time we see $n \pm 3\sqrt{n}$.

So if we want to be *sure*. . .

We want to see enough photons that we can be sure that there is something real there. Usually we say $> 3\sigma$ confidence, so only wrong 1 time in 1000. $n/\sqrt{n} = 3$, so $n > 9$. That means we need 9 photons per exposure, so we can have $\Delta t = 6\,\mathrm{ms}$ (as I said, this is very easy for a telescope).

But objects can be a lot fainter (27th mag), can have background noise, can have lower spectral width (spectra).

### XXVII.3 Other Wavelengths

Optical observing: light behaves mostly like a particle, can do from the ground. Does this change?

Consider:

$$\delta \approx \Delta\nu \left(\frac{F_\nu}{h\nu}\right) \Delta\tau A_c$$

with $\Delta\tau$ the coherence time, and $A_c$ the coherence area. Based on the uncertainty principle $\Delta\tau\Delta\nu > 1$ and $A_c \approx \lambda^2 = (c/\nu)^2$, so we get:

$$\delta \approx \frac{F_\nu}{h\nu}\left(\frac{c}{\nu}\right)^2 = c^2\frac{F_\nu}{h\nu^3}$$

From a blackbody,

$$F_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

So

$$\delta \sim \frac{2}{e^{h\nu/kT} - 1}$$

For the Sun at $T = 6000\,\mathrm{K}$, at optical wavelengths $\lambda = 500\,\mathrm{nm}$ we find $\delta = 0.02$. This is $\ll 1$, so it behaves like a particle.

At radio wavelenghts $\lambda = 1\,\mathrm{m}$, $\delta = 8 \times 10^5 \gg 1$ so it behaves like a wave. $\delta = 1$ at a wavelength of 2 $\mu$m.

### XXVII.4 Radio Telescopes

$\lambda > 1\,\mathrm{mm}$ or so, can know phase of wave. Atmosphere is transparent up to wavelengths of 10 m (beyond that is blocked by the ionosphere).

Do we need dark skies? No: need free from interference:

- 90–100 MHz: $\lambda = 3\,\mathrm{m}$, FM band
- 1 GHz (30 cm): WiFi, phones, microwaves, etc.

RFI. Better in valley than on mountain.

Telescopes can be anything from “light buckets” to coat hangars. Need a lot of area.

The surface needs to be smooth to $< \lambda/4$. Optical: 100 nm (polished glass, heavy & expensive). Radio: 1 cm (chicken wire).

So we can make things much bigger, which is good since sources are faint.

$D$ up to 300 m (Arecibo). To detect, no longer look at individual photons, but treat like a wave.

### XXVII.5 Seeing

Optical: resolution $> 1''$ by *seeing*: turbulence in the atmosphere. To do better: go to top of mountain, go to space (expensive), correct for turbulence (hard). Last is *adaptive optics*. Use laser guide stars: make a (fake) perfect star, then see how it gets distorted. Can compute how to undo that.

### XXVII.6 Resolution

Diffraction limit. We can only see things that are bigger than $\lambda/D$. In the optical this is limited in any case to $> 1''$. compare to Arecibo: $49''$ at 6 cm, so much worse. And we can't make a dish (much) bigger. But luckily we don't need to:

use multiple dishes (interferometer). Then resolution is $\lambda/B$ where $B \gg D$. Need to combine as a wave (maintain phase). Area (signal-to-noise) limited by $D$ (or area$= N \times D^2$), but resolution from $B$.

- Very Large Array: 27 dishes, $D = 25$ m each. $B$ up to 30 km, so total $A$ is $1/9$ Arecibo by $\theta$ down to $1''$ or better
- Very Long Baseline Array: 10 dishes, 25 m, $B$ up to continent (8000 km using islands). $\theta = 1$ mas

Do interferometers work at other wavelengths? Yes, but it's hard.

- IR: Keck, VLT, some others. Up to 4 telescopes, for specialized projects, within a few 10's of m. Need to know spacing between telescopes to $\lambda/4$ which is very hard.
- Others: eventually

- lion at 10 km: $100''$
- movie screen at the Moon: $0.01''$

## XXVII.7 Infrared

targets: warm things. dusty things (blocks optical light, re-emits as IR). Proto-stars, star-forming galaxies, gas clouds.

$\lambda < 5\,\mu$m from ground. After that atmosphere absorbs, need to go to space. Better from high mountain in any case.

Issues: Wien displacement, peak of BB in the IR from anything warm (300 K is 10 $\mu$m). So telescopes, people, sky all make "noise." Solution is to make the whole telescope cold, ideally in space.

## XXVII.8 UV/X-ray

Targets: hot thigs. WD, BH, NS.

atmosphere blocks so we mostly need to go to space. X-rays are even harder: cannot make a mirror to focus the light. Have to use *grazing incidence* or other tricks.

## XXVII.9 Astronomy Without Photons

neutrinos, cosmic rays, gravitational waves. All hard, in infancy.

$\nu$ and CR: indirect detection. e.g., ICECUBE. 1 km$^3$ of ice at south pole. strings of light detectors. When $\nu$ passes through, mostly goes on without interacting. Occasionally hits proton, generates $e$ or $\mu$. When it does they will be traveling faster than local $c$ (not $c$ in vacuum). This makes Cerenkov light, or a blue flash (sort of like a shock wave).

# Lecture XXVIII Extra-Galactic Astronomy

Kutner 20.1, 20.2.1

### XXVIII.2 Units and scales

Galaxies and cosmology deal with sizes and masses that are both very large and very small.

Distance units: AU, distance from Earth to Sun, $1.5 \times 10^{11}$ m. Not so useful beyond the solar system, so we use parsec (pc), distance at which 1 AU subtends an angle of 1 arcsec: 1 pc = $3.1 \times 10^{16}$ m = 3.26 light years. We are 1.3 pc from Proxima Centauri (nearest star) and 8000 pc = 8 kpc from center of Galaxy. For intergalactic distances, we use megaparsec (Mpc): 1 Mpc = $3.1 \times 10^{22}$ m. We are 0.7 Mpc from M31 (Andromeda galaxy) and 15 Mpc from the Virgo Cluster (nearest big cluster of galaxies).

Mass: standard unit of mass is solar mass, $\mathrm{M}_\odot$: 1 $\mathrm{M}_\odot$= $2.0 \times 10^{30}$ kg. Mass of Milky Way $\approx 10^{12}$ $\mathrm{M}_\odot$. Sun also provides standard unit of power or luminosity: 1 $\mathrm{L}_\odot$= $3.8 \times 10^{26}$ W. Total luminosity of Milky Way, $L_{\mathrm{gal}} = 3.6 \times 10^{10}$ $\mathrm{L}_\odot$. (Approximately how many stars are in the galaxy?)

Time: 1 year = $3.2 \times 10^7$ s. In a cosmological context we use Gyr, 1 Gyr = $10^9$ yr = $3.2 \times 10^{16}$ s. The Universe is 13.7 Gyr old.

### XXVIII.3 Olbers' Paradox: Why is the sky dark at night?

Named after Heinrich Olbers, who wrote a paper on the subject in 1826, but first proposed by Thomas Digges in 1576.

Let's suppose the universe is infinite and static, as Isaac Newton believed; a universe that isn't infinite will collapse inward due to its own self-gravity. Now let's compute how bright we expect the night sky to be in this infinite universe, where every line of sight ends at a star. Let $n$ be the average number density of stars in the universe, and let $L$ be the average stellar luminosity. The flux received here at Earth from a star of luminosity $L$ at a distance $r$ is given by an inverse square law:

$$f(r) = \frac{L}{4\pi r^2} \tag{1}$$

Now consider a thin spherical shell of stars, with radius $r$ and thickness $dr$, centered on the Earth. The intensity of radiation from the shell of stars (that is, the power per unit area per steradian of the sky) will be

$$dJ(r) = \frac{L}{4\pi r^2} \cdot n \cdot r^2 dr = \frac{nL}{4\pi} dr. \tag{2}$$

The total intensity of starlight from a shell thus depends only on its thickness, not on its distance from us, since the $1/r^2$ for the flux cancels the $r^2$ for the area. So if the Universe extends infinitely far away, we will end up with infinitely many shells, and the total light on Earth will be infinitely bright.

We have shown that the night sky is infinitely bright. Why is this wrong? Discuss.

- Stars have finite size, so we don't actually have an unobstructed line of sight to all stars. But still, each line of sight ends at a star, so sky should have the surface brightness of a typical star.
- Interstellar matter that absorbs starlight? No, because the matter would be heated by starlight until it has the same temperature as the surface of a star, and then it would emit as much light as it absorbs and glow as brightly as the stars.
- Assumed that number density and mean luminosity of stars are constant throughout the universe; distant stars might be less numerous or less luminous than nearby stars.
- Assumed that universe is infinitely large. If universe has size $r_{\max}$, then the total intensity of starlight we see in the night sky will be $J \sim nLr_{\max}/(4\pi)$. Note that this result will also be found if the universe is infinite in space, but is devoid of stars beyond a distance $r_{\max}$.
- Assumed that the universe is infinitely old. When we see stars farther away, we're also seeing stars farther back in time. If universe has finite age $t_0$, intensity of starlight will be at most $J \sim nLct_0/(4\pi)$. Also applies if stars have only existed for time $t_0$.
- Assumed that flux of light from a distant source is given by inverse square law; i.e. we have assumed that the universe obeys laws of Euclidean geometry and that the source of light is stationary with respect to the observer. Einstein showed that universe may not be Euclidean, and if the universe is expanding or contracting then the light will be red or blueshifted to lower or higher energies.
- Primary resolution: universe has a finite age, and the light from stars beyond some distance—called the horizon distance—hasn't had time to reach us yet. First person to suggest this was Edgar Allen Poe in 1848: "Were the succession of stars endless, then the background of the sky would present us an [sic] uniform density ...since there could be absolutely no point, in all that background, at which would not exist a star. The only mode, therefore, in which, under such a state of affairs, we could comprehend the voids which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all."

### XXVIII.4 Basic observations

The cosmological principle:

- The Universe is homogenous. There are no preferred locations: the universe looks the same anywhere.

- The Universe is isotropic. There is no preferred direction: the universe looks the same in all directions.

This is only true on large scales; obviously not true on the scale of a person or a planet or even a galaxy or cluster of galaxies. On scales of $\sim 100$ Mpc, Universe is homogenous and isotropic; this is roughly the scale of superclusters of galaxies and the voids between them.

Also called the Copernican principle, after Copernicus, who determined that the Earth is not the center of the Universe. There is no center.

- Galaxies show a redshift proportional to their distance.

Consider light at a particular wavelength observed from a distant galaxy: $\lambda_{\rm obs}$ is the observed wavelength of some absorption or emission feature in the galaxy's spectrum. $\lambda_{\rm em}$ is the wavelength at which that feature is measured on Earth. In general, $\lambda_{\rm obs} \neq \lambda_{\rm em}$; the galaxy has a **redshift** $z$ given by

$$z \equiv \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}}. \tag{3}$$

Most galaxies have redshifts—this is because the Universe is expanding.

In 1929 Edwin Hubble plotted galaxy redshifts against their distance (redshifts are easy to measure, but distances are hard), and showed that the redshift of a galaxy was linearly proportional to its distance. This is now known as Hubble's law:

$$z = \frac{H_0}{c} r, \tag{4}$$

where $H_0$ is a constant now called the **Hubble constant**.

If we interpret redshifts as Doppler shifts (not strictly true, but we'll talk about that later), $z = v/c$ and Hubble's law takes the form

$$v = H_0 r, \tag{5}$$

where $v$ is the radial velocity of the galaxy. Therefore we can get the Hubble constant from dividing velocity by distance, and it has the units km s$^{-1}$ Mpc$^{-1}$ (note that the actual units are inverse time). Hubble's original estimate was $H_0 = 500$ km s$^{-1}$ Mpc$^{-1}$, but he was severely underestimating the distances to galaxies. The Hubble constant has been measured with precision only in the last $\sim 10$ years; the best current value is $H_0 = 70.4^{+1.3}_{-1.4}$ km s$^{-1}$ Mpc$^{-1}$ (2010, from WMAP seven-year results, with priors from other estimates).

We can also use the Hubble constant to define a time. If galaxies are moving apart from each other, they must have been together at some point in the past. Consider two galaxies separated by a distance $r$ and moving at a constant velocity $v$ with respect to each other. The time elapsed since the galaxies were in contact is

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1}, \tag{6}$$

independent of $r$. The time $H_0^{-1}$ is called the **Hubble time**, and is an approximate timescale for the age of the Universe (it is only equal to the age of the Universe if galaxy velocities were the same at all times in the past). $H_0^{-1} = 13.8$ Gyr.

Let's use this to return to Olbers' paradox. If the universe is of finite age, $t_0 \sim H_0^{-1}$, then the night sky can be dark, even if the universe is infinitely large, because light from distant galaxies has not yet had time to reach us. Galaxy surveys tell us that the luminosity density of galaxies in the local universe is

$$nL \approx 2 \times 10^8 \ \mathrm{L}_\odot \ \mathrm{Mpc}^{-3} \tag{7}$$

This luminosity density is equivalent to a single 40 watt light bulb within a sphere 1 AU in radius. If the horizon distance is $r_H \sim c/H_0$, then the total flux of light we receive from all the stars from all the galaxies within the horizon will be

$$F_{\rm gal} \sim nL\frac{c}{H_0} \sim 9 \times 10^{11} \ \mathrm{L}_\odot \ \mathrm{Mpc}^{-2} \sim 2 \times 10^{-11} \ \mathrm{L}_\odot \ \mathrm{AU}^{-2}. \tag{8}$$

By the cosmological principle, this is the total flux of starlight you would expect at any randomly located spot in the universe. Comparing this to the flux we receive from the Sun,

$$F_{\rm sun} = \frac{1 \ \mathrm{L}_\odot}{4\pi \mathrm{AU}^2} \approx 0.08 \ \mathrm{L}_\odot \ \mathrm{AU}^{-2}, \tag{9}$$

we find that $F_{\rm gal}/F_{\rm sun} \sim 3 \times 10^{-10}$. Thus, the total flux of starlight at a randomly selected location in the universe is less than a billionth the flux of light we receive from the Sun here on Earth. For the entire universe to be as well-lit as the Earth, it would have to be over a billion times older than it is, and you'd have to keep the stars shining during all that time.

# Lecture XXIX Content & Structure of the Milky Way

Kutner 16

## XXIX.2 What is a galaxy?

A collection of stars ($\sim 10^6$ to $\sim 10^{12}$, roughly), gas, dust and dark matter, held together by gravity. Bigger than a star cluster, smaller than a cluster of galaxies. (This question isn't entirely settled; see http://arxiv.org/abs/1101.3309v1) There are hundreds of billions of galaxies, but we'll start with an overview of our galaxy, the Milky Way.

## XXIX.3 Early observations

On a dark night, the Milky Way can be seen as a luminous band of light across the sky. This was called *galaktikos kuklos* in ancient Greek, meaning "milky circle;" this is the origin of the work "galaxy." Galileo observed the band to be made up of many individual stars. A hypothesis that explains the Milky Way is that the sun is embedded in a thin disk of stars; when we look perpendicular to the disk we see few stars and the sky is dark, while we see many stars when we look into the plane of the disk. This disk of stars is a major component of the galaxy.

## XXIX.4 Star counts

First and simplest method of determining the size and shape of the galaxy.

### XXIX.4.1 The size and shape of the Galaxy from star counts

Start with some simplifying assumptions:

- All stars have the same absolute magnitude $M$. Not generally true, but we can choose to look only at a particular type of star.
- The number density of stars $n$ is constant within our Galaxy.
- There is no absorption of starlight by dust. (Dubious!)

A star of absolute magnitude $M$ will have an apparent magnitude $m$ when it is at a distance

$$d = 10^{0.2(m-M+5)} \text{ pc} \tag{10}$$

(this is just a rearrangement of the distance modulus). Every star closer than $d$ will be brighter than $m$. So the total number of stars brighter than $m$ will be

$$N(< m) = \frac{4\pi}{3} d^3 n = \frac{4\pi}{3} 10^{0.6(m-M+5)} n, \tag{11}$$

or

$$\log N = 0.6m + \text{ constant}. \tag{12}$$

By going 1 magnitude fainter you should increase the number of stars you see in a given patch of sky by a factor of $10^{0.6} \approx 4$.

Now suppose there are no stars beyond a distance $d_{\max}$. This will mean there are no stars fainter than $m_{\max}$, where

$$m_{\max} = M + 5 \log d_{\max} - 5 \tag{13}$$

(of course this requires that your survey is sensitive enough to detect stars fainter than $m_{\max}$ if they were there). So if we find $m_{\max}$ for a particular patch of sky, we can find $d_{\max}$ in that direction:

$$d_{\max} = 10^{0.2(m_{\max} - M + 5)} \text{ pc}. \tag{14}$$

This method was used by William and Caroline Herschel in the late 18th century, and more quantitatively by Jacobus Kapteyn in 1922. Both determined that the Sun was near the center of a small disk of stars approximately 5 times larger in diameter than it was thick.

Is this right? Why not? Dust!

## XXIX.5 Globular cluster distribution

Between 1915 and 1919, Harlow Shapley arrived at a better estimate of our place in the Milky Way using the distribution of globular clusters (compact, spherical clusters of old stars whose distances can be estimated because they contain variable stars whose periods depend on their absolute brightness—more on that later). Shapley noticed that globular clusters aren't distributed uniformly across the sky; instead they're concentrated in one half of the sky, centered on constellation Sagittarius. He concluded that the globular clusters were all orbiting the center of the galaxy, which lies in the direction of Sagittarius. He also measured the distances to the globular clusters to estimate the size of the galaxy; got it a bit wrong, since he thought that RR Lyrae stars were brighter than they are, but had the order of magnitude right. We are about 8 kpc from the center of the Milky Way.

Still use both of these methods, with better understandings of dust absorption and distances.

## XXIX.6 Components of the Galaxy

The Galaxy has three basic components: disk, halo and bulge.

### XXIX.6.1 The Disk of the Milky Way

- Most luminous component of Galaxy
- Radius $\sim 25$ kpc, stars to 15–20 kpc
- Gas disk of galaxy, seen in emission from neutral hydrogen, extends further, to $R \sim 25$ kpc from the Galactic center

- Thickness of disk is small compared to its radius: most stars are less than 0.5 kpc from the midplane of the disk
- Studying stellar properties as a function of $z$, the distance from the midplane, shows that the disk can be further divided into two components: a **thin disk** containing stars of all ages including stars that are currently forming, and a **thick disk** made up of stars older than $\sim 5$ Gyr.
- Near the Sun, the distribution of stars in both the thin and thick disks falls exponentially with distance $z$ from the midplane.
  - Thin disk: $n(z) = n_{\rm thin}\exp(-|z|/h_{\rm thin})$, where the scale height $h_{\rm thin} \approx 350$ pc and $n_{\rm thin}$ is the number density of thin disk stars at $z = 0$. (Scale height: the height at which the density drops by a factor of $e^{-1} \simeq 0.37$.)
  - Thick disk: $n(z) = n_{\rm thick}\exp(-|z|/h_{\rm thick})$, where the scale height $h_{\rm thick} \approx 1$ kpc.
  - In the midplane, $n_{\rm thin} \approx 10 n_{\rm thick}$.
  - Sun is member of the thin disk and is about 30 pc above the midplane.
- The disk shows spiral structure, with star formation concentrated in the spiral arms. We'll talk about this more when we discuss spiral galaxies.
- The disk also contains gas and dust. Gas mostly neutral hydrogen, and $M_{\rm dust}/M_{\rm gas} \simeq 0.007$.

### XXIX.6.2 Bulge

- Galaxy has a central bulge, about 1 kpc in radius, extending above and below the disk.
- Scale height ranges from 100 to 500 pc, depending on ages of stars measured. Younger stars have smaller scale heights.
- Tiny nucleus in the center, bright at radio wavelengths. More on that later.
- Difficult to observe because of large amounts of dust extinction.
- Contains a mixture of stars of different ages

### XXIX.6.3 Halo

- Roughly spherical distribution of stars
- Radius $\sim 100$ kpc
- Same luminosity as bulge, but volume $\sim 10^6$ times larger
- Old, low metallicity stars
  - Metallicity $Z$: fraction by mass of elements heavier than helium

- Universe is about 74% H, 24% He, 2% other elements
- Big Bang produced H, He, a bit of Li—all heavier elements formed in stars
- Stars form out of gas, create heavy elements, and return the elements to the gas when they die, so metallicity increases with each generation of star formation: good estimate of the age of a stellar population
- Population I stars are young and metal-rich ($Z \geq 0.01$), population II stars are relatively old and low in metals ($Z \leq 0.001$).
- Thin disk is Population I, halo is Population II, thick disk is intermediate.

• Globular clusters

- Spherical clusters of old stars located in halo
- At least 150
- Ages range from 11 to a little over 13 Gyr old (age of universe 13.7 Gyr).

• Dark matter. About 95% of the mass of the Galaxy. More on that later.

### XXIX.6.4 Mass-to-Light Ratios

A simple way to get information about the types of stars responsible for the generation of light. Consider the thin disk: $M \simeq 6.5 \times 10^{10}$ $\mathrm{M}_\odot$ (stars and gas), and (blue) luminosity $L_B = 1.8 \times 10^{10}$ $\mathrm{L}_\odot$. Divide these (always in solar units!) to get the mass-to-light ratio $M/L_B \approx 3$ $\mathrm{M}_\odot/\mathrm{L}_\odot$ (the units $\mathrm{M}_\odot/\mathrm{L}_\odot$ are usually implied).

Recall (or learn...) that a star's luminosity depends strongly on its mass:

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^\alpha \tag{15}$$

where $\alpha \simeq 4$ for $M \gtrsim 0.5$ $\mathrm{M}_\odot$and $\alpha \simeq 2.3$ for $M \lesssim 0.5$ $\mathrm{M}_\odot$. Solve for $M$ and substitute our observed $M/L$ to find an average mass

$$\langle M \rangle = (M/L)^{1/(1-\alpha)}\ \mathrm{M}_\odot \simeq 0.7\ \mathrm{M}_\odot \text{ for } M/L = 3 \tag{16}$$

assuming $\alpha = 4$, which indicates that the disk is dominated by stars slightly less massive than the sun. Makes sense, since low mass stars are the most common and M dwarf stars are the most common type of star observed near the Sun.

What would an $M/L$ much, much higher than this mean? What's the $M/L$ of the whole galaxy (total mass $\sim 10^{12}$ $\mathrm{M}_\odot$, total $L \sim 2 \times 10^{10}$ $\mathrm{L}_\odot$)? What would a lower $M/L$ mean?

# Lecture XXX Kinematics of the Milky Way

### XXX.2 Differential Galactic rotation

Now we'll look at rotational motion in other parts of the Galaxy. The basic principles of Galactic rotation were determined by Jan Oort in 1927. Different types of rotation:

- Rigid body rotation: rotation as a single solid body, like a wheel, with a constant angular speed and $\Theta \propto R$, so stars on larger orbits would move faster.
- Keplerian rotation: all mass concentrated at the center ($M =$ constant), $\Theta \propto R^{-1/2}$ and stars farther from the center move more slowly.
- Constant orbital speed: $\Theta =$ constant, $M \propto R$. This is a reasonable approximation for most of the Galaxy.

### XXX.3 Measuring Galactic rotation

#### XXX.3.1 21-cm H emission

It's hard to see stars to large distance (especially toward the Galactic center) because of dust extinction, so a better way to map the structure of the Milky Way is by looking at gas. We use radio telescopes to look at 21-cm emission from neutral hydrogen (digression: electron spin flip of neutral H), and by looking at the shifts in wavelength of the 21 cm line we measure the radial velocities of gas clouds along a line of sight. We have to use the more general expressions for $v_r$ and $v_t$ for this, since the gas clouds won't necessarily be near the Sun.

Toward the inner part of the Galaxy, radial velocity will be a maximum at the point along the line of sight closest to the Galactic center, and $d = R_0 \cos l$. The line of sight is tangent to the orbit at this point, so the velocity is purely radial and we are measuring the actual rotational velocity at this point. This is called the tangent point, and this is the **tangent point method** of determining Galactic rotation. We measure lots of lines of sight, and assume that clouds with the highest velocities are at the tangent point. For clouds with lower velocities, there is some ambiguity since we don't know the distance to the cloud; can try to estimate it from the angular size, but not all clouds are the same size. Also harder for outer part, since there is no unique orbit with a maximum radial velocity. Really do need to know the distance to the cloud then.

#### XXX.3.2 The rotation curve and dark matter

A plot of rotational velocity vs radius is called a rotation curve. Measurements of velocities of objects at $R > R_0$ (Cepheid variables for which we can measure distances... explain Cepheids?) show that the rotational speed $\Theta(R)$ doesn't significantly decrease with distance beyond $R_0$. This was a big surprise.... if most of the mass were concentrated in the center, stars would follow

Keplerian orbits and velocity would decrease with increasing radius as $\Theta \propto R^{-1/2}$. Instead the rotation curve is flat to the edge of the measurements. This implies that there is significant mass beyond $R_0$—the dark matter halo.

Other spiral galaxies also have rotation curves like this. We'll talk about what this implies about the distribution of mass in the outer parts of galaxies when we talk about spiral galaxies later.

Note also that near the center of the galaxy, the rotation speed rises rapidly with radius. This is consistent with rigid body rotation, and implies that that the mass is roughly spherically distributed and the density is nearly constant. Which brings us to the Galactic center.

# Lecture XXXI The Galactic Center

Kutner 16.6

- Hard to observe because of dust. 30 magnitudes of extinction ($10^{-12}$ of light gets through) in optical, so must observe either at IR wavelengths or longer (radio), or in X-rays or $\gamma$-rays.
- Observing in IR shows that number density of stars $n_* \sim 10^7$ pc$^{-3}$. Compare $n \sim 0.1$ pc$^{-3}$ near the Sun.
- Density of stars rises toward the center as $r^{-1.8}$ to a radius between 0.1 and 1 pc; this is roughly the distribution expected from dynamical consideration of the rapidly rising, "rigid body" portion of the rotation curve.
- Strong radio source called **Sagittarius A** at the center of the Galaxy
  - About 8 pc across, with more extended structure to $\sim 100$ pc
  - Synchrotron emission, produced by relativistic electrons accelerated by a magnetic field
  - Also emission from molecular and ionized gas
  - Complex structure, evidence for recent violent event
- Within Sagittarius A is a highly compact radio source called **Sagittarius A*** (Sgr A*)
  - Angular size measured from radio interferometry: $d \sim 0.8$ milliarcsec $\sim 6$ AU.
  - Sagittarius A* is also an X-ray source, and varies on a timescale $< 1$ hour, which means $d < ct$ where $t$ is 1 light-hour; so $d \lesssim 7$ AU.
  - Luminosity (Sgr A* alone) $\lesssim 3 \times 10^4$ L$_\odot$ (variable...)
- Sagittarius A* is probably an accreting supermassive black hole
  - We can measure the orbits of stars near Sgr A*: e.g. S2 has semimajor axis $a = 920$ AU, period $P = 14.5$ yr. From Kepler's 3rd law, using this and other stars near Sgr A*, we calculate the enclosed mass:

$$M_{\rm SgrA*} = 3.7 \pm 0.2 \times 10^6 \ {\rm M}_\odot \tag{17}$$

  - All stars are on Keplerian orbits, therefore mass is a single central object
  - One star comes within 45 AU of Sgr A*, so size must be smaller than that
  - Must be a black hole — only way to get that much mass in that small a volume
- We can calculate the Schwarzchild radius (radius at which the escape velocity is equal to the speed of light; the event horizon of the black hole):

$$R_{\rm SgrA*} = \frac{2GM_{BH}}{c^2} = 0.08 \ {\rm AU} = 16 \ {\rm R}_\odot \tag{18}$$

  This is below the $\sim 2$ AU resolution limit of current observations.

- Where does all the energetic radiation come from? Accretion onto the black hole. Let's check. Virial theorem: total energy of a system is equal to 1/2 the (time-averaged) potential energy, $E_{\rm tot} = 1/2\ U$. Consider a particle of mass $M$ spiraling onto a black hole from an initial radius $r_i$ to a final radius $r_f$. According to the virial theorem, the energy radiated is half the change in potential energy,

$$E = \frac{1}{2}\left(\frac{GM_{\rm BH}M}{r_f} - \frac{GM_{\rm BH}M}{r_i}\right). \tag{19}$$

  Assume $r_i \gg r_f$ and $r_f = r_S$, the Schwarzchild radius. Then

$$E = \frac{1}{2}\frac{GM_{\rm BH}M}{r_S}. \tag{20}$$

  Assume luminosity is $L = dE/dt$ and mass accretion rate is $\dot{M} = dM/dt$, and substitute the expression for $r_S$:

$$L = \frac{dE}{dt} = \frac{1}{2}\left(\frac{GM_{\rm BH}}{r_S}\right)\left(\frac{dM}{dt}\right) = \frac{1}{4}\dot{M}c^2. \tag{21}$$

  The observed accretion rate is $10^{-3}$ to $10^{-2}$ $\rm M_\odot$ yr$^{-1}$. If we assume $\dot{M} \approx 10^{-3}$ $\rm M_\odot$ yr$^{-1}$ we find $L \sim 3 \times 10^9$ $\rm L_\odot$, much higher than the observed luminosity of Sgr A* of $3 \times 10^4$ $\rm L_\odot$. So first, accretion can easily provide enough energy to account for the observed luminosity, and second, the energy release from accretion isn't as efficient as we've assumed in this order of magnitude calculation.

# Lecture XXXII Types of Galaxies and the Hubble Sequence

Kutner 17.1

### XXXII.2 Are there other galaxies outside the Milky Way?

Yes.

### XXXII.3 The Hubble sequence

Once it was known that there were other galaxies out there, we needed to classify them. Goals of a galaxy classification scheme:

- Impose order
- Reveal correlations between properties or evolution
- Classification should be complete—include every galaxy
- Classification should be economical—don't include irrelevant details (how do we know which details are irrelevant?)

In 1926, Edwin Hubble suggested a classification scheme which is still used today. This is a morphological classification scheme which divides galaxies into categories based on their overall appearance. Three general categories, with subdivisions:

- Ellipticals (E)
- Spiral, divided into normal spirals (S) and barred spirals (Sb)
- Irregular

Galaxies intermediate between spirals and ellipticals are called lenticular (S0 or Sb0).

The types of galaxies were arranged in a diagram shaped like a tuning fork. A galaxy's classification according to this scheme is called its Hubble type.

Hubble originally thought that this might be an evolutionary sequence, and therefore called elliptical galaxies *early types* and spiral galaxies *late types* (with Sc later than Sa). This isn't true, but astronomers still use the early and late terms to describe these types of galaxies.

### XXXII.3.1 Elliptical galaxies

Elliptical galaxies are classified according to their observed **ellipticity**. We measure the major axis $a$ and minor axis $b$, and define the ellipticity

$$\epsilon \equiv 1 - b/a. \quad (22)$$

We classify the galaxy as E$n$, where $n = 10(1 - b/a)$, rounded to the nearest integer.

There are no ellipticals seen with shapes flatter than E7.

Obvious problem: classification depends on our viewing angle, and the observed ellipticity may not be the actual ellipticity of the galaxy. Note though that the true flattening is always greater or equal to what we observe.

Elliptical galaxies are **triaxial**: spheroids with axes $a$, $b$ and $c$. For a sphere, $a = b = c$. Galaxies may be **oblate** or **prolate**. A perfectly oblate galaxy has $a = b$ and $c < a$: flattened sphere. A perfectly prolate galaxy has $b = c$ and $a > b$: one axis extended, like a football. These two galaxies can look the same, depending on the viewing angle.

Wide variety of elliptical galaxies: giant ellipticals are the biggest galaxies in the universe, smallest dwarfs comparable in size to globular clusters. Mass range $\sim 10^7$ to $\sim 10^{13}$ $M_\odot$.

### XXXII.3.2 Spiral galaxies

Features used to classify spiral galaxies:

- Bulge-to-disk ratio, $B/D$, the ratio of the luminosities of the bulge and the disk. Largest $L_{\rm bulge}/L_{\rm disk} \sim 0.3$.
- Smoothness of the distribution of stars. Reflects current star formation, since bright spots are regions where hot, bright young stars have just formed.
- Pitch angle of spiral arms: how tightly wound the spiral arms are

Galaxies with the largest bulge-to-disk ratios, smoothest stellar distributions, and most tightly wound spiral arms are Sa (or Sba). Sc (and Sbc) galaxies have smaller B/D ($L_{\rm bulge}/L_{\rm disk} \sim 0.05$, loosely wound spiral arms, and the spiral arms are clumpy, can be resolved into stars and HII regions (HII, ionized hydrogen—compare HI. Hot young stars ionize the gas around them, and these are called HII regions.)

Spiral galaxies are large, with masses $\sim 10^9$ to $\sim 10^{12}$ $M_\odot$, and don't vary as much as elliptical galaxies. M31 (Andromeda) is Sb. Milky Way is probably SBbc.

### XXXII.3.3 Irregular galaxies

Remaining galaxies were called irregular, placed off to the side of the diagram. These are usually not very large, but can vary widely.

### XXXII.4 General trends

- Spiral galaxies are generally blue, ellipticals red
- Spirals have more dust and gas than ellipticals
- Most current star formation occurs in spirals
- These things are related!

# Lecture XXXIII Spiral Galaxies I

Goal: understand the 3-d structure of galaxies—how do we get to this from what we observe?

## XXXIII.2 Surface brightness

Easiest to observe: the 2-d surface brightness of a galaxy.

Surface brightness $\Sigma$: luminosity per unit area (e.g. $\mathrm{L}_\odot \mathrm{pc}^{-2}$). Also used, $\mu$, magnitudes per square arcsecond. This is what we actually measure.

Surface brightness of night sky isn't zero (depends on wavelength), so hard to observe faint galaxies. May have to subtract background.

For nearby galaxies (when we can neglect cosmological effects) surface brightness does not depend on distance:

$$\Sigma = \frac{\mathrm{flux}}{\theta^2} \approx \frac{L/d^2}{\mathrm{area}/d^2} \approx \frac{L}{\mathrm{area}} \approx \text{ constant} \tag{23}$$

$\theta^2$ is the angular area of the source. Flux from a patch of sky decreases as $1/d^2$, but angular area of the patch also decreases as $1/d^2$, so surface brightness constant.

## XXXIII.3 Kinematics of spiral galaxies

Measuring the surface brightness profiles of galaxies tells us about the distribution of luminous matter, but not about the total distribution of mass. In order to measure this, we need to study the kinematics of galaxies.

We saw that the rotation curve of the Milky Way flattens at large radii, with a constant rotational velocity of $\sim 220$ km s$^{-1}$ . In the 1970s and 80s Vera Rubin showed that spiral galaxies generally have flat rotation curves (surprise!), and that therefore there is a lot of mass beyond the luminous matter we can see.

What do flat rotation curves tell us about the mass distribution? Consider a particle of mass $m$, in a circular orbit at radius $r$ around a spherically symmetric mass distribution $M$. The gravitational force on the particle is

$$F_{\mathrm{grav}} = \frac{GMm}{r^2} \tag{24}$$

and the centripetal force is

$$F_{\mathrm{c}} = \frac{mv_c^2}{r} \tag{25}$$

where $v_c$ is the (constant with radius!) circular velocity of the particle. These forces are equal:

$$v_c^2 = \frac{GM}{r}. \tag{26}$$

We can use this to determine the dark matter density profile in the outer parts of the galaxy.

Solve for $M$ and differentiate:

$$\frac{dM}{dr} = \frac{v_c^2}{G}. \tag{27}$$

We also know

$$\frac{dM}{dr} = 4\pi r^2 \rho(r). \tag{28}$$

Setting these equal,

$$\frac{v_c^2}{G} = 4\pi r^2 \rho(r) \tag{29}$$

and

$$\rho(r) = \frac{v_c^2}{4\pi G r^2}. \tag{30}$$

So we've shown that the density of dark matter goes as $r^{-2}$ at large radii! This is called an **isothermal sphere—**constant velocities inside sphere.

It"s often suggested that the density profile of dark matter halos may be universal, i.e. the same over a wide mass range. Still a matter of debate, but a commonly used form is the **NFW profile** (Navarro, Frenk and White, 1996):

$$\rho_{\rm NFW}(r) = \frac{\rho_0}{(r/a)(1+r/a)^2}. \tag{31}$$

This behaves like an $r^{-2}$ profile over much of the halo, but is shallower ($\sim 1/r$) near the center and steeper ($\sim 1/r^3$) near the edge.

## XXXIII.4 The Tully-Fisher relation

Vera Rubin's work also showed that the maximum rotational velocity of spiral galaxies depends on the galaxy type and luminosity: spirals of earlier type have larger $v_{\rm max}$, and more luminous galaxies have larger $v_{\rm max}$.

The correlation between the luminosity and maximum rotational velocity of spiral galaxies is known as the **Tully-Fisher relation** (1977). In luminosity terms, this is approximately $L \propto v_{\rm max}^4$, though the details depend somewhat on the type of galaxy and the band. As usual in astronomy, we plot this logarithmically to make it a straight line:

$$M_B = -10.2 \log v_{\rm max} + 2.71 \tag{32}$$

for Sb galaxies (other types in C & O). Recall $M = M_{\rm Sun} - 2.5\log(L/L_\odot)$, so $L \propto v^4$ is equivalent to $M = -10 \log v +$ const.

Better at IR wavelengths: less affected by dust, and the light comes from stars that are a better tracer of the overall luminous mass distribution ($B$-band light is mostly from young stars in regions of recent star formation).

Note also: distance indicator! If we can measure the maximum rotational velocity of a spiral galaxy and have reason to believe it should fall on the Tully-Fisher relation, we can estimate its absolute magnitude and thus its distance.

Can we understand where this relationship comes from?

We saw above that for flat rotation curves,

$$v_c^2 = \frac{GM}{r}. \tag{33}$$

We write $M = (M/L) \times L$, and $L = \Sigma\pi r^2$, where $\Sigma$ is the average surface brightness (luminosity per unit area of the galaxy), so $r^2 = L/(\Sigma\pi)$. Then square both sides and substitute:

$$v_c^4 = \frac{G^2 M^2}{r^2} = G^2 (M/L)^2 L^2 \frac{\Sigma\pi}{L} = \left[G^2 (M/L)^2 \Sigma\pi\right] L, \tag{34}$$

or $L \propto v_c^4$. Notice that we've assumed that all galaxies have the same $M/L$ and the same average surface brightness, neither of which is true—so the fact that this is actually observed is somewhat surprising. Another way to think about this: the radius in our original expression for velocity is the radius enclosing all the mass, while the radius we've used for the luminosity (surface brightness) is the radius enclosing all the light—these aren't the same. So the fact that this correlation is observed suggests that the total mass-to-light ratios don't actually vary all that much.

This also explains some of the variation with Hubble type and observed band, since $M/L$ and surface brightness depend on these things.

# Lecture XXXIV Spiral Galaxies II

Kutner 17.3

The spiral structure of spiral galaxies varies.

The classic spiral galaxy is called a **grand design spiral**, with two symmetric and well-defined arms. Others have more than two arms, or arms that appear fragmented. Galaxies without well-defined arms that can be traced over a large angular distance are called **flocculent spirals**.

Grand design: $\sim 10$%
Multiple arm: $\sim 60$%
Flocculent: $\sim 30$%

Optical images of spiral galaxies are dominated by the arms, especially in blue light, because massive, hot (O and B) stars are found in the spiral arms. These stars live for only $\sim 10$ Myr, so their presence indicates active star formation. The bulk of disk of the galaxy is dominated by older, redder stars. Spiral arms also have gas and dust.

## XXXIV.2 Trailing and leading spiral arms

There are two possibilities for how the spiral arms can be oriented with respect to the rotation of the galaxy. The tips of **trailing arms** point in the opposite direction from the direction of rotation, and **leading arms** are the opposite. Intuitively, it looks as if the arms should be trailing, and this is the case in most galaxies in which it can be measured. A few galaxies have a combination of leading and trailing arms, however; this is probably a result of an encounter with another galaxy.

## XXXIV.3 The winding problem

An obvious suggestion is that spiral structure is due to galactic rotation. Differential rotation, with stars closer to the center of the galaxy moving faster, will naturally generate spiral arms. Problem: after a few orbits, the arms will be too tightly wound to be observed (recall Sun has orbited Milky Way $\sim 20$ times). This is called the **winding problem**. This also shows that the spiral arms can't be fixed components of the same stars and gas.

## XXXIV.4 The origin of spiral structure: density waves

Leading theory for the origin of spiral structure is the **Lin-Shu density wave theory**. This says that spiral structure is due to long-lived **quasistatic density waves**. These are regions in the galactic disk where the mass density is ~10 to 20% higher than average. Stars and gas clouds move through these regions of enhanced density as they orbit around the center of the galaxy. This is like cars moving through a traffic jam—the density increases in the traffic jam, and the cars slowly move through it, but the traffic jam itself doesn't move (much).

Define the **global pattern speed** $\Omega_{gp}$: this is the angular speed of the spiral pattern. Viewed in a noninertial reference frame rotating with $\Omega_{gp}$, the spiral pattern is stationary.

The stars aren't necessarily stationary, however. Stars near the center of the galaxy can have orbital speeds shorter than the density wave pattern ($\Omega > \Omega_{gp}$), so they will overtake a spiral arm, move through it, and continue on until they reach the next arm. Stars far from the center of the galaxy will be moving more slowly than the density wave pattern ($\Omega < \Omega_{gp}$), so they will be overtaken by the spiral arm. At some distance from the center the stars and the density wave will have the same angular speed. This is called the **corotation radius** ($R_c$). In this noninertial reference frame, stars with $R < R_c$ will appear to pass through the arms moving in one direction, and stars with $R > R_c$ will appear to move through in the opposite direction.

This theory explains observations:

- Star formation is concentrated in spiral arms: as gas in the galaxy passes through the density wave it's compressed, and this increase in density makes it more likely to collapse and form stars.
- Distribution of star formation: collapse of gas into stars takes some time, so star formation will be observed somewhat downstream from the leading edge of the spiral arm.
- O and B stars concentrated in spiral arms, red stars distributed throughout disk: The most massive O and B stars don't live very long, so they don't have time to pass entirely through the spiral arm and become generally distributed throughout the galaxy. Longer-lived, redder stars do.

Where does the density wave come from, and how is it maintained?

Very generally speaking....

- Stellar orbits about the center of the galaxy aren't perfectly circular (we saw before that the Sun has a peculiar velocity (the solar motion) with respect to the local standard of rest).
- We can describe these orbits as motion about an equilibrium position that is moving in a perfectly circular orbit.
- This is simple harmonic motion—oscillations superimposed on a perfectly circular orbit.
- The resulting motion of the star is a non-closing rosette pattern, when viewed in an inertial frame.
- In a noninertial frame (rotating with pattern speed $\Omega_{gp}$), the orbits are closed (they overlap, the star returns to the same place it was before) and elliptical. These orbits can have major axes aligned, producing a bar structure, or each one can be rotated relative to those next to it. This produces a spiral patterned density enhancement which is stationary in the noninertial frame. Pattern depends on the number of oscillations per orbit.

# Lecture XXXV Elliptical Galaxies

According to the Hubble sequence, elliptical galaxies are classified only by their degree of observed ellipticity. This turns out to be not particularly useful; unlike the properties used to classify spirals, the observed ellipticity shows very little correlation with other properties of the galaxies.

Elliptical galaxies have a much wider range in mass than spirals. A lot of different types:

- **Cd galaxies**. These are the biggest and most massive galaxies in the universe. Rare, found near the centers of large galaxy clusters. Can be nearly 1 Mpc across, have masses between $10^{13}$ and $10^{14}$ $\mathrm{M}_{\odot}$. Very high mass-to-light ratios, implying lots of dark matter.
- **Normal ellipticals**. Centrally condensed, high central surface brightness. Mass range $10^8$ to $10^{13}$ $\mathrm{M}_{\odot}$, sizes from $< 1$ kpc to almost 200 kpc. $M/L$ ranges from 7 to $> 100$.
- **Dwarf elliptical (dE)**. Lower surface brightness than normal ellipticals, mass range $10^7$ to $10^9$ $\mathrm{M}_{\odot}$, sizes $\sim$ 1–10 kpc.
- **Dwarf spheroidal (dSph)**. Very low luminosity, low surface brightness; detected only near the Milky Way. Masses $\sim 10^7$–$10^8$ $\mathrm{M}_{\odot}$, diameters 0.1 to 0.5 kpc.
- **Blue compact dwarf galaxies (BCD)**. Small and unusually blue, indicating rapid star formation. Masses $\sim 10^9$ $\mathrm{M}_{\odot}$, sizes $< 3$ kpc. Large gas masses and low $M/L$, consistent with their high star formation rates.

### XXXV.2 Dynamics of elliptical galaxies

Unlike spiral galaxies, elliptical galaxies generally aren't coherently rotating—the stars are on **random orbits**. To measure the velocities of a system like this, we use the **velocity dispersion**:

$$\sigma^2 \equiv \frac{1}{N}\sum_{i=1}^{N}(v_i - \langle v\rangle)^2 \tag{35}$$

This should look familiar; this is the expression for the variance of a population ( $v_i - \langle v\rangle$ is the difference between the velocity of the $i$th star and the mean velocity of the system). So the velocity dispersion $\sigma$ is equivalent to the standard deviation of the velocity distribution.

This is measured from the widths of absorption or emission lines in the spectrum of a galaxy—the light from each star will be Doppler shifted due to the velocity of that star, so for a wider range of stellar velocities, the total width of a line produced by stars will be broader.

How do we estimate the mass of a system like this?

**Virial theorem**

$$-2\left\langle KE\right\rangle = \left\langle PE\right\rangle. \tag{36}$$

For a galaxy of $N$ identical stars of mass $m$ and with no coherent motions, $\langle v \rangle = 0$, the kinetic energy is

$$\begin{aligned} \langle KE \rangle &= \sum_{i=1}^{N} \frac{1}{2} m v_i^2 && (37) \\ &= \frac{N}{2} m \sigma^2 && (38) \\ &= \frac{M_{\rm tot} \sigma^2}{2} && (39) \end{aligned}$$

Potential energy?

Consider the potential energy of a spherical shell at radius $r$, surrounding a mass $M(< r)$. The mass of the shell is $dm = 4\pi r^2 dr \times \rho(r)$. The potential energy is

$$\langle PE \rangle = -\frac{3}{5} \left( \frac{G M_{\rm tot}^2}{R} \right) \tag{40}$$

The $3/5$ factor is a little tricky, but the rest is easy.

Now return to the virial theorem:

$$-2 \left\langle KE \right\rangle = \left\langle PE \right\rangle. \tag{41}$$

So

$$-2 \left[ \frac{M_{\rm tot} \sigma^2}{2} \right] = \frac{3}{5} \left( \frac{G M_{\rm tot}^2}{R} \right) \tag{42}$$

for a spherical, constant density galaxy (can work out more accurate versions for any $\rho(r)$).

Rearranging:

$$M_{\rm tot} = \frac{5R}{G} \left[ \frac{\sigma^2}{3} \right] \tag{43}$$

Bigger mass $\Rightarrow$ Larger velocity dispersion.

Now consider the total velocity dispersion. For a spherical system, the stellar velocities have three components:

$$\sigma^2 = \sigma_r^2 + \sigma_\theta^2 + \sigma_\phi^2 \tag{44}$$

$\sigma_r^2$ is the **radial velocity dispersion** and is the only one we can measure (the other two are perpendicular to our line of sight). In general,

$$\sigma_r^2 \neq \sigma_\theta^2 \neq \sigma_\phi^2. \tag{45}$$

However, if $\sigma_r^2 = \sigma_\theta^2 = \sigma_\phi^2$ (**isotropic** velocity dispersion),

$$\sigma^2 = 3\sigma_r^2 \tag{46}$$

and

$$M_{\rm tot} = \frac{5R\sigma_r^2}{G}. \tag{47}$$

The mass obtained this way is called the **virial mass**.

Problem: velocity dispersions aren't necessarily isotropic ⇒ significant uncertainty in measuring $M$.

### XXXV.2.1 What flattens elliptical galaxies?

Rotation, as with disks? Or an anisotropic velocity distribution? Clearly it's not rotation at least some of the time. Example: Luminous E4 galaxy NGC 1600 (recall classification of ellipticals, ellipticity $\epsilon \equiv 1 - b/a$, where $a$ is major axis and $b$ is minor axis; classification is E$n$, where $n = 10(1 - b/a)$. So an E4 galaxy has ellipticity 0.4) has $V_{\rm rot} = 1.9 \pm 2.3$ km s$^{-1}$ and $V_{\rm rot}/\sigma < 0.013$—no significant rotation, and yet significantly flattened. Can formalize this a bit. Expect that for a rotationally flattened system wtih ellipticity $\epsilon$:

$$\frac{V_{\rm rot}}{\sigma} \approx \left(\frac{\epsilon}{1-\epsilon}\right)^{1/2} \tag{48}$$

So if an E4 galaxy is rotationally flattened, it should have $V_{\rm rot}/\sigma \approx 0.8$, much higher than the observed $V_{\rm rot}/\sigma < 0.013$. So NGC 1600 and similar luminous ellipticals are not flattened because of rotation; must have some other cause.

Ellipticals that aren't flattened by rotation may be flattened by **velocity anisotropy**. Example: start with a rotating, flattened galaxy. Take half the stars and rotate them in the opposite direction. Kinetic energy is the same, and the galaxy is flattened by the same amount, but there is no net rotation, and the circular velocity dispersion is now much bigger than the vertical one. Things like this are actually observed—"counter-rotating cores"—and probably indicate a prior merger or accretion of another galaxy. Ellipticals can have complicated orbital structure—different fractions of stars on radial and various circular orbits.

Low mass ellipticals are generally flatted by rotation, high mass by velocity anisotropy.

Galaxies are considered to be **rotationally supported** or **pressure supported**, depending on whether their stellar velocities are mostly ordered rotation or random motions.

## XXXV.3 Scaling relations for elliptical galaxies

There is a relationship for elliptical galaxies similar to the Tully-Fisher relation for spirals, relating the velocity dispersion to the luminosity rather than the rotational velocity. This is the **Faber-Jackson relation**:

$$L \propto \sigma^4. \tag{49}$$

This relationship has quite a bit of scatter, and it's found that we can reduce the scatter by including another parameter, the effective radius $r_e$ (remember that this is the radius that encloses half the light of the galaxy). Observationally, we find

$$L \propto \sigma^{2.65}\, r_e^{0.65} \tag{50}$$

for the whole family of elliptical galaxies. This is called the **fundamental plane**. Both this and the Faber-Jackson relation can be roughly reproduced by considering relationships between mass and velocity (from the virial theorem), surface brightness and the mass-to-light ratio, as we showed with the Tully-Fisher relation. The fact that this correlation is observed for such a wide range of different galaxy types is clearly telling us something important about how galaxies form.

# Lecture XXXVI Gas, Dust and Star Formation In Galaxies

In addition to stars, galaxies contain **gas** and **dust**, which are both very important to the formation of stars in galaxies. The gas and dust in galaxies is called the **interstellar medium (ISM)**. The ISM and the process of star formation are complex and inter-connected; they both affect each other.

Gas in galaxies is primarily hydrogen, and comes in three basic forms:

- **Molecular**: $H_2$—forms in cold, dense regions
- **Atomic/Netural**: "HI"—single atoms of neutral hydrogen
- **Ionized**: "HII"—ionized hydrogen

All forms coexist in galaxies.

Almost all galaxies have gas, but in amounts varying from nearly zero to being the dominant state of matter (i.e. nearly all gas, few stars). Ellipticals have less gas and spirals have more gas.

## XXXVI.2 Phases of the Interstellar Medium

### XXXVI.2.1 Molecular gas

- Very dense ($10^8$ to $10^{11}$ atoms m$^{-3}$) and cold ($T \sim 20$ K)
- Usually found in clumps called molecular clouds or Giant Molecular Clouds (GMCs)
- This is where stars form—gas that is dense and cold can collapse more easily to form stars
- Hard to detect because it has no easily observed emission lines. Usually detected indirectly by using carbon monoxide (CO) as a tracer, which has emission lines that can be detected at submillimeter wavelengths.
- Other, more exotic molecules also seen: $H_2O$, $H_2CO$ (formaldehyde), $H_2SO_4$ (sulfuric acid), $CH_4$ (methane), $NH_3$ (ammonia), many others

### XXXVI.2.2 Atomic gas

- Less dense, warmer than molecular gas. $n \sim 10^6$ atoms m$^{-3}$, $T \sim 100$ – 1000 K
- Easily detected through the 21-cm emission line with radio telescopes—electron spin flip transition between the ground state hyperfine levels of hydrogen
- Confined to a thin layer in the plane of disk galaxies
- Extends to much larger radii than starlight ⇒ good for studying the dynamics of galaxies at large radii, through Doppler shifting of the 21-cm line

### XXXVI.2.3 Ionized gas

- Hot ($T \gtrsim 1000$ K)
- Heated by some combination of
  - ionizing radiation, emitted in the ultraviolet (UV), usually by very hot and short-lived stars. Hydrogen ionizing photons have energies greater than 13.6 eV, or wavelengths shorter than 912 Å (1 Å= $10^{-10}$ m, this is how astronomers measure wavelengths of optical light).
  - kinetic energy: shocks. These can come from supernovae or rapid mass loss in the late stages of stellar evolution
- Detected by recombination emission lines (free electrons recombining with nuclei), usually detected in the optical. One of the strongest and most common: H$\alpha$, the $n = 3$ to $n = 2$ transition, seen at 6563 Å.
- Three main physical components:
  - HII regions—compact, ionized regions near young stars (OB associations). These are bright spots in optical galaxy images.
  - Diffuse or Warm Ionized Medium—widely distributed, heated by "escaping" UV radiation
  - Hot Ionized Medium—usually in halo, heated by shocks (galaxy halo... get more on this)

These phases are largely in pressure equilibrium with each other.

$P = nkT \Rightarrow$ if in equilibrium, $nT \sim$ constant, so high densities and low temperatures can be in balance with low densities, high temperatures.

The ISM is dynamic—phases change and evolve, largely in response to star formation.

And... the total amount of gas changes with time.

**Sources** (processes which add gas):

- Infall of fresh gas from outside the galaxy
- Return of gas from evolving stars (mass loss from stellar winds in the late stages of stellar evolution, supernovae)
- Accretion of other galaxies

**Sinks** (processes which remove gas):

- Gas condenses into stars

- Supernovae, stellar winds blow gas out of the galaxy
- Gas stripped by interactions with nearby galaxies
- Metals in gas condense into dust

The phases of the ISM respond to the current state of star formation, and the rate of star formation responds to the state of the gas. Very useful to consider the **star formation rate (SFR)** in a galaxy.

$$\text{SFR} \equiv \text{mass in new stars formed per unit time, expressed in } \mathrm{M}_\odot \text{ yr}^{-1} \tag{51}$$

Within galaxies and from galaxy to galaxy, there is a close relationship between the density of the gas and the amount of star formation. Because most star formation takes place in disk galaxies, this can be expressed in terms of the surface denisties of star formation and gas (gas mass per unit area $\Sigma_{\rm gas}$, star formation rate per unit area $\Sigma_{\rm SFR}$). The relationship is called the Schmidt law:

$$\Sigma_{\rm SFR} \propto \Sigma_{\rm gas}^{a}, \tag{52}$$

where $a$ is an exponent that tells us about the efficiency of star formation (how good galaxies are at turning their gas into stars).

# Lecture XXXVII The Stellar Populations of Galaxies

How do we understand the *integrated* light of galaxies? The colors and spectra of galaxies vary from galaxy to galaxy—why?

Galaxy light comes from stars, so we can understand the evolution of galaxies by understanding the evolution of stars.

The **stellar population** of a galaxy refers to the different types of stars which make up its light.

Consider the evolution of a single burst of star formation:

1. A bunch of stars are born simultaneously, with a range of masses—lots of low mass stars, few high mass stars
2. When these stars are born, they occupy the zero-age main sequence (ZAMS)
3. As time progresses, high mass stars use up their fuel and evolve off the main sequence. They climb the red giant branch (RGB) and asymptotic giant branch (AGB), before either blowing up as a supernova or fading to a white dwarf.
4. As more time passes, progressively lower mass stars will evolve off the MS

Steps 3 and 4 change the proportions of red and blue stars. An observer looking at all the stars at once will see the colors change with time.

## XXXVII.2 The initial mass function

The distribution of stellar masses in a population of newly-formed stars is called the **initial mass function (IMF)**, $\xi(M)$.

$$\text{Number of stars with masses between } M \text{ and } M + dM = N_0\, \xi(M)\, dM \tag{53}$$

where $N_0$ depends on the size of the burst of star formation.

This is usually written as a power law:

$$\xi(M) = \frac{dN}{dM} = CM^{-(1+x)} \tag{54}$$

where $x$ may take different values for different mass ranges and $C$ is a normalization constant. Popular value: $x = 1.35$ — Salpeter IMF.

The initial mass function is very difficult to measure! It's often assumed to be universal — the same for all bursts of star formation in all environments and at all times. There is evidence both for and against this, and the question really isn't settled. The IMF probably flattens out at low masses.

**Low mass stars are much more common than high mass stars, and most mass is in low mass stars.**

## XXXVII.3 Stellar evolution and the color magnitude diagram

How do we interpret the colors and luminosities of a collection of stars? Start by analyzing the **color-magnitude** diagram for a **single stellar population —** a bunch of stars that all formed at the same time, for example in a globular cluster.

The color-magnitude diagram is a plot of the color (e.g. $B - V$) vs magnitude of a bunch of stars. This is effectively the H-R diagram, since color is directly related to temperature and magnitude is a measure of luminosity (we'd like absolute magnitude, but if the stars are all at the same distance, as in a cluster, we can use apparent magnitude instead). Reminder of what this looks like, and the nomenclature.

We analyze evolving stellar populations through **isochrones**. An isochrone is the locus in the color-magnitude diagram of a single stellar population at a single point in time. By considering isochrones of different ages we can see that **as a stellar population ages it gets redder**.

## XXXVII.4 Stellar populations in galaxies

Galaxies have more complex star formation histories—they have had more than one burst of star formation in the past. We think of these as **sums of different single stellar populations with different ages**. The relative amounts of older and more recent star formation affect the colors of the galaxy in the same way:

More star formation long ago $\Rightarrow$ redder, fainter (higher M/L)

More star formation recently $\Rightarrow$ bluer, brighter (lower M/L)

The past star formation rate as a function of time in a galaxy is called the **star formation history**. Any star formation history can be represented as the sum of single stellar populations of different ages. Combining these sums of stellar population and comparing them with the observed colors of a galaxy (in as many different bands as possible) is called **population synthesis modeling**, and is one of the major ways that we estimate how old galaxies are.

**Important:** Other things also change the colors of galaxies.

Dust preferentially scatters and absorbs blue light $\Rightarrow$ dusty galaxies look redder.

Metallicity. Higher metallicity stars are also redder. (Opacity is higher in stars; photons take longer to reach the surface. Star puffs up, gets redder. Line blanketing.)

These things, especially dust, need to be included when attempting to match the colors of galaxies with models.

# Lecture XXXVIII Galaxy Spectra

The colors of a galaxy (the integrated light) can tell us about its stellar population, but we can learn even more information from its spectra. Galaxies contain gas and stars; unsurprisingly, the spectrum of a galaxy looks like some mixture of the spectra of stars and clouds of gas, and it varies depending on the type of the galaxy.

### XXXVIII.2 Review: the production of spectral lines — the Bohr atom

Before the reasons for it were understood, it was observed that hydrogen gas emitted lines with wavelengths given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \tag{55}$$

where $m$ and $n$ are integers with $m < n$ and $R_H$ is the experimentally determined Rydberg constant for hydrogen.

Based on simple quantum mechanics, we can write the kinetic energy:

$$\frac{1}{8\pi\epsilon_0}\frac{e^2}{r} = \frac{1}{2}\mu v^2 = \frac{1}{2}\frac{(\mu v r)^2}{\mu r^2} = \frac{1}{2}\frac{(n\hbar)^2}{\mu r^2}. \tag{56}$$

We can solve this for $r$ to see that only certain values are allowed:

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}n^2 = a_0 n^2, \tag{57}$$

where $a_0 = 5.29 \times 10^{-11}$ m $= 0.0529$ nm is the Bohr radius. Only certain orbits are allowed, with $r = a_0$, $4a_0$, $9a_0$, etc.

We then insert this expression for the radius into our expression for the total energy of the atom to find the allowed energies:

$$E_n = -\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = -13.6 \text{ eV } \frac{1}{n^2}. \tag{58}$$

The integer $n$ is the principal quantum number. When the electron is in the ground state, $n = 1$ and it takes at least 13.6 eV to ionize the atom. When it's in the first excited state, $n = 2$ and the atom's energy is higher, $E_2 = -13.6/4\text{eV} = -3.40$ eV.

Electrons can move between energy levels. If an electron goes from a higher state to a lower state, it emits a single photon with energy $E = E_{\text{high}} - E_{\text{low}}$, or

$$E = \frac{hc}{\lambda} = E_{n,\text{high}} - E_{n,\text{low}}. \tag{59}$$

Using the expression for $E_n$, we find

$$\frac{1}{\lambda} = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c}\left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right), \tag{60}$$

which is the observed relationship we started with, with the combination of constants equal to the Rydberg constant. Spectral lines of hydrogen appear in different series: the Balmer series were the first observed, with $n_{\rm low} = 2$. The Lyman series has $n_{\rm low} = 1$ and the Paschen series has $n_{\rm low} = 3$. The most commonly observed lines are the Balmer lines in the optical: H$\alpha$ is the strongest at 6563 Å, then H$\beta$ at 4861 Å, H$\gamma$, etc.

Electrons can also move from a lower to a higher energy level by absorbing a photon with energy equal to the energy difference between two levels.

With this information we can understand the production of spectra from hot sources, clouds, and hot sources seen through clouds.

1. A hot dense gas or a hot solid object produces a **continuous spectrum** with no spectral lines. This is blackbody radiation, and the spectrum is determined by the temperature of the object.

2. A hot, diffuse gas produces bright **emission lines**, when electrons make downward transitions from higher to lower orbits. If the cloud is near a strong source of radiation, the electrons will be continually excited by photons, and emit photons as they then fall back to lower orbits. These are called **recombination lines**.

3. If diffuse gas is seen in front of the source of radiation, it will produce **absorption lines** in the continuous spectrum of the hot object. These occur when the atom absorbs a photon and the electron makes a transition from a lower to a higher energy level. This is the case with a star: diffuse gas in the stellar atmosphere produces an absorption line spectrum against the continuous blackbody spectrum of the star itself.

(These are Kirchoff's laws.)

## XXXVIII.3 The real world — applications to galaxies

Spectra of galaxies look like spectra of stars and gas. Examples:

Spectra of elliptical galaxies have strong absorption lines, due mostly to metals in the atmospheres of cool, low mass, low luminosity stars.

Compare the spectrum of a low mass star:

Elliptical galaxies have weak or no emission lines, because they don't have much gas.

Spectra of star-forming galaxies (spiral, irregular) look different. They have emission lines from gas ionized by hot young stars, blue light from those young stars, and red light and absorption features from the underlying older stellar population.

Spectra of gas-rich galaxies with high star formation rates are dominated by emission lines:

## XXXVIII.4 What can we learn from spectral lines?

What wavelength corresponds to an energy of 13.6 eV? How hot is a star that produces lots of photons with energy of at least 13.6 eV?

$$\lambda = \frac{hc}{E} = 912 \text{ Å}, \tag{61}$$

and

$$T = \frac{0.0029 \text{ m K}}{\lambda} = 32,000 \text{ K}. \tag{62}$$

What type of star is this? Somewhere between O8 and B0. How long do these stars live? $\sim 5$ Myr. This is instantaneous, considering the lifetime of a galaxy. $\Rightarrow$ The presence of emission lines from ionized hydrogen indicates current star formation (unless they're from shocks or an AGN, but we can tell that by looking at line ratios and line widths).

We can get useful information from this! The number of hydrogen ionizing photons is directly proportional to the number of massive stars, which means that if we can measure how much radiation there is from ionized gas, via the H$\alpha$ emission line for example, we can measure the **current star formation rate** of the galaxy! This is one of the mostly widely used measurements of the star formation rate.

Other methods: measure the continuum luminosity in the UV, since this is also produced by massive stars.

Problem with both of these, especially the UV continuum: dust. If a galaxy is very dusty (and dusty galaxies often have the highest star formation rates) then neither of these methods will work. Then what?

Measure the thermal emission from dust in the IR. Dust absorbs light from massive stars and is heated, so measuring the radiation from the dust can tell us how many massive stars there are.

Note that all of these methods tell us about the number of massive stars only — to get the overall star formation rate from this, we have to assume an initial mass function. Usually assumed universal, but this is an important systematic uncertainty.

# Lecture XXXIX Galaxy Clusters

Kutner 18

Galaxies are not distributed randomly throughout the universe; they are usually found in associations called **groups** or **clusters**. In both cases, the galaxies are gravitationally bound to each other and orbit the system's center of mass.

## XXXIX.2 Classification of clusters

### XXXIX.2.1 Galaxy groups

- Usually have less than 50 members
- About 1.4 Mpc across
- Galaxies of the group have velocity dispersion $\sim 150$ km s$^{-1}$ (this now refers to the velocity of galaxies relative to each other, **not** to the velocity of stars within galaxies)
- Mass of an average group $\sim 2 \times 10^{13}\ \mathrm{M}_{\odot}$, from the virial theorem
- Typical mass-to-light ratio $\sim 260\ \mathrm{M}_{\odot}/\mathrm{L}_{\odot} \Rightarrow$ lots of dark matter

### XXXIX.2.2 Galaxy clusters

- Contain between $\sim 50$ to thousands of galaxies
  - Few galaxies $\Rightarrow$ **poor** cluster
  - Lots of galaxies $\Rightarrow$ **rich** cluster
- Higher velocity dispersion than in a group. Characteristic velocity dispersion is 800 km s$^{-1}$, may exceed 1000 km s$^{-1}$ for very rich clusters
- Typical virial mass $\sim 10^{15}\ \mathrm{M}_{\odot}$
- Typical mass-to-light ratio $\sim 400\ \mathrm{M}_{\odot}/\mathrm{L}_{\odot} \Rightarrow$ even more dark matter than in groups
- Further classified as **regular** (spherical and centrally condensed) and **irregular**

## XXXIX.3 The Local Group

About 35 galaxies are known to lie within $\sim 1$ Mpc of the Milky Way — this collection of galaxies is called the Local Group. Most prominent members are the three spiral galaxies, the Milky Way, Andromeda (M31), and M33. The next most luminous are the Large and Small Magellanic Clouds, irregular galaxies near the Milky Way; the LMC is 48 kpc away, and the SMC is about 60 kpc away (can see them from the Southern hemisphere). They are two of the 13 irregular galaxies in the Local Group. The remaining galaxies are dwarf ellipticals or dwarf spheroidals, very small and very faint (which means they're hard to see; there may be some we haven't found yet). Also notable is the Magellanic Stream, a long ribbon of gas tidally stripped from the Magellanic Clouds.

Most of the galaxies in the Local Group are clustered around the Milky Way and Andromeda (see Figure 1), which are on opposite sides of the Local Group about 770 kpc apart. The center of mass of the group is between them.

Andromeda and the Milky Way are approaching each other with a velocity of 119 km s$^{-1}$ (the gravitational attraction between them is strong enough to overcome the Hubble flow, i.e. the expansion of the universe). We don't know whether or not they will actually collide because we can't measure the transverse velocity of Andromeda; indirect constraints suggest it's $\sim 100$ km s$^{-1}$ or less. Neglecting the transverse velocity, we can calculate that they will collide in about $t_c = d/v = 6.3$ billion years (more careful estimates suggest that at least the dark matter halos of the galaxies will collide). This is an overestimate because the galaxies will accelerate as they approach.

Let's use the distance and relative velocity of the two galaxies to estimate their combined mass. From conservation of energy, the orbital speed and separation are related by

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \tag{63}$$

where $M = m_1 + m_2$ is the total mass of the two galaxies (this expression is derived in Section 2.3). The orbit's semimajor axis $a$ is related to its period $P$ by Kepler's third law,

$$P^2 = \frac{4\pi^2}{GM}a^3. \tag{64}$$

Combining these to eliminate $a$, we find

$$v^2 = \frac{2GM}{r} + \left(\frac{2\pi GM}{P}\right)^{2/3}. \tag{65}$$

In this equation, $r = 770$ kpc and $v = 115$ km s$^{-1}$. What's $P$? Let's assume that the galaxies started out together at the Big Bang, so when they collide they will have returned to their original configuration and one period will have elapsed. So we'll assume

$$P = t_H + t_c. \tag{66}$$

Recall $t_H$ is the Hubble time, $1/H_0$, the approximate age of the universe, $\sim 13.7$ Gyr for $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$. With this value of $P$, the total mass is $4 \times 10^{12}$ M$_\odot$. Note from Equation 3 that a smaller period gives a larger mass; we have overestimated $P$, so we have underestimated $M$.

There are other groups of galaxies within 10 Mpc of the Local Group—about 20 small groups of galaxies closer to us than the Virgo cluster. Most galaxies live in small groups and poor clusters; probably at most 20% of galaxies live in rich clusters like the Virgo cluster.

## XXXIX.4 The Virgo Cluster and the Coma Cluster

The nearest rich clusters are the Virgo cluster and the Coma cluster.

### XXXIX.4.1 The Virgo cluster

- Covers $10^\circ \times 10^\circ$ region of the sky. Huge!
- Center about 16 Mpc away.
- Contains about 250 large galaxies and $> 2000$ smaller ones, within a region about 3 Mpc across.
- Mix of galaxy types: 4 brightest are giant ellipticals, spirals dominate overall, but ellipticals become increasingly common near the center of the cluster. Why?
- M87, giant E1 elliptical, brightest galaxy in the Virgo cluster. Mass $\sim 3 \times 10^{13}\ \mathrm{M}_\odot$, $M/L \simeq 750\ \mathrm{M}_\odot/\mathrm{L}_\odot$. Over 99% dark matter! Not a normal elliptical galaxy.

### XXXIX.4.2 The Coma Cluster

Another rich cluster, about five times farther away than Virgo. Provided first evidence of dark matter—in 1933 Fritz Zwicky measured the radial velocities of galaxies in the Coma cluster, calculated the velocity dispersion, which we now know is $\sigma = 977$ km s$^{-1}$, and used the virial theorem to estimate the cluster's mass. With radius 3 Mpc:

$$M \approx \frac{5\sigma^2 R}{G} = 3.3 \times 10^{15}\ \mathrm{M}_\odot \tag{67}$$

Luminosity is about $5 \times 10^{12}\ \mathrm{L}_\odot$, so $M/L \approx 660\ \mathrm{M}_\odot/\mathrm{L}_\odot$. Zwicky understood that this was strange, and wrote that the mass of the cluster considerably exceeds the sum of the masses of the individual galaxies. This was the first recognition of dark matter.

(More Zwicky: with Walter Baade, proposed the existence of neutron stars in 1934, only a year after the discovery of the neutron, and proposed that supernovae were transitions of normal stars into neutron stars. Also proposed that galaxy clusters could be gravitational lenses. And other, crazier stuff.)

### XXXIX.5 Hot, Intracluster Gas

Some of the missing mass in galaxy clusters was discovered when the first X-ray satellites were launched (1977). It was observed that many clusters emit X-rays, from much of their volume. This revealed the **intracluster medium**: a diffuse, irregular collection of stars, and **hot intracluster gas**.

The X-rays are produced by **thermal bremsstrahlung emission** (braking radiation, also called free-free emission). This is emission which occurs when a free electron passes near an ion, emits a photon, and slows down (the ion is necessary for conservation of energy and momentum). Bremsstrahlung radiation has a characteristic, easily identifiable spectrum, and the luminosity density (luminosity per unit volume) depends on the electron density and the temperature:

$$\mathcal{L}_{\mathrm{vol}} = 1.42 \times 10^{-40}\, n_e^2 T^{1/2} \text{ W m}^{-3}. \tag{68}$$

(Important parts: $\mathcal{L}_{\mathrm{vol}} \propto n_e^2 T^{1/2}$) The temperature can be measured from the X-ray spectrum of the gas; for the Coma cluster it is $8.8 \times 10^7$ K. The total X-ray luminosity of the cluster is

$$L_x = \frac{4}{3}\pi R^3 \mathcal{L}_{\mathrm{vol}}. \tag{69}$$

We can measure $R$, $L_x$ and $T$, so these two equations can be combined to solve for $n_e$. For the Coma cluster, $n_e = 300$ m$^{-3}$. Very diffuse!

We can then compute the total mass of the gas:

$$M_{\mathrm{gas}} = \frac{4}{3}\pi R^3 n_e m_H \tag{70}$$

since there is one proton for every electron. For the Coma cluster, $M_{\mathrm{gas}} = 1.05 \times 10^{14}\ \mathrm{M}_\odot$. This is a lot of gas, but much less than the total cluster mass $3.3 \times 10^{15}\ \mathrm{M}_\odot$ we computed earlier.

Summary: visible light (1%), gas (9%), dark matter (10%)

(next lecture: types and colors of galaxies in clusters, galaxy interactions in clusters, galaxy interactions generally)

# Lecture XL Galaxy Interactions

Things we observed about galaxy clusters:

- In large clusters like the Virgo cluster, elliptical galaxies become increasingly common toward the center of the cluster
- Rich, regular centrally condensed clusters also have a higher fraction of elliptical galaxies than irregular clusters
- Clusters contain large amounts of hot gas between galaxies, the intracluster medium

These things are evidence for galaxy interactions, which are more likely in the densely populated centers of clusters.

Other observations:

- At least 50% of disk galaxies have warped disks (seen with radio observations of HI disks)
- Elliptical galaxies often have discrete shells of stars, or populations of stars with orbits different from those of the rest of the stars in the galaxy

### XL.2 What happens when galaxies collide?

- The stars don't physically collide, but may interact gravitationally
- Gas clouds collide, triggering star formation
- Entropy increases: disk galaxies with stars on ordered, nearly circular orbits will likely become elliptical galaxies, with stars on random orbits
- Galaxy collisions are inelastic: some of the orbital kinetic energy is converted to internal energy, in the form of random motions of stars. So even galaxies that start out on orbits with velocities greater than the escape velocities of the galaxies can end up gravitationally bound
- Extended tidal structures are produced

Now more detail about some of the important processes involved in galaxy interactions.

### XL.3 Tidal interactions

Even if galaxies don't physically collide, they can be disrupted by tidal forces. The same analysis that is used to describe the interactions of close binary stars is useful here. Recall that when two stars are in a close orbit they can be distorted by tidal forces (the differential force of gravity across the star), and mass can be transferred from one star to the other.

For now we will just look at the **tidal radii** of the two interacting galaxies. If stars or gas clouds extend beyond the tidal radius they are likely to be stripped from the galaxy; this is probably the cause of the Magellanic Stream, the long ribbon of gas associated with the Magellanic Clouds. The process is called **tidal stripping**.

The tidal radii are the distances from each galaxy to the inner Lagrangian point $L_1$, the point at which the gravitational and centrifugal forces balance.

Approximate expressions for the distances from $L_1$ to $M_1$ and $M_2$, called $l_1$ and $l_2$ respectively:

$$l_1 = a\left[0.500 - 0.227\log\left(\frac{M_2}{M_1}\right)\right] \quad (71)$$

$$l_2 = a\left[0.500 + 0.227\log\left(\frac{M_2}{M_1}\right)\right], \quad (72)$$

where $a$ is the distance between $M_1$ and $M_2$. The tidal radii will constantly change as the galaxies move with respect to each other.

Tidal interactions are very important in creating features seen in interacting galaxies. The most well-known are **tidal tails**, which are long, curved tails of gas and stars pulled out of interacting galaxies by tidal forces. See Figure 2, the Mice galaxies in the Coma cluster.

### XL.4 Ram-pressure stripping

Early interactions between galaxies in clusters probably caused the galaxies to lose most of their gas, from bursts of star formation triggered by the interactions and tidal stripping. Once the process is underway, it is enhanced by **ram pressure stripping**. Ram pressure is the pressure exerted on something moving through a fluid medium, and it creates a strong drag force. As you'd expect, it depends on the velocity at which the object is moving and the density of the medium:

$$P = \rho v^2. \quad (73)$$

In the case of galaxies moving through the intracluster medium, the ram pressure can be strong enough to strip most of the gas out of galaxies. So this is another reason why most galaxies in the centers of clusters are ellipticals with little gas.

### XL.5 Dynamical friction

Now we'll consider another gravitational effect, one that is important to satellite galaxies, globular clusters, and general interactions of small galaxies with large ones.

Consider a small galaxy or cluster of mass $M$ moving through an infinite collection of stars, gas clouds and dark matter, with constant mass density $\rho$. There are no collisions, and the masses of the individual objects are too small to deflect $M$, so it continues forward. However, the gravitational force of the large object pulls the smaller objects toward it, causing a density enhancement behind it along its path. There is then a net gravitational force that opposes its motion. This is called **dynamical friction**, and it results in a transfer of kinetic energy from $M$ to the surrounding material.

Not going to derive an expression for the force of dynamical friction here, but we'll write down what it looks like:

$$f_d = C\,\frac{G^2 M^2 \rho}{v_M^2} \tag{74}$$

$C$ isn't a constant; it's a function that depends on how $v_M$ compares with the velocity dispersion of the surrounding medium. As we'd expect, the force is stronger when the mass $M$ is larger and when the density of surrounding material is higher. To understand the inverse squared dependence on velocity, think about the impulse of the encounter: the impulse is the integral of the force with respect to time, and is equal to the change in momentum.

$$I = \int F\,dt = \Delta p \tag{75}$$

The faster the object is moving, the shorter the time the force will be applied. If it's moving twice as fast, it will spend half as much time near a given object and the impulse will be half as large. Then the density enhancement will develop only half as fast, and $M$ will be twice as far away by the time the enhancement arises.

This means that slow encounters are much more effective at decreasing the speed of an interacting object than fast ones.

Dynamical friction affects globular clusters in the halos of galaxies, causing them to lose energy and spiral in to the centers of galaxies. This may be why the Andromeda galaxy has no massive globular clusters. Satellite galaxies are also affected. The Milky Way has already swallowed some satellite galaxies this way, and the Magellanic Clouds will probably merge with the Milky Way in another 14 billion year or so.

# Lecture XLI Starburst Galaxies

It was observed in 1972 that interacting galaxies tend to be bluer than isolated galaxies of the same type. This is attributed to increased star formation triggered by tidal interactions and collisions of gas clouds.

Starburst galaxies:

- The most luminous galaxies in the local universe
- Most of the star formation occurs in very dusty regions, so most of the light from star formation is absorbed by dust and reradiated in the IR.
  - LIRGs (Luminous IR Galaxies)
  - ULIRGs (Ultra-Luminous IR Galaxies). Some ULIRGS may also be powered by black hole accretion
  - Up to 98% of the total energy produced by these galaxies is radiated in the IR—compare about 30% for the Milky Way, and a few percent for Andromeda
- Often compact, with much of the star formation taking place within 1 kpc of the center—these are called circumnuclear starbursts
- Gas is mostly molecular, fueling the star formation—may be $\sim 10^9$ to $10^{12}$ $\mathrm{M}_\odot$ of gas
- Star formation rates are very high, $\sim 10$–$300$ $\mathrm{M}_\odot$ yr$^{-1}$. Compare $\sim 1$ $\mathrm{M}_\odot$ yr$^{-1}$ in the Milky Way.
- Enough gas to sustain the star formation for $\sim 10^8 - 10^9$ years, though a given strong burst of star formation may last only about 20 Myr

Recall homework problem using the Schmidt law to study star formation efficiency and gas consumption timescales. The galaxies in that sample with the highest surface densities of gas and star formation rate were starburst galaxies, and we saw that they are more efficient at turning their gas into stars than galaxies with lower gas densities. These galaxies also have shorter dynamical times, because the star formation is occurring in a compact region.

Not all starburst galaxies are interacting. Disk starbursts are also seen, and in this case it's not clear what triggers strong star formation simultaneously across a galaxy.

### XLI.2 Galactic outflows

The intense star formation in starburst galaxies drives strong outflows of gas. Also called **galactic winds**, **galactic superwinds**, and **feedback**, because they probably play an important role in regulating star formation by limiting the gas supply. Observationally, superwinds are ubiquitous in galaxies with star formation rates per unit area $\Sigma_{\rm SFR} > 0.1$ $\mathrm{M}_\odot$ yr$^{-1}$ kpc$^{-2}$. Local starbursts and (as we will see) nearly all star-forming galaxies at high redshifts meet this criterion, but the disks of normal spirals do not.

Energy and momentum from supernovae and stellar winds is injected into the interstellar medium of the galaxy, driving gas to velocities of hundreds of km $s^{-1}$ . Winds are observed to be **multi-phase**, i.e. containing gas with different temperatures and ionization states. Hot gas is detected in X-rays and $\mathrm{H}\alpha$ emission. This is often metal-enriched and is probably directly associated with the supernova ejected. There is also cooler, neutral gas seen in absorption — blueshifted absorption lines seen against the stellar continuum of the galaxy. This is the interstellar medium of the galaxy entrained in the outflow, also with high velocities. Estimates of the **mass outflow rate** from the winds suggest that it is of the same order as the star formation rate: starburst galaxies are driving as much gas out in winds as they are forming into stars.

An important question is whether or not this gas escapes the galaxy, or falls back down. Do the velocities of outflows reach the escape velocities of galaxies? In many cases they do, and this is probably the source of much of the metals in the intracluster and intergalactic medium.

# Lecture XLII High Redshift Galaxies

(Note: most of this material isn't in the textbook, which unfortunately neglects the properties of galaxies over most of the history of the universe.)

### XLII.2 Review of redshift

Redshift:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm rest}}{\lambda_{\rm rest}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm rest}} - 1 \tag{76}$$

Because the universe is expanding, objects with higher redshifts are farther away, and because of the finite speed of light, when we look at objects farther away we are looking back in time.

**The relationship between redshift and time is nonlinear.** According to our current best cosmological model, the universe is 13.7 Gyr old. When we look at objects at a given redshift, we are looking at the universe when it was younger. Summary of the relationship between redshift and the age of the universe:

| $z$ | Age of universe at $z$ | Fraction of total age of universe |
|---|---|---|
| 0.0 | $1.37 \times 10^{10}$ | 1.00 |
| 0.2 | $1.12 \times 10^{10}$ | 0.82 |
| 0.5 | $8.64 \times 10^{9}$ | 0.63 |
| 0.7 | $7.36 \times 10^{9}$ | 0.54 |
| 1.0 | $5.92 \times 10^{9}$ | 0.43 |
| 2.0 | $3.33 \times 10^{9}$ | 0.24 |
| 3.0 | $2.18 \times 10^{9}$ | 0.16 |
| 4.0 | $1.56 \times 10^{9}$ | 0.11 |
| 5.0 | $1.19 \times 10^{9}$ | 0.09 |
| 6.0 | $9.41 \times 10^{8}$ | 0.07 |
| 7.0 | $7.70 \times 10^{8}$ | 0.06 |
| 8.0 | $6.45 \times 10^{8}$ | 0.05 |
| 9.0 | $5.50 \times 10^{8}$ | 0.04 |
| 10.0 | $4.76 \times 10^{8}$ | 0.03 |

(Will talk about how and why this works when we talk about cosmology later.)

### XLII.3 The $K$-correction

Because the light from galaxies is redshifted, some or all of the light that would normally fall in (for example) the $B$ band is redshifted, and falls at redder wavelengths. We need to account for this in our observations of galaxies—this is called the $K$-correction. Example: if we observe the optical (4000 - 7000 Å) spectrum of a galaxy at $z = 3$, we are actually observing the rest-frame UV:

$$\lambda_{\rm rest} = \frac{\lambda_{\rm obs}}{1+z} \tag{77}$$

So a 4000–7000 Å observation of a galaxy at $z = 3$ corresponds to 1000–1750 Å in the rest frame. Note that the bandwidth of the observation also changes: we cover 3000 Å in the observed frame and 750 Å in the rest frame. The difference is a factor of $1 + z$.

## XLII.4 How do we find high redshift galaxies?

Most obvious method: Take deep images, and then take spectra of galaxies that look far away (small, faint) to see what their redshifts are. This works, and this is how most galaxies with low to moderate ($z \lesssim 1$) redshifts are found.

Problem: this is inefficient. Hard to distinguish local dwarf galaxies from high redshift galaxies, and taking lots of spectra takes a lot of time.

### XLII.4.1 Color selection techniques

Searching for high redshift galaxies will be much more efficient if we can take spectra of galaxies that we already know are likely to be at high redshift from their colors in deep images. The most widely used method takes advantage of the nearly complete absorption of hydrogen ionizing photons by neutral hydrogen in galaxies.

Take images in three different filters, chosen so that the bluest one is blueward of the Lyman break at the redshift you want to target – then galaxies at that redshift will be present in the two redder filters, but drop out of the bluest one. This is also called the dropout technique. Pioneered for galaxies at $z \sim 3$, but can be used at higher redshifts just by selecting a redder set of filters. This is how candidates for the most distant galaxies known ($8 < z < 10$) are selected.

The color selection technique is very efficient (most of the galaxies selected are in the expected redshift range), but spectroscopy is needed to confirm the redshifts and carry out more detailed studies.

(Other color selection techniques have been developed based on the typical colors of galaxies in various filter sets and at various redshifts...)

### XLII.4.2 Photometric redshifts

If you want a really large sample of high redshift galaxies (many thousands), it takes too long to get spectra of them all, even if you have an efficient technique for selecting the ones that are probably at the redshift you want.

In these cases you have to use **photometric redshifts**: take images of the galaxies in as many filters as possible, and try to determine the redshift of the galaxy from its **spectral energy distribution**—the brightness of a galaxy at many different wavelengths. This also relies on strong breaks in the spectrum—the Lyman break at 912 Å, and a break at 4000 Åthat appears in the spectra of older galaxies. The more filters you have, the better this works. With a lot of filters ($> 10$) it works very well, although the precision is never as good as what you get from a spectrum. Most surveys don't have that many filters.

### XLII.4.3 Searches for $\mathrm{Ly}\alpha$ emission

Another popular method, especially for the highest redshift galaxies, is to search for $\mathrm{Ly}\alpha$ emission.

Many (but not all!) high redshift galaxies have very strong emission lines from $\mathrm{Ly}\alpha$ at 1215 Å (rest frame). This is the $n = 2$ to $n = 1$ transition of hydrogen, produced in the H II regions around massive stars but subject to complicated scattering because it's a **resonance line**.

Resonance line: a transition between the ground state and an excited level. Most atoms are in their ground state, so a photon with a wavelength corresponding to a resonance transition can be absorbed by an atom, excite an electron, and then be re-emitted as the electron falls to a lower state. This process is called **resonant scattering**. If the atoms doing the scattering are moving, the emitted and absorbed photons have Doppler shifts corresponding to the atoms' velocities. This means that photons of these wavelengths get scattered many times, in both space and frequency. Because of this long path length, they are also more likely to encounter a dust molecule which will absorb them. For these reasons, many galaxies have $\mathrm{Ly}\alpha$ in absorption or in some combination of emission and absorption, and the wavelength of $\mathrm{Ly}\alpha$ emission is usually shifted with respect to that of the H II regions where it originates because of the resonant scattering.

We look for $\mathrm{Ly}\alpha$ emission by making a very narrow filter that targets $\mathrm{Ly}\alpha$ emission at a particular redshift, and look for things that are bright in that narrowband filter. Things detected this way require spectroscopy to confirm that they're really at that redshift; if the $\mathrm{Ly}\alpha$ emission is strong, this usually isn't that hard to do.

Especially at very high redshifts, a galaxy isn't considered to be securely confirmed unless its redshift has been measured from a spectrum. For the faintest galaxies at the highest redshifts ($z >$ 5–6ish), the only way to confirm the redshift is with $\mathrm{Ly}\alpha$ emission. Other strong emission lines are shifted too far into the IR, and the galaxies are too faint to identify the redshift from absorption features. ($\mathrm{Ly}\alpha$ at $z = 6$: 8500 Å).

The current record-holder for the highest reported spectroscopic redshift is $z = 8.6$ (quite a bit higher than the previous record which was $z = 6.96$). At this redshift, $\mathrm{Ly}\alpha$ is in the IR at 1.17 $\mu$m, which makes it much harder to observe (the sky background is high—and this isn't a great detection). At $z = 8.6$, the universe was 590 Myr old (!), 4% of its current age of 13.7 Gyr.

The highest reported galaxy detections are at $z \sim 10$ (at which the universe was 480 Myr old). These are selected by the dropout technique, and haven't been spectroscopically confirmed. It's obviously very interesting and important to study the first galaxies, but these things are so faint that we can't tell very much about them.

## XLII.5 Properties of high redshift galaxies

Now that we can find galaxies at high redshifts, what are they like and how are they different from local galaxies?

### XLII.5.1 Morphological changes with redshift

Spiral galaxies become more common and ellipticals less common with increasing redshift, and the fraction of irregular galaxies also increases. This also means that galaxies become bluer and have more star formation, and this is what we'd expect if elliptical galaxies form from the mergers of spirals. This is true up to $z \sim 1$ ($\sim 8$ Gyr ago, or about 40% of the age of the universe).

At redshifts higher than $z \sim 1$ we don't see the Hubble sequence any more, and it isn't useful to talk about galaxies in terms of spirals and ellipticals. Most galaxies are irregular, with a wide variety of morphologies. See Figure 3, and keep in mind that these are rest-frame UV images (from the Hubble Space Telescope, which we need to get good enough angular resolution to study these small galaxies) and that we don't have the sensitivity to detect extremely low surface brightness features.

### XLII.5.2 Star formation

We can also measure the star formation rates of high redshift galaxies, using many of the same techniques we use for local galaxies—they're just harder.

- Measure the strength of $\mathrm{H}\alpha$ emission, directly related to the ionizing flux from massive stars
- Measure the brightness of the rest-frame UV continuum, since this also comes from massive stars. Actually easier at high redshift, because it's redshifted into the optical, but requires correction for dust, which is hard
- Measure light absorbed by dust and re-radiated in the IR. Done from space, with the Spitzer Space Telescope (until it ran out of coolant) and now Herschel

Results: the average star formation rate was much higher in the past, peaking (probably) at $z \sim 2$–3. Typical galaxies in this redshift range have SFRs 10-100 times higher than normal local galaxies. Most high redshift galaxies are starbursts, and there are so many of them that the star formation is unlikely to be triggered by mergers.

Some galaxies are even more extreme. **Submillimeter galaxies**, discovered at submm wavelengths in the last 10 years and now known to be mostly at $z \sim 2$–3 (though there may be others at higher redshifts, where the redshift is harder to determine). These galaxies have extremely bright dust emission, which indicates extremely high star formation rates, estimated at $\sim 1000\ \mathrm{M}_\odot$ yr$^{-1}$ (though there are significant systematic uncertainties associated with that). These high star formation rates may be triggered by mergers, like local starbursts.

There also appear to be some **passive galaxies** at high redshift more like local ellipticals: red, old, and not forming stars. This was a surprise, since it was thought that galaxies like this wouldn't have had time to form. There don't appear to be very many of them, and they seem to be puzzlingly compact compared to local ellipticals. Topic of current interest. Note that galaxies that aren't forming stars are much harder to find: they will be faint in the rest-frame UV (observed optical), and they won't have emission lines.

### XLII.5.3 Galactic outflows

Recall from the discussion of starburst galaxies that galactic outflows are ubiquitous in galaxies with star formation rates per unit area $\Sigma_{\rm SFR} > 0.1~{\rm M}_\odot$ yr$^{-1}$ kpc$^{-2}$. What's a typical star formation rate surface density for a star-forming galaxy at high redshift? Take SFR$=10~{\rm M}_\odot$ yr$^{-1}$, which is probably less than average, size 5 kpc (galaxies tend to be more compact): $\Sigma_{\rm SFR} \approx 0.4~{\rm M}_\odot$ yr$^{-1}$ kpc$^{-2}$, and most galaxies will be higher. So we'd expect nearly all high redshift galaxies to drive outflows, and they do. How do we know? By comparing the redshifts of emission and absorption lines.

The outflow is roughly spherical in this model, as opposed to local starbursts which tend to have bipolar outflows (directed out of the plane of disk galaxies). May make sense, since most galaxies don't seem to have formed disks yet, and outflows would have to cover a large solid angle for us to see them in most galaxies.

# Lecture XLIII Active Galaxies I

Kutner 19

A few percent of galaxies are peculiar in that they produce huge amounts of energy in excess of the normal stellar light. These are called **active galaxies** or **AGN (Active Galactic Nuclei)**.

Distinctive characteristics of active galaxies:

- Large amounts of nonstellar emission, some of it nonthermal in origin. Active galaxies produce more X-ray and radio emission than would be produced by their stars.
- Much of the light is concentrated in a small, central region called an active galactic nucleus, AGN
- Light from AGNs is variable on short timescales, at virtually all wavelengths. Timescale for variability depends on luminosity and wavelength, with most rapid variability seen at short wavelengths and low luminosities. X-rays in low luminosity AGNs can vary on timescales of minutes
- Some active galaxies have jets detectable at X-ray, visible and radio wavelengths. The jets contain ionized gas flowing outward at relativistic speeds
- The UV, visible and IR spectra of AGNs are dominated by strong emission lines

Not all active galaxies have all of these features.

Accumulated evidence indicates that the activity in AGN comes from accretion onto massive black holes. Most bright galaxies have black holes in their centers, but not all bright galaxies are active galaxies. To be an active galaxy, the central black hole must be accreting gas rapidly enough to produce luminosity as bright or brighter than the galaxy's stars. We will start by discussing the different types of active galaxies. The situation initially seems very complicated, but we'll see that most of the observations can be explained by a unifying scenario.

## XLIII.2 Types of Active Galaxies

### XLIII.2.1 Seyfert galaxies

History: galaxies with broad (500–5000 km $s^{-1}$) emission lines discovered by Carl Seyfert in 1943

Properties:

- Emission lines come from center of galaxies
- $\sim 0.5$% of galaxies are Seyfert galaxies
- $\gtrsim 95$% of Seyferts are spirals

- In addition to broad emission lines, center often has a power law ($F_\nu \propto \nu^{-\alpha}$) continuum excess of light, giving Seyferts their blue color compared to normal galaxies
  - Power law continuum is non-thermal in origin—not emitted by stars. $F_\nu \propto \nu^{-\alpha}$: $\alpha$ is **spectral index**, $\nu$ is frequency

Aside: Emission lines are divided into **permitted** and **forbidden** lines.

Permitted lines have high transition probabilities. Common examples are the Balmer lines, H$\alpha$, H$\beta$, etc.

Forbidden lines have lower transition probabilities and occur in low density regions. Because of the lower transition probability, an excited electron will be collisionally de-excited before it can emit a photon if the density is too high. Forbidden lines are indicated by square brackets: [O III], [N II], etc.

Two types of Seyfert galaxies

- Seyfert I (Sy I)
  - very broad permitted lines (1000–5000 km s$^{-1}$)
  - narrower forbidden lines ($\sim 500$ km s$^{-1}$)
  - often X-ray luminous
- Seyfert II (Sy II)
  - permitted and forbidden lines both relatively narrow ($\sim 500$ km s$^{-1}$)

### XLIII.2.2 Radio galaxies

History: sky studied at radio wavelengths after WWII

- 1946: discovery of discrete radio source Cygnus A—poor resolution, no optical counterpart found
- 1949: positions to 10 arcmin achieved (still not very good!). M87 and NGC5128 found to be associated with double radio sources
- 1951: optical counterpart to Cygnus A found to be peculiar galaxy at $z = 0.06$ ($\sim 240$ Mpc! the only brighter radio sources are the Sun and the nearby (3 kpc) supernova remnant Cas A)
  - Cyg A is $\gtrsim 10^6$ times more powerful at radio wavelengths than the Milky Way
- 1953: Cyg A resolved as double radio source

Properties:

- Usually elliptical
- Non-thermal power law spectrum in radio
- Optical spectra of nucleus similar to Seyferts. Distinguish broad line radio galaxies (BLRGs) and narrow line radio galaxies (NLRGs).
- $\sim 100$ times less abundant than Seyfert galaxies
- May have extended jets and lobes

#### XLIII.2.3 QSOs (Quasi-Stellar Objects)

History:

- 1960: 3C 48 (#48 in the Third Cambridge catalog of radio sources) identified with star-like object (quasi-stellar). Thought to be a star, but spectrum very weird
- 1963: Maarten Schmidt recognizes the spectrum of another "radio star" (3C 273) as Balmer lines redshifted to $z = 0.158$. Then 3C 48's spectrum understood as redshifted to $z = 0.367$ (these objects weren't expected to be so far away because they are so bright)

Properties:

- Most QSOs radio quiet
- Look like stars. Some have optical jets, and some are the very bright nuclei of fuzzy-looking host galaxies
- Blue. UV excess called the "blue bump"
- Optical fluxes often variable on timescales of years
- Broad emission lines resemble Seyfert I galaxies
- Almost always strong X-ray emitters
- Bright! can be detected to high redshifts
- Radio-quiet QSOs found by looking for extremely blue stars. Followup spectroscopy needed for confirmation

### XLIII.3 Variability and physical size

Flux from AGN can vary significantly on very short timescales (except for Sy IIs and NLRGs)

- Luminosity of broad lines and continuum can vary by a factor of $\sim 2$ on day to month timescales
- Variations in broad emission lines typically lag behind those of continuum by $\sim 1$ month
- Variations of a few percent in visible, X-rays. X-ray flux can vary on timescales of minutes

AGN with rapid time variability are often called **BL Lac objects** (after prototype BL Lacertae) or **blazars**. Changes in flux can be up to $\sim$30% in a day, up to $\sim$100% over longer periods. Nearly devoid of emission lines (continuum dominates flux)

The timescale of variability puts limits on the size of the source. A source with size $R$ will take a time of at least $\Delta t = R/c$ to change its luminosity, since information can't travel from one side of the source to the other faster than $c$. If we observe $\Delta t = 1$ hour,

$$R \simeq c\Delta t = 7.2 \text{ AU}. \tag{78}$$

This is very small for something so energetic!

### XLIII.4 Accretion by supermassive black holes

#### XLIII.4.1 Energetics

The release of gravitational potential energy through mass accretion is a very efficient way to generate energy. Consider a mass $m$ falling a large distance $r \gg r_{\text{Sch}}$ toward the Schwarzchild radius of a black hole. Recall

$$r_{\text{Sch}} = \frac{2GM_{\text{bh}}}{c^2}. \tag{79}$$

The loss of gravitational potential energy will be

$$\Delta E = -\frac{GM_{\text{bh}}m}{r} + \frac{GM_{\text{bh}}m}{r_{\text{Sch}}} \approx \frac{GM_{\text{bh}}m}{r_{\text{Sch}}} \approx \frac{1}{2}mc^2. \tag{80}$$

If the mass doesn't stop before it reaches the Schwarzchild radius it will pass the event horizon with speed $v \sim c/\sqrt{2}$, and its kinetic energy will increase the mass of the black hole. If instead it's decelerated by an accretion disk, its kinetic energy will be converted into thermal energy and then radiation. This process isn't perfectly efficient, and we write the energy carried away by photons as

$$\Delta E_{\text{phot}} = \eta mc^2 \tag{81}$$

where $\eta$ is a dimensionless number called the efficiency of the black hole. We expect $\eta \leq 1/2$ from Equation 3 above. In practice, we think $\eta \approx 0.1$, which means that a gram of matter falling toward the black hole gives 9 trillion joules of radiation energy (!).

As gas falls into the black hole at rate $\dot{M}$, the **accretion luminosity** of the AGN is

$$L = \eta \dot{M} c^2. \tag{82}$$

We can therefore estimate an AGN's accretion rate $\dot{M}$ from its luminosity:

$$\dot{M} = \frac{L}{\eta c^2} = 0.018\ \mathrm{M}_\odot\ \mathrm{yr}^{-1} \left(\frac{L}{10^{37}\ \mathrm{W}}\right) \left(\frac{\eta}{0.1}\right)^{-1}. \tag{83}$$

We don't expect $\dot{M}$ to be constant with time, which probably accounts for some of the variability of AGN.

### XLIII.4.2 The Eddington Limit

We can't get an arbitrarily high luminosity from accretion because eventually the gas surrounding the black hole will be blown away by radiation pressure. This leads to a maximum luminosity for accreting black holes.

At a distance $r$ from the AGN, the photons have an energy flux

$$F = \frac{L}{4\pi r^2}. \tag{84}$$

The photons also have momentum $p = E/c$ and momentum flux

$$F_p = \frac{F}{c} = \frac{1}{c}\frac{L}{4\pi r^2}. \tag{85}$$

The photons can transfer momentum to particles in the ionized gas surrounding the black hole. The force exerted on a particle is the rate at which momentum is transferred to it, and the rate of momentum transfer depends on the particle's cross-section for interaction with photons. Electrons have a much larger cross-section for interaction than protons (because $\sigma \propto m^{-2}$), and the relevant cross-section is the Thomson cross-section

$$\sigma_e = \frac{8\pi}{c}\left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^2 = 6.65 \times 10^{-29}\ \mathrm{m}^2. \tag{86}$$

The force transferred is the product of the momentum flux and the cross-section:

$$F_{\mathrm{rad}} = \sigma_e F_p = \frac{\sigma_e L}{4\pi c r^2}. \tag{87}$$

As each electron is accelerated, it drags a proton along with it. The gravity of the black hole provides the inward force on the proton-electron pair

$$F_{\mathrm{grav}} = -\frac{G M_{\mathrm{bh}}(m_p + m_e)}{r^2} \simeq -\frac{G M_{\mathrm{bh}} m_p}{r^2} \tag{88}$$

since $m_p \gg m_e$.

The maximum possible luminosity (the **Eddington luminosity** or **Eddington limit**) of the black hole is the luminosity at which the radiation pressure balances the gravitational force:

$$\frac{\sigma_e L_E}{4\pi c r^2} = \frac{G M_{\rm bh} m_p}{r^2} \tag{89}$$

The Eddington luminosity for a black hole of mass is

$$L_E = \frac{4\pi G m_p c}{\sigma_e} M_{\rm bh} = 1.3 \times 10^{29}\ \mathrm{W} \left(\frac{M_{\rm bh}}{10^8\ \mathrm{M}_\odot}\right) \tag{90}$$

$$= 3.3 \times 10^{12}\ \mathrm{L}_\odot \left(\frac{M_{\rm bh}}{10^8\ \mathrm{M}_\odot}\right) \tag{91}$$

This leads to a maximum accretion rate for black holes,

$$\dot{M}_E = \frac{L_E}{\eta c^2} = 2\ \mathrm{M}_\odot\ \mathrm{yr}^{-1} \left(\frac{M_{\rm bh}}{10^8\ \mathrm{M}_\odot}\right) \left(\frac{\eta}{0.1}\right)^{-1}. \tag{92}$$

It's sometimes useful to express the accretion rate in terms of the Eddington rate,

$$\dot{m} = \frac{\dot{M}}{\dot{M}_E}. \tag{93}$$

This is the **Eddington ratio**.

### XLIII.4.3 Accretion disks

Accreting matter can't go directly into the black hole because it has angular momentum. General consensus is that the black hole is surrounded by an accretion disk in which matter slowly spirals inward. Viscosity (internal friction) causes the matter to lose its angular momentum and converts kinetic energy into random thermal motion. This heats the accretion disk to temperatures of $10^5$ K.

The spectrum of a blackbody with $T \sim 10^5$ K peaks in the UV, and the blue bump in the SED of AGN is probably thermal emission from the accretion disk.

# Lecture XLIV Active Galaxies II

### XLIV.2 The AGN unification model

The AGN zoo of Seyfert galaxies, radio galaixes, QSOs and BL Lac objects seems complicated, with variations in morphologies and spectra. However, most of the properties of various types of AGN can be explained by a unified model in which the structure of AGN is flattened, not spherical, and the strength of various components depends strongly on the angle of the AGN axis relative to our line of sight.

Review of the major components of an AGN, starting at the center:

- **Supermassive black hole**. Suspected for many years based on arguments of energetics and time variability, more recent evidence from high speed motions of stars and gas in galactic nuclei. The Schwarzchild radius of a $10^8\ \mathrm{M}_\odot$ black hole is $\sim 3 \times 10^{11}$ m $\sim 2$ AU.

- **Accretion disk**. Surrounds the black hole, responsible for the UV and visible continuum emission of AGN. For a $10^8\ \mathrm{M}_\odot$ black hole, the UV and visible emission arises on a scale of $10^{12}$–$10^{13}$ m (from considerations of gravitational potential energy being converted into heat in the disk). X-rays apparently produced in a hot corona surrounding the accretion disk.

- **Jets**. Ionized gas is ripped from the accretion disk by electromagnetic fields, and spirals along magnetic field lines away from the disk, producing a jet. Accelerated electrons in the ionized gas emit synchrotron radiation, accounting for the radio emission from the jet.

- **Broad-line region**. Size is measured by timing the delay between flux variations in the UV and visible continuum and the response of the emission lines. The delay is due to the light travel time across the broad line region. This is called **reverberation mapping**. The size of the broad-line region scales with luminosity:

$$\left(\frac{R_{\mathrm{BLR}}}{10^{15}\ \mathrm{m}}\right) \approx 0.26 \left(\frac{L_{\mathrm{bol}}}{10^{37}\ \mathrm{W}}\right)^{1/2} \tag{94}$$

  The broad-line region may be the outer edge of the accretion disk.

- **Obscuring torus**. Outer edge of the broad-line region is defined by the **dust sublimation radius**, the closest point to the continuum source (i.e. the accretion disk) where dust grains can survive the UV radiation. At smaller radii, where equilibrium blackbody temperature exceeds $\sim 1500$ K, dust is vaporized. Dust is important because it provides the opacity that blocks our direct view of the inner regions from some directions. The dust sublimation radius is the inner edge of a dusty structure with a larger scale height than the inner regions—this is the obscuring torus or dusty torus (this may actually be a cool, dense wind arising from the accretion disk).

- **Narrow-line region**. At about the same radius as the obscuring torus, but can extend out to hundreds of pc along the jets. Morphology is often wedge-shaped or conical, and along the axis of the black hole/accretion disk system. Seems to be the interstellar medium of the host galaxy: interstellar gas that isn't shielded from the central source by the obscuring torus is photoionized by the UV radiation from the AGN.

The type of AGN we see depends on the orientation of the outer, dusty torus relative to the line of sight. Looking directly along the jet, we see primarily synchrotron emission from the jet: the featureless continuum of BL Lac objects. Looking at an angle close to the jet gives a good view of the accretion disk and the broad-line region, so the AGN is classified as a Seyfert I. At an angle closer to the disk the broad-line region is hidden by dust and we see only the narrow-line region: Seyfert II. For strong radio sources, a quasar is seen from an angle near the jet, a blazar when looking directly along the jet, and a radio galaxy when the torus hides the active nucleus from view.

### XLIV.3 Relativistic jets and superluminal velocities

Radio lobes are produced by jets of charged particles ejected from the nucleus at relativistic speeds, probably accelerated and collimated by magnetohydrodynamic effects. The absence of emission lines in the power-law synchrotron spectrum of the jets makes velocities difficult to measure, but the best evidence for relativistic speeds involves radio observations of material ejected from AGN with apparently **superluminal velocities**.

Example: Radio images of the core of the quasar 3C 273 show a blob of emission moving away form the nucleus with proper motion $\mu = 0.0008''$ yr$^{-1}$ (long baseline radio interferometry is required for this kind of spatial resolution). 3C 273 is at a distance $d = 620$ Mpc, so, if the clump is moving perpendicular to the line of sight, its apparent velocity is

$$v_{\rm app} = d\mu = 2.4 \times 10^9 \text{ m s}^{-1} = 7.8c. \tag{95}$$

This is obviously unphysical, and it can be explained by material moving toward the observer at relativistic speeds.

A source is moving at speed $v$ (actual, not apparent speed) at an angle $\phi$ from our line of sight. A photon is emitted at time $t = 0$ when the object is at distance $d$. At a later time $t_e$ another photon is emitted, when the distance to the object is $d - vt_e \cos\phi$. The first photon reaches Earth at time $t_1$,

$$t_1 = \frac{d}{c}. \tag{96}$$

The second photon arrives at time $t_2$,

$$t_2 = t_e + \frac{d - vt_e \cos\phi}{c}. \tag{97}$$

The time between the reception of the two photons is

$$\Delta t = t_2 - t_1 = t_e\left(1 - \frac{v}{c}\cos\phi\right), \tag{98}$$

which is *shorter* than $t_e$, the actual time between when the two photons were emitted. The apparent transverse velocity measured on Earth is then

$$v_{\rm app} = \frac{v t_e \sin\phi}{\Delta t} = \frac{v \sin\phi}{1 - (v/c)\cos\phi}. \tag{99}$$

We can solve this for $v/c$:

$$\frac{v}{c} = \frac{v_{\rm app}/c}{\sin\phi + (v_{\rm app}/c)\cos\phi} \tag{100}$$

Requiring $v/c < 1$ then puts constraints on the angle $\phi$ and on the minimum value of $v/c$. In the case of 3C 273, the measured apparent velocity requires that $\phi < 14.5°$, and the lower limit on the speed of the knot is $v_{\rm min} = 0.992c$ (more detail in text and in HW problem 3). This provides strong evidence that the cores of AGN can accelerate material to relativistic speeds.

## XLIV.4 Quasars over cosmic history

We can tell from the masses of black holes today that quasars must have relatively short lifetimes. Most luminous quasars have $L \approx 3 \times 10^{14}$ L$_\odot$ and must have accretion rates $\dot{M} \approx 200$ M$_\odot$ yr$^{-1}$ to maintain that luminosity. The biggest supermassive black holes today have masses $M_{\rm bh} \sim 4 \times 10^9$ M$_\odot$. To grow to that mass at the accretion rate of the most luminous quasars would take

$$t \approx \frac{M_{\rm bh}}{\dot{M}} \approx \frac{4 \times 10^9 \ {\rm M}_\odot}{200 \ {\rm M}_\odot \ {\rm yr}^{-1}} \approx 20 \ {\rm Myr}. \tag{101}$$

Not long! If quasars maintained their luminosity for a long time the black holes would be much more massive than observed.

There were more than 1000 times as many bright quasars per Mpc$^3$ (comoving, i.e. corrected for the expansion of the universe) at $z \sim 2$ than there are today. This appears to be an evolutionary effect caused by a decrease in quasar luminosity with time. Beyond $z \sim 3$, the density of quasars declines again.

## XLIV.5 Black holes and the growth of galaxies

Most bright galaxies have supermassive black holes in the center, and may have been AGN at some point in the past. The growth of black holes and the growth of galaxies appear to be closely related. There is a correlation (the "$M - \sigma$ relation") between the mass of supermassive black holes and the velocity dispersion of the stars in the surrounding galaxy—stars that are far enough away from the black hole that it has a negligible effect on their velocity:

$$M_{\rm bh} \propto \sigma^\beta, \tag{102}$$

with $\beta \approx 4$. This implies that the black holes and the galaxies grow together and somehow know about each other, but the origin of this relationship, and whether it's the galaxy that sets the black hole mass or vice versa, are not well understood.

# Lecture XLV  The Intergalactic Medium

There is gas between galaxies: the **intergalactic medium (IGM)**. We see this via absorption lines in the spectra of quasars—absorption lines from gas clouds along the line of sight to the QSO.

$\mathrm{Ly}\alpha$ photons can be easily absorbed by neutral hydrogen, so if the light from a quasar passes through a cloud of gas on the way from the quasar to us, a $\mathrm{Ly}\alpha$ absorption line will be produced in the spectrum of the quasar *at the redshift of the intervening cloud*. There are many clouds along the line of sight, especially for quasars at high redshifts, so this produces a dense series of narrow absorption lines blueward of the $\mathrm{Ly}\alpha$ emission from the quasar. This is called the **Ly$\boldsymbol{\alpha}$ forest.** See Figure 2.

Absorption from ionized metals is also seen: C IV, Mg II, many other elements. This indicates that the material has been processed by stars and enriched. These lines correspond to the stronger $\mathrm{Ly}\alpha$ absorption systems, and can be used to determine the metallicity of the gas clouds, which shows a wide range from very low to solar. This variation may depend on how close the line of sight passes to the center of a galaxy.

The relationship between galaxies and absorption systems isn't fully understood. At least some absorption systems are caused by the line of sight passing through a galaxy or a galaxy halo, but the absorption systems don't seem to have the same large scale structure as galaxies; more randomly distributed, rather than grouped into clusters and voids.

Absorption systems can also be used to probe the effects of galaxies on the IGM, by studying correlations between the positions of galaxies and absorption systems. This is a way to estimate the distance to which a galaxy enriches its environment. Very much an area of active research.

Note: most of the gas in the universe is ionized, was ionized by the light from the first galaxies sometime around $z \sim 7$. Absorption comes from the small neutral fraction.

# Lecture XLVI Gravitational Lensing

- 1919: Eddington measures deflection of starlight near the Sun during an eclipse, confirms general relativity
- 1920s: Astronomers discuss using a gravitational lens to focus starlight, possibility of gravitational lensing producing multiple images of the same source
- 1937: Fritz Zwicky proposes that gravitational lensing by a galaxy much more likely than by individual stars
- 1979: Quasar Q0957+561 discovered to appear twice in the sky: two images separated by $6''$, both showing a quasar with redshift $z = 1.41$. Both images have the same emission and absorption lines and the same radio core and jet structure. Gravitational lens is a giant cD galaxy at $z = 0.36$ that lies between the two images of the quasar.
- Gravitational lensing can magnify and object and increase its brightness.

### XLVI.2 The geometry of gravitational lensing

Light follows the straightest possible path as it travels through the curved space around a massive object. Analogous to the normal refraction of light by a glass lens, as light passes from one index of refraction to another. The index of refraction $n$ is the ratio of the speed of light in a vacuum to the speed of light in the medium, $n \equiv c/v$.

Near a spherical object of mass $M$, the effective "index of refraction" is

$$n \simeq 1 + \frac{2GM}{rc^2} \tag{103}$$

assuming $2GM/rc^2 \ll 1$. At a distance of $10^4$ pc from a galaxy with mass $10^{11}$ $\mathrm{M}_\odot$, the effective index of refraction is $n = 1 + 9.6 \times 10^{-7}$; the deviation of light from a straight line will be very small.

Figure 1 shows the situation: light from source $S$ is deflected through an angle $\phi$ by the gravitational lens due to a point mass $M$ at location $L$. The light arrives at the observer at position $O$.

The angular deviation of a photon passing a distance $r_0$ (very nearly the distance of closest approach) from a mass $M$ is

$$\phi = \frac{4GM}{r_0 c^2} \text{ rad.} \tag{104}$$

The distance to the source is $d_S/\cos\beta \simeq d_S$, where $\beta \ll 1$, and $d_L$ is the distance to the lens. Can then show that the angle $\theta$ between the lensing mass and the image of the source is

$$\theta^2 - \beta\theta - \frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right) = 0, \tag{105}$$

where $\theta$ and $\beta$ are measured in radians.

This is a quadratic equation, and for the geometry shown in the figure there are two solutions for $\theta$, so two images will be formed by the gravitational lens. These solutions are $\theta_1$ and $\theta_2$; the angles can be measured observationally and then used to find the values of $\beta$ and $M$:

$$\beta = \theta_1 + \theta_2 \tag{106}$$

and

$$M = -\frac{\theta_1 \theta_2 c^2}{4G}\left(\frac{d_S - d_L}{d_S d_L}\right). \tag{107}$$

The two images are formed on opposite sides of the gravitational lens.

Gravitational lensing is a good way to determine the masses of lensing objects. For example, we can calculate the mass of the galaxy responsible for the lensing of quasar Q0957+561. The two angles are $5.35''$ and $-0.8''$, and we use the Hubble law $v = H_0 d$ and the redshifts of the quasar and lens to calculate their distances (2990 Mpc and 1250 Mpc). The resulting mass is $1.1 \times 10^{12}$ $\mathrm{M}_\odot$. This has assumed that the lensing galaxy is a point mass; we can do better by assuming a more realistic mass distribution, and in this case the lens mass is about 10% larger.

Other geometries and lensing mass distributions can produce more images, rings, or arcs. If the background object is exactly along the line of sight to the lens, a ring will be produced; this is called an **Einstein ring**. Partial rings and arcs are seen for slightly off-center alignments. If the lensing galaxy has an asymmetric mass distribution (an ellipsoid rather than a spheroid) three or five images are produced; this can produce a cross shape called an **Einstein cross**.

Galaxy clusters can also produce lensing arcs, if they are centrally concentrated enough. This is another way to estimate the masses of clusters, and further evidence that they contain large amounts of dark matter. The cluster Abell 370 produces many arcs, and from mass determined by the lensing model we find $M/L \geq 1000$ $\mathrm{M}_\odot/\mathrm{L}_\odot$. Lots of dark matter!

Gravitational lensing is also useful for studying faint galaxies at high redshift. If lensed background galaxies can be found, we can take advantage of their increased brightness to study them in much greater detail than would be possible for similar, unlensed galaxies at the same redshifts. Much of our most detailed knowledge about high redshift galaxies comes from studying lensed galaxies.

# Lecture XLVII The Extragalactic Distance Scale

In order to study the structure of the universe, we need to be able to measure how far away things are. We also need to measure distances in order to measure the Hubble constant: $v = H_0 d$, so we need to measure both $v$ (easy, for things that are far enough away so that peculiar velocities don't matter) and $d$ (hard).

There are many methods of measuring distances, making up what's called the **extragalactic distance scale** or **cosmological distance ladder**. For objects within $\sim 1$ kpc we can measure **trigonometric parallaxes**, but beyond that we need other methods. There are many; we'll cover some of the more important here.

### XLVII.2 Extragalactic distance indicators

Most astronomical distance indicators are what are called "standard candles." A standard candle is an object with a fixed luminosity, or with some other property we can measure that will tell us what its luminosity is. If we measure the apparent brightness (the flux) and know the absolute brightness (the luminosity) we can determine the distance.

Also used but less common are "standard rulers," objects with a fixed physical size; if we can measure the angular size and know the physical size, we can determine the distance.

#### XLVII.2.1 Cepheid variables

Review: variable stars whose period is directly related to their luminosity, discovered by Henrietta Swan Leavitt in the early 20th century. Stars pass through the **instability strip** on the HR diagram during late phases of stellar evolution. Also measure a stars's color, to account for the width of the instability strip, so we have a period-luminosity-color relation,

$$M_V = -3.53 \log P_d - 2.13 + 2.13(B - V), \tag{108}$$

where $M_v$ is the absolute $V$ magnitude, $P_d$ is the pulsation period in days, and $B - V$ is the color index. Can measure $P_d$, $B - V$ and apparent magnitude, use $P - L$ relation to calculate distance modulus. **Need to know extinction!**

Most distant known Cepheids are 29 Mpc away.

#### XLVII.2.2 Expanding photosphere method

This method uses measurements of the photosphere of a supernova in two ways. If the supernova is close enough, we can measure the angular size of the photosphere $\theta(t)$, and determine the angular velocity of the expansion of the photosphere by comparing angular size measurements taken at different times, $\omega = \Delta\theta/\Delta t$. The transverse velocity of the expanding photosphere is $v_t = \omega d$, where $d$ is the distance to the supernova. Assuming the expansion is spherically symmetric, the transverse velocity is equal to the radial velocity of the supernova ejecta $v_{\rm ej}$, which we can measure from Doppler shifts of spectral lines. The distance to the supernova is therefore

$$d = \frac{v_{\rm ej}}{\omega}. \qquad (109)$$

Most supernovae are too far away for this to work, so we use another method. We assume that the expanding shell of hot gas radiates as a blackbody, so that the supernova's luminosity is given by the Stefan-Boltzmann law:

$$L = 4\pi R^2(t)\sigma T_e^4, \qquad (110)$$

where $R(t)$ is the radius of the expanding photosphere and $t$ is the age of the supernova. Assuming the radial velocity of the ejecta is nearly constant, $R(t) = v_{\rm ej}t$. With measurements of the temperature from the blackbody spectrum, the age, and the radial velocity, we can determine the luminosity and therefore the distance. Problems: expansion is neither spherical nor a perfect blackbody, and (as for all standard candles) we need to be able to correct for dust. Uncertainties are 10–25%.

### XLVII.2.3 Type Ia supernovae lightcurves

The most important extragalactic distance indicator, because it reaches to the largest distances. Review: what's a Type Ia supernova? Standard model (in textbook) is that a CO white dwarf accretes material from a companion which pushes it over the Chandrasekhar limit and causes it to explode. Recent work suggests that there are some problems with this scenario; almost certainly have to do with explosion of a CO white dwarf, but it could be a merger of two or some other scenario.

Whatever the cause, the useful fact for measuring distances is that the brightnesses and light curves (a plot of luminosity vs. time) of Type Ia SNe are very similar. They have average absolute magnitudes at maximum light of $\langle M_B \rangle \simeq \langle M_V \rangle \simeq -19.3 \pm 0.03$. Note: small scatter, and very bright; this is as bright as an entire galaxy.

What we actually measure is the brightness of the supernova at various points in time, and there is a well-defined inverse correlation between the maximum luminosity and the rate of decline of the light curve: brighter supernova take longer to decline. We can use this to determine the intrinsic peak luminosity.

Type Ia supernovae can be used to measure distances to $> 1000$ Mpc ($z \sim 0.25$) with an uncertainty of $\sim 5$%. We will return to these, since they are very important to the measurement of dark energy and the acceleration of the expansion of the universe.

### XLVII.2.4 Summary

This has not been a comprehensive discussion! There are more methods in Chapter 18 of the text, and even more that aren't in the book. Main message: the different methods work best for different types of galaxies and to different distances, and have different uncertainties associated with them. We need to use as many different methods as possible to calibrate the various methods and understand their uncertainties.

# Lecture XLVIII The Expansion of the Universe

### XLVIII.2 The expansion of the universe

In 1929 Hubble published the famous paper "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae," showing that galaxies had recessional velocities proportional to their distance, $v = H_0 d$ and that therefore the universe is expanding. What does this actually mean?

Suppose the Earth doubles in size in an hour. Right now, from Milwaukee, it's 500 miles to Pittsburgh, 1000 miles to Dallas, and 2000 miles to Seattle. After the expansion, it will be 1000 miles to Pittsburgh, 2000 miles to Dallas, and 4000 miles to Seattle, so the expansion velocities we will observe will be 500 miles an hour for Pittsburgh, 1000 miles an hour for Dallas, and 2000 miles an hour for Seattle. The velocity of expansion is directly proportional to the distance; this is a result of expansion that is isotropic and homogeneous. Note that we would observe the same thing no matter where we were, because every point is moving away from every other point. The expansion has no center.

Galaxy redshifts are often described as Doppler shifts, but this isn't strictly correct. The redshift is due not to the galaxy moving through space, but because of the expansion of space itself. It is a **cosmological redshift**, not a Doppler shift; the redshift is produced by the expansion of the universe, as the wavelength of light is stretched along with space. The motion of galaxies due to the expansion of the universe is called the **Hubble flow**. Galaxies also have **peculiar velocities**, which are their velocities through space, independent of the Hubble flow. Also important to note that gravitationally bound structures (galaxies, clusters of galaxies) do not participate in the expansion.

#### XLVIII.2.1 The Hubble constant: review

The Hubble constant is measured using as many different independent distance indicators as possible, and through other tests of cosmological parameters we'll talk about later. For most of the 20th century, the Hubble constant was only known to be between 50 and 100 km $\mathrm{s}^{-1}$ $\mathrm{Mpc}^{-1}$ (and there were two distinct camps with very strong opinions about whether it was 50 or 100). For this reason, it's common to define the dimensionless parameter $h$:

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (111)$$

This is incorporated into measurements of quantities that involve the Hubble constant. For example, the mass of a typical galaxy group is $s \times 10^{13} h^{-1}\ \mathrm{M}_\odot$. We have better measurements of $H_0$ now, so we adopt $h_{\mathrm{WMAP}} = 0.71^{+0.04}_{-0.03}$.

In conventional units, the Hubble constant is inverse time:

$$H_0 = 3.24 \times 10^{-18} h \text{ s}^{-1}, \quad (112)$$

so

$$H_0 = 2.30 \times 10^{-18} \text{ s}^{-1}, \quad (113)$$

To estimate how long ago the Big Bang occurred, we assume (incorrectly) that the recessional velocities of galaxies is constant. We call the time since the Big Bang $t_H$; this is the time required for a galaxy to travel to a distance $d$ at speed $v$. So

$$d = vt_H = H_0 d t_H \tag{114}$$

and the **Hubble time** $t_H$ is

$$t_H = \frac{1}{H_0} = 4.35 \times 10^{17} \text{ s} = 1.38 \times 10^{10} \text{ yr}. \tag{115}$$

So the universe is about 13.8 Gyr old. This is a pretty good estimate.

# Lecture XLIX Newtonian Cosmology I

### XLIX.2 Newtonian cosmology

Now we will describe the evolution of the universe mathematically. This will take some time, but it isn't too complicated when we consider only gravitational forces; the evolution of the universe is essentially a battle between expansion and gravity.

The universe we consider is homogeneous and isotropic, with constant density $\rho$ (the textbook calls this a universe of pressureless dust, and it's a simple, special case). Consider, as we often do, a spherical shell of radius $r$ and mass $m$. The shell is expanding due to the expansion of the universe, with velocity $v$.

As the universe expands, the mass inside the shell $M(< r)$ is **constant**—the expansion is uniform, so mass that starts within the shell will always be within the shell. So $\rho r^3 =$ constant.

Because the mass is uniformly distributed, mass outside the shell doesn't matter.

The total energy of the shell as it expands is the sum of its kinetic and potential energies:

$$\frac{1}{2}mv^2 - \frac{GM(< r)m}{r} = E \tag{116}$$

Now let's parameterize the total energy of the shell:

$$E = -\epsilon\,\frac{1}{2}mc^2, \tag{117}$$

where $\epsilon$ is just a dimensionless number which we'll rewrite later. So

$$\frac{1}{2}mv^2 - \frac{GM(< r)m}{r} = -\epsilon\,\frac{1}{2}mc^2. \tag{118}$$

Next we substitute $M(< r) = (4\pi/3)r^3\rho$, cancel the shell mass $m$, and multiply by 2:

$$v^2 - \frac{8\pi}{3}G\rho r^2 = -\epsilon c^2. \tag{119}$$

Now let's return to our dimensionless number $\epsilon$, and rewrite it as

$$\epsilon \equiv k\varpi^2. \tag{120}$$

More on $\varpi$ later. $k$ contains information about whether the shell is **gravitationally bound or unbound**.

- Bound: $k > 0$. Total energy $E < 0$, so shell can recollapse. We call this **closed**.
- Unbound: $k < 0$. Total energy $E > 0$, so kinetic energy wins and shell keeps expanding. We call this **open**.

- $k = 0$: **Critical** or **flat**. Perfect balance between expansion and gravitational resistance.

So, we have

$$v^2 - \frac{8\pi}{3} G\rho r^2 = -kc^2\varpi^2. \tag{121}$$

Now we write the radius $r$ in terms of a global **scale factor** $R(t)$, which describes the overall expansion of the universe:

$$r(t) = R(t)\varpi \tag{122}$$

$R(t)$ is called the "scale factor" of the universe, and $\varpi$ is called the **comoving coordinate**. The comoving coordinate describes the distance between objects, but it stays constant as the universe expands; this is a useful way to talk about distances when the whole universe is expanding.

For instance, at time $t_1$, two galaxies are separated by a distance $r(t_1) = R(t_1)\varpi$, and at $t_2$ they are separated by a distance $r(t_2) = R(t_2)\varpi$. Their comoving separation stays the same, but their physical separation $r$ increases due to the increasing scale factor $R(t)$.

Now we rewrite the expansion velocity

$$v = \varpi V \tag{123}$$

where $V$ is the rate at which $R$ is changing. Substitute this and $r = R\varpi$:

$$\varpi^2 V^2 - \frac{8\pi}{3} G\rho R^2 \varpi^2 = -kc^2\varpi^2. \tag{124}$$

We cancel $\varpi$:

$$\boxed{V^2 - \frac{8\pi}{3} G\rho R^2 = -kc^2} \tag{125}$$

This is an expression for the evolution of only the scale factor $R$ — no more references to specific shells or coordinates!

Now we'll do a few more things. Divide out $R^2$ from the left side:

$$\left[\left(\frac{V}{R}\right)^2 - \frac{8\pi}{3} G\rho\right] R^2 = -kc^2 \tag{126}$$

We can choose the normalization of $R(t)$ in any way we like. For convenience, let's set the scale factor today equal to 1: $R(t_0) = 1$ ($t_0$ = time today—0 subscript refers to the value of something at the present time, e.g. $H_0$ is the value of the Hubble constant (which isn't constant) today, and $\rho_0$ is the density of the universe today). So

$$\left[\frac{V(t_0)}{R(t_0)}\right]^2 - \frac{8\pi}{3} G\rho_0 = -kc^2 \tag{127}$$

at the present time.

Now let's look at the first term. According to the Hubble law,

$$v(t) = H(t)r(t) = H(t)R(t)\varpi. \tag{128}$$

Also,

$$v(t) = \varpi V(t) \tag{129}$$

Equating these two expressions for $v$, we find an expression for the Hubble constant at any time $t$:

$$H(t) = \frac{V(t)}{R}. \tag{130}$$

So in general,

$$H(t) = \frac{V(t)}{R(t)}, \tag{131}$$

and at the present day

$$H_0 = \frac{V(t_0)}{R(t_0)}. \tag{132}$$

We recognize this as the left term in Equation 17 and substitute it in:

$$\boxed{H_0^2 - \frac{8\pi}{3} G\rho_0 = -kc^2} \tag{133}$$

In order from left to right, the three terms in this equation describe the kinetic, potential and total energies.

Now let's return to that "critical" universe with $k = 0$, where expansion and gravity are perfectly balanced. We can calculate the density we need to make this happen. This is the **critical density** $\rho_c$. If $k = 0$,

$$H_0^2 - \frac{8\pi}{3} G\rho_c = 0 \tag{134}$$

and

$$\boxed{\rho_c = \frac{3H_0^2}{8\pi G}} \tag{135}$$

This is about $9.5 \times 10^{-27}$ kg m$^{-3}$. For comparison, our best estimate of the density of baryonic[1] matter is $4.17 \times 10^{-28}$ kg m$^{-3}$; this is about 4% of the critical density.

If $\rho_0 < \rho_c$, there isn't enough mass to reverse the expansion and the universe is open. If $\rho_0 > \rho_c$, gravity wins and the universe is closed.

We parameterize the density of the universe in terms of a dimensionless number $\Omega$:

$$\Omega_0 \equiv \frac{\rho_0}{\rho_c} \tag{136}$$

- $\Omega_0 = 1$: flat universe with $k = 0$
- $\Omega_0 < 1$: open universe with $k < 0$

[1]Baryons are particles made of three quarks. Protons and neutrons are the most common. Baryons make up nearly all of the visible matter in the universe. (Electrons are leptons—not composed of quarks.) Most dark matter is probably non-baryonic.

- $\Omega_0 > 1$: closed universe with $k > 0$

We can then also rewrite Equation 23 as

$$H_0^2(\Omega_0 - 1) = kc^2 \tag{137}$$

at the present day.

# Lecture L Newtonian Cosmology II

## L.2 Evolution of the scale factor

We return to our equation for the evolution of the scale factor $R(t)$,

$$V(t)^2 - \frac{8\pi}{3}G\rho R(t)^2 = -kc^2. \tag{138}$$

Remember that the mass within the original shell was constant,

$$\rho(t)r^3(t) = \text{ constant.} \tag{139}$$

Writing this in terms of the scale factor,

$$\rho(t)R^3(t)\varpi^3 = \text{ constant,} \tag{140}$$

where the comoving coordinate $\varpi$ is also constant. So

$$\rho(t)R^3(t) = \rho(t_0)R^3(t_0) = \rho_0 \tag{141}$$

since $R(t_0) = 1$.

Therefore

$$V(t)^2 - \frac{8\pi G}{3}\frac{\rho(t)R(t)^3}{R(t)} = -kc^2 \tag{142}$$

which can be written

$$\boxed{V(t)^2 - \frac{8\pi G\rho_0}{3R(t)} = -kc^2.} \tag{143}$$

This is a equation for $R(t)$ and $V(t)$, and we can solve it to get an expression for the evolution of the size of the universe.

### L.2.1 Flat universe

We'll start with the case in which the universe is flat, with $k = 0$ and density equal to the critical density,

$$\rho_0 = \rho_{c,0} = \frac{3H_0^2}{8\pi G}. \tag{144}$$

$$\boxed{R(t) = \left(\frac{3}{2}\right)^{2/3}\left(\frac{t}{t_H}\right)^{2/3}} \tag{145}$$

where $t_H \equiv 1/H_0$ is the Hubble time.

So, for a critical, flat universe ($\Omega_0 = 1$), $R(t) \propto t^{2/3}$. Note that this expands forever. Also note that, since $R(t_0) = 1$,

$$t_0 = \frac{2}{3H_0}. \tag{146}$$

This is the age of the universe.

### L.2.2 Closed universe

Things get more complicated if the universe isn't flat. For a closed universe with $\Omega_0 = \rho_0/\rho_c > 1$ we see that the Universe starts expanding, but then slows down and recollapses! (These are equations for a cycloid.).

What's the current age of the universe in this model? Messy! $R_0 = 1$, so

$$t_0 = \frac{\pi}{2H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \left[ \cos^{-1}\left[\frac{2}{\Omega_0} - 1\right] - \sqrt{1 - \left[\frac{2}{\Omega_0} - 1\right]^2} \right] \tag{147}$$

### L.2.3 Open universe

For an open universe with $k < 0$ and $\Omega_0 < 1$, $R$ is monotonically increasing, so the universe expands forever.

For all of these expressions, you can see that the age of the universe depends on $\Omega_0$. More dense universes are younger. [for fixed $H_0$; can we explain this qualitatively?]

## L.3 Cosmological redshift

The evolution of the scale factor affects more than galaxies. The expansion of the universe also changes the wavelength of light; light stretches as the universe expands.

Suppose light is emitted with wavelength $\lambda_1 = \lambda_{\rm em}$ at some time $t_1 << t_0$, when the universe was smaller than its current size—say $R_1 = 1/3$. At some later time $t_2$ the universe has doubled in size to $R_2 = 2/3$, changing the wavelength of light along with it, so the wavelength of the light is also twice as big, $\lambda_2 = 2\lambda_{\rm em}$. We detect the light at time $t_0$, when $R_0 = 1 = 3R_1$, and the wavelength of the light is now $\lambda_3 = 3\lambda_{\rm em}$. So, recalling the definition of redshift,

$$\frac{\lambda_{\rm obs}}{\lambda_{\rm em}} \equiv (1+z) = \frac{R(t_0)}{R(t_1)}. \tag{148}$$

For this example, the redshift $z = 2$.

So the redshift and scale factor are directly related, and when we talk about the history of the universe we usually just parameterize it in terms of the redshift we would measure today for photons which were emitted at some earlier time $t < t_0$ when the universe was more compact.

$$(1+z) = \frac{R(t_0)}{R(t_1)}, \tag{149}$$

so

$$R(t) = \frac{1}{(1+z)}, \tag{150}$$

independent of any of the cosmological parameters like $H_0$ or $\Omega_0$. So, for example, at a redshift $z = 3$, the universe was 1/4 of its current size.

For a flat universe we derived

$$R(t) = \left( \frac{t}{\frac{2}{3} t_H} \right)^{2/3} \tag{151}$$

which can be rearranged to

$$t(z) = \left( \frac{2}{3} t_H \right) \left( \frac{1}{(1+z)^{3/2}} \right). \tag{152}$$

So as we found before, today at $z = 0$ $t_0 = (2/3)t_H$, and at $z = 3$, $t = t_0/8$. If $\Omega_0 = 1$ and we observe a galaxy at redshift $z = 3$, we are observing a galaxy at 1/8 the current age of the universe.

### L.4 Lookback time

It's also useful to define the "lookback time" to redshift $z$, the amount of time which has passed between when a redshifted photon was emitted and when we detect it today.

$$t_{\rm lookback} \equiv t_0 - t(z) \tag{153}$$

In other words, this is just the difference in age between the universe today and the universe at time $t(z)$. The lookback time depends on $\Omega_0$ and $H_0$.

# Lecture LI Newtonian Cosmology III

### LI.2 The expansion rate of the universe and the flatness problem

The rate at which the universe is expanding is a function of time—the value of the Hubble "constant" isn't constant, $H(z) \neq H_0$ and

$$H(z) = \frac{V(t)}{R(t)} \neq \frac{V(t_0)}{R(t_0)} \tag{154}$$

We return to our equation for the evolution of the scale factor

$$\left[H^2(z) - \frac{8\pi}{3}G\rho\right]R^2(t) = \left[H_0^2 - \frac{8\pi}{3}G\rho_0\right]R^2(t_0). \tag{155}$$

Since $R(t_0) = 1$ and $R(t) = 1/(1+z)$,

$$H^2(z) - \frac{8\pi}{3}G\rho = \left[H_0^2 - \frac{8\pi}{3}G\rho_0\right](1+z)^2. \tag{156}$$

From the definition of the critical density

$$\Omega_o = \frac{\rho_0}{\rho_c} = \frac{8\pi G\rho_0}{3H_0^2}, \tag{157}$$

$$\frac{8\pi G\rho_0}{3} = H_0^2\Omega_0, \tag{158}$$

and because mass is conserved,

$$\rho(z)R^3(t) = \rho_0 R^3(t_0) \Rightarrow \rho(z) = \rho_0(1+z)^3. \tag{159}$$

The universe was denser at higher redshifts. So

$$\frac{8\pi G\rho(z)}{3} = H_0^2\Omega_0(1+z)^3 \tag{160}$$

and we have

$$H(z)^2 - H_0^2\Omega_0(1+z)^3 = H_0^2[1-\Omega_0](1+z)^2 \tag{161}$$

and

$$H(z)^2 = H_0^2[\Omega_0(1+z) + 1 - \Omega_0](1+z)^2 \tag{162}$$

which simplifies to

$$\boxed{H(z)^2 = H_0^2(1+z)^2(1+\Omega_0 z)}. \tag{163}$$

This tells us about the evolution of the expansion rate of the universe.

It is also useful to calculate the evolution of the density of the universe. As noted above, the universe used to be smaller and denser, so $\Omega(z) \neq \Omega_0$. We can calculate $\Omega(z)$ starting from the same point at which we calculated $H(z)$. The algebra is left as an exercise for the reader, but the result is

$$\boxed{1-\Omega(z)=\frac{1-\Omega_0}{1+\Omega_0 z}} \tag{164}$$

The general behavior of this is shown in the Figure, for an open universe with $\Omega_0 = 0.3$ and a closed universe with $\Omega_0 = 1.5$. In both cases, $\Omega(z) \to 1$ at high redshift. Flat universes are always flat, but open and closed universes also used to be very close to flat. This is a problem which we'll talk more about later. $\Omega = 1$ is unstable: if the universe was even slightly more dense than $\rho_c$, then at late times it's much denser than $\rho_c$, and if it were slightly less dense than $\rho_c$, it should now be much less dense than $\rho_c$. So the current matter density of the universe ($\Omega_0 \sim 0.3$) is very unlikely! In order to have this density today, the universe would have to have formed with a density within a very tiny fraction of the critical density—one part in $10^{62}$ or less. This is called the **flatness problem**. This is one of the main motivations for inflation, and one of the reasons why theorists favored a flat universe even when observations favored an open universe.

### LI.3 Adding pressure to the model of the universe

So far we have only considered the gravitational effect of matter in our model of the universe. We will now broaden this a bit. We start with our now familiar differential equation for the evolution of the scale factor (the Friedmann equation)

$$\left[\left(\frac{V}{R}\right)^2-\frac{8\pi G\rho}{3}\right]R^2=-kc^2. \tag{165}$$

We will expand the definition of $\rho$ to include relativistic particles like photons and neutrinos as well as normal matter. For normal matter, $\rho$ is just the usual mass density. For relativistic particles, we make use of the equivalence of mass and energy: $\rho$ is the energy density divided by $c^2$.

Remember that this equation is essentially a statement of the conservation of energy; it says that the sum of the gravitational potential energy and the kinetic energy of expansion of the universe is constant. We will use another expression of conservation of energy to calculate the effects of components of the universe that produce pressure. Our universe is now filled with a fluid of density $\rho$ (this is now the equivalent mass density), temperature $T$, and pressure $P$. Can get:

$$\boxed{a(t)=-\frac{4\pi G}{3}\left(\rho+\frac{3P}{c^2}\right)R(t)} \tag{166}$$

This is the **acceleration equation**, $a(t)$ is the acceleration of $R(t)$. Note that the effect of pressure is to slow down the acceleration; for positive matter density and pressure, the acceleration is negative, so the universe is slowing down.

We'll write down the Friedmann equation again, so that we have all three important equations together:

$$\boxed{\left[\left(\frac{V}{R}\right)^2-\frac{8\pi}{3}G\rho\right]R^2=-kc^2} \tag{167}$$

These equations have three unknowns, $R$, $\rho$ and $P$, but they are not independent: we can use any two to derive the third, as we just did for the acceleration equation. So to solve for $R$, $\rho$ and $P$ we need another equation: an **equation of state** that links the variables. We write the equation of state as

$$P = w\rho c^2, \tag{168}$$

where $w$ is a constant, so the pressure is proportional to the energy density of the fluid. For matter with no pressure, as in our first model of the universe, $w = 0$. A fluid of photons or other massless particles is relativistic, and has the equation of state

$$P = \frac{1}{3}\rho c^2, \tag{169}$$

so $w = 1/3$. We will return to this question of pressure and the equation of state when we discuss the cosmological constant.

# Lecture LII The Cosmic Microwave Background

### LII.2 Cooling of the universe after the Big Bang

A key point of the Big Bang theory is that the early universe was very dense and hot. We expect that this hot, dense universe would have been in thermodynamic equilibrium , and that therefore the radiation field had a blackbody spectrum. We can compute the cooling of this radiation as the universe expands.

The energy density of blackbody radiation is

$$u = at^4, \tag{170}$$

where $a$ is the radiation constant $a \equiv 4\sigma/c$. By replacing $P$ in the fluid equation with the equation of state $P = w\rho c^2$ we find

$$R^{3(1+w)}u = R^4 u = u_0, \tag{171}$$

since $w = 1/3$ for photons and $R_0 = 1$. This tells us that the energy density of the universe today is smaller by a factor of $R^4$ than it was at some earlier time with scale factor $R$. A factor of $R^3$ comes from the change in volume of the universe, and an additional factor of $R$ comes from the lower energy of the longer wavelength photons we see today, because of the cosmological redshift. Therefore

$$R^4 aT^4 = aT_0^4 \tag{172}$$

and the current temperature of the blackbody radiation is related to the temperature at an earlier time by

$$RT = T_0. \tag{173}$$

When the universe was half as large it was twice as hot. Recalling that

$$R = \frac{1}{1+z} \tag{174}$$

we can also write

$$T = (1+z)T_0 \tag{175}$$

for the dependence of the temperature of the radiation on redshift.

We can make an order of magnitude estimate of the current temperature of the blackbody radiation by considering the conditions needed to produce helium in the early universe. The early universe was hot and dense enough for nuclear reactions to take place, and the heaviest element that was formed in these reactions was He (and a very small amount of Li). This fusion requires approximately $T \simeq 10^9$ K and $\rho_b \simeq 10^{-2}$ kg m$^{-3}$, where the $b$ subscript refers to the baryon density. (If the temperature were higher the deuterium nuclei needed for the reaction would photodisassociate, and if the temperature were lower it would be too difficult to overcome the Coulomb barrier. The density is needed to produce the observed amount of He.) We can therefore estimate the value of the scale factor at the time of helium formation:

$$R \simeq \left(\frac{\rho_{b,0}}{\rho_b}\right)^{1/3} = 3.5 \times 10^{-9} \tag{176}$$

We can then combine this with the temperature required, $T(R) = 10^9$ K, to determine the current temperature of the radiation:

$$T_0 = RT(R) \simeq 3.5 \text{ K} \tag{177}$$

We will see that this simple calculation gives a very good estimate of the actual temperature of the radiation. Also note that this was predicted in 1948, well before it was discovered.

## LII.3 Discovery and measurement of the cosmic microwave background radiation

- Discovered in 1963 by Arno Penzias and Robert Wilson, working at Bell Labs
- Couldn't get rid of background hiss in their signal... even after removing the pigeons from the antenna
- Knew that a 3 K blackbody would produce the signal, but didn't know of any possible source until they heard about recent work at Princeton (Robert Dicke and Jim Peebles) calculating the temperature of relic radiation from the Big Bang
- Penzias and Wilson published the discovery with the title "A Measurement of Excess Antenna Temperature at 4080 Megacycles per Second," a very modest title for an extremely important discovery!
- The spectrum of the CMB was measured by the COBE satellite in 1991, and proved to be a spectacular confirmation of the prediction—a nearly perfect blackbody with temperature 2.725 K
- The CMB radiation fills the universe and is isotropic. An observer moving with respect to the Hubble flow (the general expansion of the universe) will see a Doppler shift in the CMB. This change in wavelength can be expressed as a change in temperature using Wien's law. The Sun's peculiar velocity produces a **dipole anisotropy** in the CMB: the temperature depends on the peculiar velocity of the Sun in the direction we're looking. This allows us to determine the peculiar velocity of the Sun with respect to the Hubble flow—it's $370.6 \pm 0.4$ km s$^{-1}$ . We can decompose this into motions of the Sun around the Galaxy, the Milky Way within the Local Group, and the Local Group with respect to the Hubble flow.
- Once we remove the dipole, the CMB is incredibly isotropic. However, it does have small variations in temperature: on scales of $1^\circ$ or less, the temperature departs from the average value by about one part in $10^5$. These anisotropies produce the famous map of the CMB you've probably seen (from the WMAP satellite), and measurements of these anisotropies are the primary source of our precise measurements of cosmological parameters. More on that later.

## LII.4 A two-component model of the universe

A complete model of the universe needs to include the effects of the CMB as well as matter. The CMB has a negligible effect on the dynamics of the universe now, but at very early times it was dominant. Lots of detail in the book—just a few important points here. Note: we will only talk about photons, but this applies to neutrinos and other relativistic particles as well.

We already saw that

$$R^{3(1+w)}u = R^4 u = u_0, \tag{178}$$

since $w = 1/3$ for photons and $R_0 = 1$. Converting the energy density into the equivalent mass density $\rho_{\rm rel} = u/c^2$, we see that

$$R^4 \rho_{\rm rel} = \rho_{\rm rel,0}. \tag{179}$$

Compare this with the evolution of matter density with the scale factor,

$$R^3 \rho_m = \rho_m. \tag{180}$$

As the scale factor becomes smaller, $\rho_{\rm rel}$ increases more rapidly than $\rho_m$, which means that although matter dominates now, there must have been a time in the early universe as $R \to 0$ when radiation (all relativistic particles) was dominant. We can find out when this was by seeing when $\rho_{\rm rel} = \rho_m$.

This is done in the book; the result is that the universe was **radiation dominated** until a redshift of $z = 3270$, at which time the temperature was $T = 8920$ K. After this, the universe was **matter dominated**.

We can also look at how the universe expands during the radiation era. Including the contributions of both matter and relativistic particles,

$$\left[\left(\frac{V}{R}\right)^2 - \frac{8\pi G}{3}(\rho_m + \rho_{\rm rel})\right] R^2 = -kc^2 \tag{181}$$

Writing $\rho_m$ and $\rho_{\rm rel}$ in terms of their values today, we find

$$\left[\left(V\right)^2 - \frac{8\pi G}{3}\left(\frac{\rho_{m,0}}{R} + \frac{\rho_{\rm rel,0}}{R^2}\right)\right] = -kc^2 \tag{182}$$

We can set $k = 0$ because the early universe was extremely close to flat. We can then show that the time of matter and radiation equality was $t_{r,m} = 5.5 \times 10^4$ yrs, and that when $R \ll R_{r,m}$, $R \propto t^{1/2}$. This should be compared with the matter-dominated era, when $R \gg R_{r,m}$; in this case $R \propto t^{2/3}$, as we already showed for a flat universe containing only matter. So the universe expanded more slowly in the radiation era.

## LII.5 The origin of the CMB

What are we actually seeing when we look at the CMB radiation?

After the Big Bang, the universe was hot and dense, and filled with free electrons, free protons, and photons. The photons scattered off the free electrons, and could only travel short distances between scatterings. The frequent scatterings kept the electrons and photons in thermal equilibrium (they had the same temperature). As the universe expanded, the electrons became farther apart and the photons could travel longer between scatterings. This process, of the expansion diluting the density of a particle until it no longer interacts with other particles, is called **decoupling**, and it occurred for other particles such as neutrinos—early in the history of the universe, neutrinos stopped interacting with other particles because the distances between them became too great.

In the case of photons, something else happened: the universe became cool enough for the free electrons and free protons to combine into neutral atoms, so the photons no longer had electrons to interact with, and matter and radiation began to evolve independently. This completed the decoupling of radiation and matter. The formation of neutral atoms is called **recombination**, even though that doesn't actually make sense since the electrons and protons were never combined to begin with. Decoupling is a better word.

Once there were no free electrons, the opacity of the universe was much lower, and the photons could stream freely without scattering. The CMB photons we see were last scattered during recombination.

For this reason we define the **surface of last scattering**. This is a spherical "surface," centered on the Earth, from which all the CMB photons come. Because the universe was opaque before recombination, this is the farthest redshift we can possibly observe. The surface of last scattering actually has a thickness $\Delta z$, because recombination didn't happen all at once.

The redshift at which recombination occurred, and the temperature of the universe at that time, can be calculated, and measured from the WMAP CMB observations. The result is

$$z_{\rm dec} = 1089 \pm 1 \tag{183}$$

and

$$T_{\rm dec} = T_0(1 + z_{\rm dec}) = 2970\ \text{K}. \tag{184}$$

This corresponds to an age of $t_{\rm dec} = 379,000$ years.

# Lecture LIII Relativistic Cosmology

We've been talking about different scenarios for the evolution of the universe: open, flat or closed, depending on the density of mass and energy. This is very closely related to the geometry of the universe, which is described by general relativity. To understand the curvature of the universe, we need to consider some of the principles of non-Euclidean geometry.

### LIII.2 Euclidean, Elliptic and Hyperbolic Geometries

In about 300 BC, Euclid worked out 5 postulates from which all the rules of geometry could be derived—these rules lay out the basic behaviors of straight lines, right angles, etc.

5th postulate: Given a line and a point not on the line, there is exactly one parallel line which passes through the point.

In the18th century, mathematicians realized it was possible to make fully consistent definitions of geometry using Euclid's first 4 postulates, but modifying the 5th. These correspond to **curved spaces**.

These examples are curved 2-dimensional surfaces embedded in 3-d spaces—it's not too hard to understand the geometry of something when you can leave the space to look at it. But we can't do that with the universe—we can't pop off into a 4th spatial dimension to look at the curvature. Instead we have to try to understand it using measurements entirely within the space itself. These are called "inner properties."

The inner properties of a curved space are closely related to how distances are measured within that space.

- On a flat Euclidean plane, a circle has circumference $C = 2\pi r$.
- On a sphere of radius $R$, the radius of the circle $r = R\theta$, but the circumference is

$$\begin{aligned} C &= 2\pi(R\sin\theta) && (185)\\ &= 2\pi r\left(\frac{\sin\theta}{\theta}\right) && (186)\\ &< 2\pi r && (187) \end{aligned}$$

  So, an observer on the surface of a sphere could measure the radius and circumference of a circle, and deduce that they were living in a positively curved world.

- Similarly, in a negatively curved space, $C > 2\pi r$.

In our Newtonian derivation of the Friedman equation, $k$ referred to the total energy of the universe, but when this equation is derived from general relativity, $k$ is the curvature constant defined above.

# Lecture LIV The Cosmological Constant

Consider again the now familiar equation

$$\left[\left(\frac{V}{R}\right)^2 - \frac{8\pi}{3}G\rho\right]R^2 = -kc^2. \tag{188}$$

We derived this equation from Newtonian physics, by considering the kinetic and potential energy of the universe, but this equation is also a solution to Einstein's field equations for an isotropic, homogeneous universe. In 1922 the Russian mathematician Aleksandr Friedmann solved the field equations and obtained this equation for a non-static universe. We've been calling it the Friedmann equation all along, but strictly speaking this refers to the equation as derived from general relativity. As we've seen, the constant $k$ refers to the curvature of the universe.

### LIV.2 The cosmological constant

Einstein developed his field equations before Hubble's discovery of the expanding universe, and he believed that the universe was static. In their original form, his field equations couldn't produce a static universe, so Einstein added an additional term (a constant of integration) in order to make the universe static, $V = 0$. This term is the cosmological constant $\Lambda$, and with this addition the general solution to Einstein's field equations is

$$\left[\left(\frac{V}{R}\right)^2 - \frac{8\pi}{3}G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2. \tag{189}$$

In our original Newtonian derivation, this would result from adding an additional potential energy term

$$U_\Lambda \equiv -\frac{1}{6}\Lambda m c^2 r^2 \tag{190}$$

to our equation for the energy balance of the universe. The result of this new potential is a force

$$F_\Lambda = \frac{1}{3}\Lambda m c^2 r \tag{191}$$

which is radially outward for $\Lambda > 0$: a repulsive force on the mass shell countering gravity, which allowed Einstein to balance the universe in an (unstable) equilibrium.

### LIV.3 Effects of the cosmological constant

After Hubble's discovery of the expanding universe, Einstein called the inclusion of this term the "biggest blunder" of his life. However, recent results have indicated that the universe is actually dominated by some sort of energy which behaves like the cosmological constant. We call this **dark energy**, and we'll now look at its effect on the dynamics of the universe.

We write the Friedmann equation in a form that makes it clear that we're now dealing with a three-component universe of mass, relativistic particles and dark energy:

$$\left[\left(\frac{V}{R}\right)^2 - \frac{8\pi}{3}G(\rho_m + \rho_{\rm rel}) - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2. \tag{192}$$

The Friedmann equation and the fluid equation can be combined to produce the acceleration equation:

$$a = \left[-\frac{4\pi G}{3}\left[\rho_m + \rho_{\rm rel} + \frac{3(P_m + P_{\rm rel})}{c^2}\right] + \frac{1}{3}\Lambda c^2\right]R \tag{193}$$

We now define the equivalent mass density of dark energy

$$\rho_\Lambda \equiv \frac{\Lambda c^2}{8\pi G} = \text{constant} = \rho_{\Lambda,0} \tag{194}$$

so that the Friedmann equation becomes

$$\left[\left(\frac{V}{R}\right)^2 - \frac{8\pi}{3}G(\rho_m + \rho_{\rm rel} + \rho_\Lambda)\right]R^2 = -kc^2. \tag{195}$$

Note that because $\rho_\Lambda$ remains constant as the universe expands, more and more dark energy must appear to fill the larger volume.

We can calculate the pressure due to dark energy from the fluid equation

$$P_\Lambda = -\rho_\Lambda c^2 \tag{196}$$

which is the equation of state for dark energy. In the general equation of state $P = w\rho c^2$, $w = -1$ for dark energy. The pressure due to the cosmological constant is *negative*, while the equivalent mass density is positive. We can now substitute expressions for $\rho_\Lambda$ and $P_\Lambda$ into the acceleration equation (Eq 7):

$$a = \left[-\frac{4\pi G}{3}\left[\rho_m + \rho_{\rm rel} + \rho_\Lambda + \frac{3(P_m + P_{\rm rel} + P_\Lambda)}{c^2}\right]\right]R \tag{197}$$

The Friedmann equation can also be written in terms of the Hubble constant and the density parameter $\Omega$ (we showed this for the single component universe a while back) as

$$H^2[1 - (\Omega_m + \Omega_{\rm rel} + \Omega_\Lambda)]R^2 = -kc^2 \tag{198}$$

where

$$\Omega_m = \frac{\rho_m}{\rho_c} = \frac{8\pi G\rho_m}{3H^2} \tag{199}$$

$$\Omega_{\rm rel} = \frac{\rho_{\rm rel}}{\rho_c} = \frac{8\pi G\rho_{\rm rel}}{3H^2} \tag{200}$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda c^2}{3H^2}. \tag{201}$$

We define the **total density parameter**

$$\Omega \equiv \Omega_m + \Omega_{\rm rel} + \Omega_\Lambda. \tag{202}$$

Note that $\Omega$ without a subscript refers to the total density parameter for all of the components of the model under consideration. The Friedmann equation is then

$$H^2(1 - \Omega)R^2 = -kc^2. \tag{203}$$

A flat universe with $k = 0$ requires $\Omega(t) = 1$.

The WMAP measurements of the cosmic microwave background give us values for the three components of the universe today:

$$\Omega_{m,0} = 0.27 \pm 0.04 \tag{204}$$
$$\Omega_{\rm rel,0} = 8.24 \times 10^{-5} \tag{205}$$
$$\Omega_{\Lambda,0} = 0.73 \pm 0.04 \tag{206}$$

Within our ability to measure it, the universe is flat and currently dominated by dark energy.

We can also define the **deceleration parameter** $q(t)$:

$$q(t) \equiv -\frac{R(t)[a(t)]}{[V(t)^2]} \tag{207}$$

The name and the minus sign, which gives a positive value for a decelerating universe, reflect the once-common belief that the universe had to be decelerating. The deceleration parameter can also be written in terms of the density parameters for the different components of the universe:

$$q(t) = \frac{1}{2}\Omega_m(t) + \Omega_{\rm rel}(t) - \Omega_\Lambda(t). \tag{208}$$

With current values,

$$q_0 = -0.60, \tag{209}$$

telling us (because of the minus sign) that the universe is currently accelerating.

## LIV.4 The $\Lambda$ era

We've already discussed the dependence of the density of radiation and matter on the scale factor: $\rho_m \propto R^{-3}$ and $\rho_{\rm rel} \propto R^{-4}$. Because the radiation density decreases more quickly as the universe expands, the universe was dominated by radiation at early times but then became matter-dominated at a redshift of $z_{r,m} = 3270$. We now have another component to consider, $\rho_\Lambda$, which is *constant*. This means that at some point, as the matter density decreases as the universe expands, the universe will become dominated by the cosmological constant. As it turns out, this has already happened. The scale factor at the equality of matter and dark energy is

$$R_{m,\Lambda} = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3} = 0.72, \tag{210}$$

which corresponds to a redshift $z_{m,\Lambda} = 0.39$.

We can also use the acceleration equation to find when the acceleration of the universe changed from negative to positive. This is one of the problems on this week's problem set, and the result is

$$R_{\rm accel} = \left(\frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}}\right)^{1/3} = 0.57, \tag{211}$$

corresponding to a redshift $z_{\rm accel} = 0.76$. So the universe began to accelerate *before* the equality of matter and dark energy; this is because dark energy has pressure as well as equivalent mass density which affects the dynamics of the universe, and (non-relativistic) matter does not. Note also that unlike all of the other cosmological times we've calculated, these are relatively recent. We routinely observe objects at these redshifts, and can use them to understand the dynamics of the universe. More on that when we discuss observational cosmology.

## LIV.5 What is dark energy?

We don't really know, but the leading candidate is **vacuum energy**. In classical physics there is no such thing, but quantum physics allows for energy in a vacuum. The Heisenberg uncertainty principle allows particle-antiparticle pairs to spontaneously appear and then annihilate in an otherwise empty vacuum. The total energy and lifetime of these particles must satisfy the relation

$$\Delta E \Delta t \geq h. \tag{212}$$

There is an energy density associated with these particle-antiparticle pairs, and this energy density is a quantum phenomenon that doesn't care at all about the expansion of the universe or the passage of time. However, calculating the actual value of this energy density is an exercise in quantum field theory that hasn't yet been completed. One suggestion is that the natural value for the vacuum energy density is the Planck energy density,

$$u_{\rm vac} = \frac{E_P}{l_P^3}. \tag{213}$$

However, the Planck energy is large (by particle physics standards; $E_P = \sqrt{\hbar c^5/G} = 1.96 \times 10^9$ J) while the Planck length is very small ($l_P = 1.6 \times 10^{-35}$ m), and the resulting energy density is 124 orders of magnitude larger than the current critical density of the universe (!). So the question of what dark energy is and why it has the value it does is very much still open.

# Lecture LV Observational Cosmology: Cosmological Parameters

Values of the various cosmological parameters (the density parameters for different components of the universe, the Hubble constant, etc) are measured in many different ways. We will discuss the two that are currently most important: measurement of the acceleration of the universe from Type Ia supernovae, and measurement of anisotropies in the CMB.

Based on measuring something distance that is far away. We measure how far it actually is (based on other information), and then adjust the cosmological parameters until the distance matches the distance at that redshift.

## LV.2 Type Ia supernovae

Type Ia supernovae are useful because they are standard candles and they're extremely bright. Their intrinsic luminosity can be estimated from their light curves, which allows us to determine their distance. Because they're so bright, we can see them to extremely large distances, $z > 1$. To see how this is useful in constraining cosmological parameters, we can look at how the luminosity distance changes for different cosmologies.

Note that for $z < 1.5$, where our current measurements are made, the luminosity distance is largest for the flat, $\Lambda$-dominated universe, and smaller for both of the matter-only models. This larger luminosity distance means that the supernovae will appear fainter than they would in matter-only cosmologies, and that is indeed what is observed. Note also that the differences are small, particularly between the flat, $\Lambda$ model and the open, matter-only model. Very precise photometric measurements are required!

- The dashed line on the main plot shows the best fit cosmological model, which has $\Omega_m = 0.29$ and $\Omega_\Lambda = 0.71$. The inset shows the differences between the binned data and various models (after an empty universe model with $\Omega = 0$ has been subtracted).
- **Gray dust.** We won't discuss all of the models shown on the inset plot, but one that is important to note is the high-z gray dust model. An important initial concern with the supernova data, when the SNe were found to be fainter than expected, was that there might be some sort of "gray" dust in the way, absorbing the light equally at all wavelengths and making the SNe fainter. Normal dust affects blue light more than red light, so we can see that it changes the color of objects; gray dust would be very difficult to detect because it would make objects fainter without changing their color. We don't actually know of any gray dust, but it didn't seem to be any less plausible than the cosmological constant when the fainter-than-expected supernovae were initially discovered. However, if gray dust were making the SNe fainter, they would continue to be fainter than expected as they got farther and farther away, and that's not what happened as more distant SNe were discovered. As can be seen from the solid black (best-fit) line in the inset panel, the supernovae do indeed get fainter than expected with redshift, but then they turn over and start getting brighter again, at about

the redshift when the universe changed from being matter-dominated to being $\Lambda$-dominated. This trend rules out the gray dust. In other words, *the supernovae strongly favor a model with recent acceleration and previous deceleration.*

- **Evolution.** Another early counter-argument to the accelerating universe was that the Type Ia supernovae might not actually be standard candles: there might be something different about high redshift supernovae which makes them intrinsically fainter—lower metallicity, for example. However, like the gray dust case, if the intrinsic brightness of the SNe were evolving with redshift we would expect them to continually get fainter with distance, and they don't. The turnover in observed brightness also makes such evolutionary effects very unlikely. This is the "evolution $\sim z$" model on the plot.

- There were a lot of doubts about the SNe results when they first started coming out over 10 years ago, but subsequent data has made the accelerating universe model much more robust.

### LV.3 Temperature fluctuations in the CMB

The other very important method of determining cosmological parameters is through measurements of the temperature fluctuations in the CMB. As we've seen, these are very small, with

$$\frac{\delta T}{T} \sim 10^{-5}. \tag{214}$$

These small variations tell us that the universe wasn't perfectly homogeneous at the time of last scattering ($z \sim 1100$), and the angular size of the fluctuations is related to the physical size of the density and velocity fluctuations at this time. We study these fluctuations by plotting the strengths of the fluctuations as a function of angular scale; this is called the angular power spectrum.

The power spectrum is somewhat analogous to the waveform of a musical instrument displayed on an oscilloscope; the largest peak is the fundamental frequency, and the smaller peaks are harmonics. In the case of the CMB, the peaks are due to *acoustic oscillations* (standing sound waves) in the photon-baryon fluid of the universe at the time of recombination. The location of the first peak corresponds to the angular diameter of the largest region that could have supported a standing wave at the time of decoupling; roughly the size of the universe at that time. The peak appears at an angular scale of $\sim 1°$, corresponding to a physical size of $\sim 0.2$ Mpc. This is approximately the Hubble distance ($d_H = c/H(z)$) at the time.

The location of this peak is related to the curvature of the universe, through the relationship between curvature and angular size. In a positively curved universe the angular size of an object of known proper size is larger than it is in a negatively curved universe. This means that if the universe were negatively curved, the first peak would be seen at an angle $< 1°$, while it would be at an angle $> 1°$ if it were positively curved. The observed position of the peak tells us that the universe is very close to flat. The second and third peaks tell us about the densities of baryonic and dark matter; the fact that the third peak is about as strong as the second tells us that most of the matter in the universe is dark.

We often plot constraints on cosmological parameters from different methods on a plot of $\Omega_m$ vs. $\Omega_\Lambda$. This plot summarizes our model of the universe, which is often called the Concordance Model or Consensus Model.