Astron 211 Problem Set 10

Given: Nov 29. Due: Dec 6 at the beginning of class

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Reading: Kutner Chapter 16, 17, 18, 19

Problem 1 Mass-to-Light Ratios

a. From information given in the Table below and in Kutner Chapter 16, determine the approximate mass-to-light ratio of the Galaxy interior to a radius of 25 kpc from the center.

Component	Mass	Luminosity	Radius	Shape
Neutral Gas	$5 \times 10^9 M_\odot$	0	25 kpc	disk
Thin Disk	$6 \times 10^{10} M_\odot$	$1.8 \times 10^{10} L_\odot$	25 kpc	disk
Thick Disk	$3 \times 10^9 M_\odot$	$2 \times 10^8 L_\odot$	25 kpc	disk
Bulge	$1 \times 10^{10} M_\odot$	$3 \times 10^9 L_\odot$	4 kpc	sphere
Stellar Halo	$3 \times 10^9 M_\odot$	$1 \times 10^9 L_\odot$	> 100 kpc	sphere
Dark-Matter Halo	$1.9 \times 10^{12} M_\odot$	0	> 230 kpc	sphere

b. Repeat your calculation for a radius of 230 kpc. What can you conclude about the effect that dark matter might have on the average mass-to-light ratio of the universe?

Problem 2 Stars, Supermassive Black Holes and Tidal Forces

In this problem we will calculate whether or not stars swallowed up by the supermassive black hole at the center of the Milky Way are ripped apart before they pass the event horizon of the black hole. The tidal force is the *differential* force across a body due to an external gravitational force; the gravitational force will act more strongly on the side of the body closer to the massive external object. If the tidal force is stronger than the force holding the object together, the object will be ripped apart.

Consider a star of mass $M_\star$ and radius $r_\star$ a distance r from a black hole of mass $M_{\rm BH}$. If the differential tidal force across the star is

$$\Delta F = \frac{GM_{\rm BH}M_\star}{r^3} r_\star \tag{1}$$

and the gravitational force holding the star together is

$$F_{\rm grav} = \frac{GM_\star^2}{r_\star^2}, \tag{2}$$

how close can the star come to the black hole before it is ripped apart? Consider a star with the mass and radius of the Sun, and a black hole of mass $M_{\rm BH} = 3.7 \times 10^6 M_\odot$. What about a star with mass $60M_\odot$ and radius 13 $R_\odot$? Do these stars cross the Schwarzchild radius of the black hole before they are ripped apart?

If a star is swallowed whole by a black hole it doesn't produce a major outburst of radiation before it crosses the event horizon, but if it's ripped apart first its gas forms a hot and bright accretion disk around the black hole. Would we expect to see an accretion disk around the black hole at the center of the Milky Way? [Note that this process was likely responsible for a new class of explosion seen recently. See http://www.youtube.com/watch?v=azLDH9ZPbVs

Problem 3 Kutner Problem 17.9

Calculate the mass of a galaxy with a flat rotation curve with $v = 300$ km/s out to $r = 20$ kpc. Express your answer in $M_\odot$.

Problem 4 Timescales in Galaxies and Clusters

a. How long does it take the Sun to orbit the center of the Galaxy? Assume that we are moving at 220 km s^{-1}, and are 8 kpc from the Galactic center.

b. How many times would the Sun have completed an orbit during the age of the universe? Assume 13.7 Gyr for the age of the universe.

c. As an observational aside, how long would we have to wait to observe a star in Andromeda move through an angle of 1 arcsecond, i.e. to directly detect its rotation? Assume a rotation speed of 250 km s^{-1} and a distance to Andromeda of 750 kpc. You can neglect the inclination of the galaxy. Will you live this long?

   1 arcsecond would be easily measurable with ground-based telescopes, but the Hubble Space Telescope could measure a change of 0.1 arcsecond. Will you live long enough to see an 0.1 arsecond change?

d. Now consider a cluster of galaxies. The Coma cluster has a velocity dispersion of roughly $\sigma = 1000$ km s^{-1}, a core radius of 0.3 Mpc, and a total radius of about 3 Mpc. How long would it take a galaxy to completely cross the Coma cluster, if it were moving at the typical velocity dispersion?

Problem 5 Order of Magnitude: Colliding Galaxies

Consider a disk galaxy with radius 15 kpc, thickness 1 kpc, and mass in stars $M_* = 5 \times 10^{10}\,M_\odot$. Estimate the average number density of stars in this galaxy (the number of stars per unit volume). Assume all of the stars have the same mass, and make a reasonable estimate of that mass.

Now consider two such galaxies, colliding. How far can a star in either of these galaxies travel before hitting another star? Do you think stars are likely to hit each other when galaxies collide?