Astron 211 Midterm Review

Monday, Oct 23, 2015

Exam Policy: Please bring a calculator (not just a phone). You will also be given the list of formulas and constants attached to this review. **You cannot work with other people**. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Do not spend all your time on one problem: try to look at each problem quickly first.

Exam Format:

- True/False
- Multiple choice (like activities)
- Short answer (like Kutner problems on problem set)
- One order-of-magnitude

Topics: you should have at least general familiarity with these areas.

- Seasons, orbit of the Earth, coordinates
- Parallax, small-angle approximation
- Magnitudes, luminosities, inverse-square law
- Orbits, binary stars, reduced mass, Kepler's laws
- Time-scales for the Sun: free-fall, Kelvin-Helmholtz
- Stellar energy sources, basics of fusion
- Getting energy out of the Sun, random walks, optical depth
- Ideal gas law, hydrostatic equilibrium
- Blackbodies, light, radiation pressure

- Virial theorem
- Stellar scalings (how temperature, radius, luminosity depend on mass and why)
- Evolution of the Sun, main sequence, HR diagrams, star clusters

Formulas:

Distance $d[\mathrm{pc}] = 1/\mathrm{parallax}[\mathrm{arcsec}]$

magnitudes $m = C - 2.5\log_{10} F$; $m_1 - m_2 = -2.5\log_{10}(F_1/F_2)$, $M - m = 5 - 5\log_{10}(d/\mathrm{pc})$

Doppler shift $\Delta\lambda/\lambda = -\Delta\nu/\nu = v_{\mathrm{rad}}/c$

Gravity and Tides accel$= GM/r^2$, $\mathrm{accel}_{\mathrm{tide}} \approx (GM/r^2)(2R/r)$; escape speed $v_{\mathrm{esc}} = \sqrt{2GM/R}$

Kepler $GM = \Omega^2 a^3 = \left(\frac{2\pi}{P}\right)^2 a^3$; $\frac{a_1}{a} = \frac{v_1}{v} = \frac{m_2}{M}$; reduced mass $\mu = m_1 m_2/(m_1 + m_2)$, total mass $M = m_1 + m_2$; for circular orbit $v = \sqrt{GM/a}$, $L = \mu\sqrt{GMa}$; for Solar units (years, AU, $M_\odot$): $P^2 = a^3/M$

Virial Theorem $E_{\mathrm{kin}} = -\frac{1}{2}E_{\mathrm{pot}}$; $E_{\mathrm{tot}} = E_{\mathrm{kin}} + E_{\mathrm{pot}} = \frac{1}{2}E_{\mathrm{pot}}$, [where $E_{\mathrm{pot,binary}} = -\frac{Gm_1m_2}{a}$ and $E_{\mathrm{pot,star}} \approx -\frac{GM^2}{R}$]

Ideal Gas $P = nk_BT = \frac{\rho}{\mu m_H}k_BT$; typical KE per particle is $\frac{3}{2}k_BT$; energy density $u = \frac{3}{2}nk_BT = \frac{3}{2}P$

Degenerate Gas $\Delta x\Delta p \sim \hbar$; $E_{\mathrm{F}} = \frac{1}{2}\frac{p_{\mathrm{F}}^2}{m_e} \propto n_e^{2/3}$; $P \propto n_e E_{\mathrm{F}} \propto n_e^{5/3} \propto (\rho/\mu)^{5/3} \rightarrow R \propto M^{-1/3}$ [non-relativistic]

Photon Propagation $l_{\mathrm{mfp}} = \frac{1}{n\sigma} = \frac{1}{\kappa\rho}$; $t_{\mathrm{randomwalk}} = \frac{R}{l_{\mathrm{mfp}}}\frac{R}{c}$

Blackbody $L = 4\pi R^2\sigma T_{\mathrm{eff}}^4$, $F = \sigma T_{\mathrm{eff}}^4$; $\lambda_{\mathrm{peak}} = 0.29\,\mathrm{cm}/T$

Light $c = \lambda\nu$, $E = h\nu = hc/\lambda$, $p = E/c$, energy density $u = aT^4$, pressure $P = (a/3)T^4$

Hydrostatic Equilibrium $\frac{\Delta P}{\Delta r} = \rho\frac{GM}{r^2} = -g\rho \rightarrow P \propto M^2/R^4$

Stars $T_c \propto M/R$, $\rho_c \propto M/R^3$, $P_c \propto M^2/R^4$

Timescales $\tau_{\mathrm{free-fall}} \sim \sqrt{1/G\rho}$; $\tau_{\mathrm{Kelvin-Helmholtz}} \sim \frac{GM^2/R}{L}$

Hydrogen Fusion $E = \Delta mc^2 \approx 0.7\%c^2$

Hydrogen Atom $E_n = -13.6\,\mathrm{eV}/n^2$

Jeans mass $M_J = 3k_BTR/2G\mu m_{\mathrm{H}}$; $\rho_J = (3/4\pi M^2)(3k_BT/2G\mu m_{\mathrm{H}})^3$

Equilibrium Temperature $T_p = T_s(1-A)^{1/4}(R_s/2d)^{1/2}$

Diffraction Limit $\theta = \lambda/D$

Poisson noise uncertainty on n counts is $\sqrt{n}$

Constants:

bolometric absolute mag of the Sun $M_{\mathrm{bol},\odot} = 4.74$

Solar Mass $M_\odot = 2 \times 10^{30}\,\mathrm{kg}$

Solar Luminosity $L_\odot = 4 \times 10^{26}\,\mathrm{W}$

Solar Radius $R_\odot = 7 \times 10^{8}\,\mathrm{m}$

Earth Mass $M_\oplus = 6 \times 10^{24}\,\mathrm{kg}$

Earth Radius $R_\oplus = 6.4 \times 10^{6}\,\mathrm{m}$

AU $1.5 \times 10^{11}\,\mathrm{m}$

parsec $3.1 \times 10^{16}\,\mathrm{m} = 206265\,\mathrm{AU}$

year $3.16 \times 10^{7}\,\mathrm{s}$

c $3 \times 10^{8}\,\mathrm{m\,s^{-1}}$

G $6.7 \times 10^{-11}\mathrm{N\,m^2\,kg^{-2}}$

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$

Permittivity of free space $\epsilon_0 = 1/\mu_0 c^2$

Electric Charge $e = 1.6 \times 10^{-19}\,\mathrm{C}$

Electron volt $\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$

Planck's constant $h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$, $\hbar = h/2\pi$

Boltzmann's constant $k_B = 1.4 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

Radiation constant $a = 4\sigma/c$

Proton mass $m_p \approx m_n \approx m_H = 1.7 \times 10^{-27}\,\mathrm{kg}$

Electron mass $m_e \approx m_p/1800 = 9.1 \times 10^{-31}\,\mathrm{kg}$