

Astronomy 211

Lecture I Preliminaries

Course Description: *quantitative* astronomy. Emphasis on the “why” and the “how” rather than just the “what.” There will be some math(!) and physics. There will be no calculus.

If you don't want that, look at ASTRON 103. If you want a lab, take ASTRON 104. Here we will cover:

- Celestial mechanics
- The nature of light and its interaction with matter
- Telescopes
- The structure and evolution of single stars
- The evolution of binary stars
- The end-products of stellar evolution
- The Solar System
- Extra-solar planets
- Galaxies & quasars
- Expansion of the universe & dark matter
- The big bang

The textbook will be Kutner: *Astronomy: A Physical Perspective*

Evaluation will be:

- Weekly problem sets (50%), with the best 10 of 11 counting.
- Weekly D2L quizzes (5%)
- Weekly in-class problems (5%)
- Midterm exam (20%)
- Final exam (20%)

I.2 Set the Stage

Astrophysics mostly starts next lecture.

I.3 Physics Synopsis

The level of physics and math that you are expected to be familiar with (but not necessarily know in detail) is:

Newton's Laws : most importantly, $F = ma$

Kinetic Energy : $K = \frac{1}{2}mv^2$

Gravitation : $F = GM_1M_2/r^2$ (on the surface of the Earth, $F = gm$)

Potential Energy : $U = -GM_1M_2/r$ (from gravity; on the surface of the Earth, $U = gmh$)

Centripetal Acceleration : $a = v^2/r$

Ideal Gas Law : $PV = NRT$

Circumference of a Circle : $2\pi r$

Area of Circle : πr^2

Surface Area of a Sphere : $4\pi r^2$

Volume of a Sphere : $\frac{4}{3}\pi r^3$

Radians : $180^\circ = \pi$ radians, $\sin(\pi/2) = 1$, $\sin \pi = 0$, etc.

Small Angles : $\sin x \approx x$ for x very small and measured in radians. Also, $\tan x \approx x$, and $\cos x \approx 1$ (**draw these**)

Scientific notation : $A \times 10^a \cdot B \times 10^b = (AB) \times 10^{a+b}$

I.3.1 Calculus

Not required. However, it is in the book. **Don't Panic!** If you see:

$$\frac{dx}{dt}$$

or

$$\int dx f(x)$$

just read around it. Or ask questions.

I.3.2 Greek

If I use a symbol you don't recognize or can't read, **ask!**

I.4 Precision

We often do not know things very precisely. So we use \sim and \approx and related symbols. \sim is for when we know something to *an order of magnitude*. So we if we know that $x \sim 5$, we know that x is between $5/3$ and $5 * 3$, where 3 is roughly $\sqrt{10}$. This means that the possible range for x is in total a factor of 10. We will also sometimes use \sim to mean *scales as*. For example, if you were to estimate the height of a person as a function of their weight (for a wide range of people), you might expect that as you double the weight, the height changes by $2^{1/3}$. We could write $\text{height} \sim \text{weight}^{1/3}$. There will be a lot of variation, but this is roughly correct.

\approx means more precision. It doesn't necessarily have an exact definition. But generally, if we say $x \approx 5$, that means that 4 is probably OK but 2 is probably not.

Finally, we have \propto , which means *proportional to*. This is more precise than the *scales as* use of \sim . So while for a person $\text{height} \sim \text{weight}^{1/3}$ is OK, for a sphere (where we know that volume is $4\pi/3 r^3$) we could write $\text{volume} \propto r^3$: we take this as correct, but leave off the constants ($4\pi/3$ in this case).

I.4.1 Order of Magnitude Problems

There are many problems — in Astronomy, Physics, or Life — where we know only the basics. But we want to estimate something. So we do an “order of magnitude” estimate (also known as a “Fermi problem”: see http://en.wikipedia.org/wiki/Fermi_problem). Basically, we want to know whether something is 1, 10, or 100, but we do not care whether it is 20 or 30. These make heavy use of the \sim symbol. We will come back to these.

I.4.2 Small Angles

For small angles θ , $\sin \theta \approx \tan \theta \approx \theta$ and $\cos \theta \approx 1$. We need θ to be in radians. But we also often deal with fractions of a circle. A circle has 360° . We break each degree into 60 minute (or *arcminutes*): $1^\circ = 60'$. And each arcminute into 60 seconds (or *arcseconds*): $1' = 60''$, so $1^\circ = 3600''$. But we also know that 2π radians is 360° , so we can convert between radians and arcsec. This will come up frequently: $1'' = 360 \times 3600 / 2\pi \approx 1/206265$ radians.

I.5 Example Problem

Consider all of the people on the UWM campus:

- How much do all of the people on the UWM campus weigh?
- How many buses would you need to transport them all?
- If they all jumped in to Lake Michigan, how much would the water level change (numbers: 300 miles long by 118 miles wide, average depth 279 feet, volume 1180 cubic miles)? What about all of the people in the world?

- 30,000 people, 60 kg each, 1,800,000 kg or 1800 m³
- 5×10^{12} m³
- So answer is 0
- for whole world answer is 1 cm (400,000,000 m³ total).

Lecture II Celestial Sizes, Distances, and Coordinates

II.1.1 Units

Astronomy emphasizes *natural* units (\odot is for the Sun, \oplus is for the Earth):

- $M_{\odot} = 2 \times 10^{30}$ kg (solar mass)
- $R_{\odot} = 7 \times 10^8$ m (solar radius)
- $M_{\oplus} = 6 \times 10^{21}$ kg (earth mass)
- $L_{\odot} = 4 \times 10^{26}$ W (solar luminosity or power)
- light year = 10^{16} m: the *distance* light travels in one year (moving at $c = 3 \times 10^8$ m s⁻¹)
- parsec = parallax second (we will understand this later) = pc = 3×10^{16} m
- Astronomical Unit = AU = 1.5×10^{11} m (distance between earth and sun)

And then we use usual metric-style prefixes to get things like kpc, Mpc, etc.

II.2 Distances

Kutner 2.6.

How far away/big are things? Use meter stick to draw centimeter, meter, 10 meters.

Chicago : 144 km

circumference of Earth : 40,000 km

distance to Moon : 380,000 km

distance to Sun : 1.5×10^{11} m = 1 AU

solar system (orbit of Neptune): 30 AU in radius

Oort cloud : 50,000 AU

Nearest star (Proxima Centauri): 1.29 pc = 4×10^{16} m

Center of Milky Way : 8.5 kpc = 2.6×10^{20} m

Andromeda Galaxy : 0.6 Mpc = 1.8×10^{22} m

and so on. There is *a lot of empty space!*

How do we measure distances to stars? First step: parallax = geometry. $\tan \theta = 1 \text{ AU}/d \rightarrow d = 1 \text{ AU} \tan \theta$. But stars are very far away, so θ is very small. Again, if θ in radians $\tan \theta \approx \theta$ so $d \approx 1 \text{ AU} \theta$ (which puts d in AU too if θ in radians).

But remember, 1 radian is 206265". So if θ is in arcsec now, $d = 206265 \text{ AU}(\theta/\text{arcsec})$. 206265 AU has a special name: it is 1 parallax second or 1 parsec (or 1 pc or 3×10^{16} m).

How big is an arcsecond? For a quarter to be 1" across (diameter of 25 mm) need to be 5 km away. And we can measure much smaller angles.

II.2.1 Planetary Motion and the Copernican Model

Kutner, Chapter 22.

The geocentric model for the Universe is wrong. The Earth is not the center of the solar system, the galaxy, the universe, etc. Partly this was uncovered through observations of *retrograde motion*: stars appear the same every night, but some objects (often bright ones) move relative to the stars. These are known as *planets* (literally wanderers). Mostly the planets move from West to East. Except when they don't — then they go the other way, which is called retrograde motion. This was a 2000-year old puzzle. **Mars 1994 from Astron 103 week 3.**

In the geocentric view, it was complicated and elaborate (*epicycles*). But the heliocentric view (from Copernicus) has an elegant solution. Taking inner orbits as faster (we'll see why later), we find retrograde motion happens occasionally for the planets that are further out than the Earth.

We can then define two periods:

sidereal period (or sidereal time) is the period relative to the fixed background stars. This is close to (but not exactly the same) as a year.

synodic period is the time between when planets are closest together

and we can relate these by asking how long until the planets line up again. We define the angle of the Earth $\theta_E = 2\pi t/P_E$ which goes around from 0 and one revolution happens at $t = P_E$ (θ in radians). We can do the same for Venus $\theta_V = 2\pi t/P_V$. $P_V < P_E$, so θ_V goes faster. They line up when $\theta_E = \theta_V - 2\pi$: the 2π is since Venus will have gone around one extra time. So we write:

$$\frac{2\pi t}{P_E} = \frac{2\pi t}{P_V} - 2\pi$$

and can solve for $1/t = 1/P_V - 1/P_E$, and we identify t with the synodic period. For planets outside the Earth's orbit, the sign is opposite.

II.2.2 Motion of the Earth

The Earth rotates around its axis once every 24 hours. So each hour is then $360^\circ/24 = 15^\circ$.

This rotation is what makes the Sun and the stars appear to move over the course of a day. The star to which the Earth's axis appears to point is the North Star (Polaris): it's not a special star, we just point near it. Because of "precession", we point to different stars over the course of about 24,000 years.

This rotation can help you figure out how long you have (for example) until the Sun goes down:

- Your hand is roughly 10° across when you hold your arm out
- Your finger is roughly 1° across

And then, just like all of the planets, the Earth goes around the Sun. Each planet takes its own time. For the Earth, this is 1 year.

II.2.3 Seasons and the Changing Sky

The rotation axis of the Earth is tilted 23.5° to the plane of its orbit. This means that as the Earth orbits around the Sun, the Sun appears to trace a path in the sky. We call this the *ecliptic* (constellations in the ecliptic are the zodiac). The plane of the orbit extending out to infinity is the *ecliptic plane*.

During the northern summer, the North pole of the Earth points towards the Sun. This is roughly June 21. On the northern winter solstice, what points towards the Sun? Seasons depend on what hemisphere you are in.

solstice : roughly June (northern summer) or December (northern winter), when the "sun stands still." Longest/shortest day. Flips in the southern hemisphere.

equinox : roughly March/September 21. When the night and day are of equal length.

Just like the equator on the Earth, we can extend it into the sky to make the *celestial equator*. This divides the sky into northern & southern halves. We do the same thing with the poles. The Sun crosses the celestial equator twice per year, on the equinoxes.

winter (in north) : Sun is low in the sky, days short

summer (in north) : Sun is high in the sky, days long

spring/fall : in the middle.

II.2.3.1 Weather (Hot vs. Cold)

The Sun puts out energy at a roughly constant rate: L_\odot . It is changing the way that we receive this energy that makes seasons. How does this work? Do we move closer to or further from the Sun?

NO! This wouldn't work since the northern Summer = southern winter. Instead we change the area over which the Sun's energy is spread.

Draw a circle perpendicular to the path of the Sun's light with area 1 m^2 . This defines a *flux* which is power per area: $F = L/\text{area}$. The total area over which the Sun's light is spread is the area of a sphere $4\pi R^2$, so at the Earth $F_{\odot} = L_{\odot}/4\pi R^2$ with $R = 1 \text{ AU}$. When the Sun is high overhead (summer), this power gets spread over a patch of the ground with the same area. So each 1 m^2 of the ground gets heated with the full F_{\odot} of flux. This makes it hot. But in the winter the Sun is low in the sky. So the circle that is perpendicular to the Sun's light gets spread over a wide area on the ground. So we still have F_{\odot} of flux, but it gets spread over (for example) 2 m^2 of area. This means that each patch of the ground gets heated by less power, which makes it colder.

II.3 How bright are stars?

Kutner 2.1

Historical method: magnitudes (messy and annoying, but has its uses).

lower = brighter, higher = fainter. Used to be $m = 1$ is roughly the brightest. But now we have quantified this on a *logarithmic scale*:

- 2.5 mag fainter = 1/10 the brightness
- 5.0 mag fainter = 1/100 the brightness
- 10 mag fainter = 1/10,000 the brightness

We look at things from the Sun ($m = -26.8$, as it appears from the Earth) to $m \approx 30$.

But remember that we are measuring how bright things appear! This would change if they were closer/farther. We are essentially measuring *flux*: energy per time per area. Or (energy per time) per area. (energy per time) is the same as power, and we measure this in Joules per second or Watts. So we measure flux in Watts per area, or Watts per m^2 .

If instead we look at total power put out (independent of where you are) that is *Luminosity*, L . L is measured in Joules/s or W. How can we go between these? Take a star of luminosity L , and surround it by a sphere with radius R . The flux is $F = L/4\pi R^2$, since the area of that sphere is $4\pi R^2$. This is the *inverse square law* for fluxes, and it should be familiar (when you go away from a lightbulb, it gets fainter, etc.).

$L_{\odot} = 4 \times 10^{26} \text{ W} = 1L_{\odot}$. $R = 1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$. So the flux on the Earth $F_{\odot} = 1300 \text{ W m}^{-2}$. In comparison, on Neptune at $R = 30 \text{ AU}$, the flux is 1/900 as much.

So we have magnitudes. How do we tie magnitudes (as a measure of brightness) to real fluxes?

II.3.1 Absolute Magnitudes

$=M$ =the apparent magnitude m of a star if it were 10 pc away.

From the definition of a magnitude, a change of 2.5 mag is a factor of 10 change in F . So we can take two magnitudes m_1 and m_2 and put them with two fluxes F_1 and F_2 :

$$\frac{F_2}{F_1} = 10^{-(m_2-m_1)/2.5}$$

The negative sign comes from having a lower magnitude mean brighter. Or:

$$m_1 - m_2 = -2.5 \log_{10} \frac{F_1}{F_2}$$

To put in absolute magnitudes, we use the inverse square law: $F = L/4\pi d^2$. So $F_1 = L/4\pi d_1^2$ and $F_2 = L/4\pi d_2^2$, and we divide:

$$\frac{F_1}{F_2} = \left(\frac{d_1}{d_2}\right)^{-2}$$

with $d_2 = 10$ pc. So $F_1/F_{10\text{pc}} = (d_1/10\text{ pc})^{-2}$. This is the same as:

$$\frac{F_1}{F_{10\text{pc}}} = 10^{-(m_1-M)/2.5}$$

we can work this through to find: $m - M = 5 \log_{10}(d/10\text{ pc})$ which is known as the *distance modulus*: how much being far away changes the apparent magnitude of something.

So if we know m and M , we can get d . Or if we know m and d , can get M . For the Sun: $m_{\odot} = -26.83$, $d_{\odot} = 1\text{ AU} = 1/206265\text{ pc}$. We get: $M_{\odot} = m_{\odot} - 5 \log_{10}(d/10\text{ pc}) = +4.74$. This is pretty modest compared to other stars (the Sun is only remarkable by being close).

Knowing what M_{\odot} is, if we are then given the absolute magnitude of a star we can calculate its flux and actual luminosity. That is because for two objects at the same distance (10 pc in this case) the relation between magnitudes and fluxes can work with luminosities too. So we can say:

$$F = F_{\odot} 10^{-(M-M_{\odot})/2.5}$$

$$L = L_{\odot} 10^{-(M-M_{\odot})/2.5}$$

where F_{\odot} is now the flux of the Sun as perceived from 10 pc away.

Lecture III Gravity & Celestial Mechanics

And finally some physics. Kutner 5.3, 5.4

Johannes Kepler: used data from Tycho Brahe to determine 3 “laws”.

1. Orbits are ellipses with the Sun at a focus. Semi-major axis is a , semi-minor axis is b . The equation of an ellipse says that the distance from the planet to the Sun + the distance from the planet to the other focus is a constant.
2. Planet-sun line traverses equal areas in equal time. (**Kepler’s 2nd law from Astron 103 week 3**)
3. $P^2 = a^3$, with P the period of the orbit. For the Sun, this works with P in years and a in AU.

III.2 Elliptical Motion

aphelion = far from star, *perihelion* = close to star. Eccentricity e between 0 (circle) and 1. We find $b^2 = a^2(1 - e^2)$, with perihelion at $a(1 - e)$ and aphelion at $a(1 + e)$. Planets have only slightly eccentric orbits. For the Earth, $e = 0.0167$. So maximum distance from Sun - average is $ae = 0.0167 \text{ AU} \approx 400R_{\oplus}$. This means a change in the solar flux of about $\pm 3\%$.

III.3 Newton’s Laws

1. Inertia
2. $\vec{F} = m\vec{a}$
3. $\vec{F}_{12} = -\vec{F}_{21}$

(here \vec{F} is a force, not a flux). Also have gravitation: $F = GMm/r^2$.

For a circular orbit of something with mass m around something with mass M that is not always the sun, Kepler’s third law says: $P^2 \propto r^3$, but the constant of proportionality can change: $P^2 = kr^3$. Let’s derive k . $P = 2\pi r/v$, since it has to traverse a distance $2\pi r$. Putting this in gives us

$$\frac{4\pi^2 r^2}{v^2} = kr^3$$

We need a centripetal force $F = mv^2/r$ to keep the planet in orbit: $F = GmM/r^2$. We put these together and re-arrange to get $k = 4\pi^2/GM$.

III.4 Escape: Work and Energy

Potential energy $U = -GMm/r$. It depends just on the start and stop points, not the path.

Kinetic energy $K = 1/2mv^2$.

Total energy $E = K + U = 1/2mv^2 - GMm/r$. Start at r , move away until $v = 0$ at $r = \infty$. $K(\infty) = 0$, $U(\infty) = 0$, so $E(\infty) = 0$. This will always be true (no external source of work), so $K = -U$. We can write $1/2mv^2 = GMm/r$ and get:

$$v = \sqrt{2GM/r} = v_{\text{escape}}$$

is the escape speed. For the Earth this is 11 km/s: if you go this fast and point up, you will escape the Earth's gravity.

III.5 Consequences of Gravity

III.5.1 Linear Momentum

If the net force on a system is 0, then momentum is conserved.

III.5.2 Angular Momentum

is conserved also.

III.5.3 Energy

is conserved also.

III.6 2 Body Problem

This is more general than what we did before, and we no longer require $m \ll M$. This applies to binary stars, binary asteroids, black holes, planets, etc. We can write.

Since momentum is conserved, the two bodies will orbit their common center of mass. The COM can move, but it will move at a constant velocity. This is defined such that:

$$m_1 r_1 = m_2 r_2$$

Both stars must go around in the same time such that the line between them always goes through the COM:

$$P = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$

(assuming circular motion). Or $r_1/v_1 = r_2/v_2$. Combining these we get:

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

We also have the force:

$$F = G \frac{m_1 m_2}{(r_1 + r_2)^2}$$

As before, this must be the force needed to keep an object in a circular orbit. So on object 1, $F = m_1 v_1^2 / r_1$:

$$\frac{m_1 v_1^2}{r_1} = G \frac{m_1 m_2}{(r_1 + r_2)^2}$$

Divide both sides by m_1 , and use $P = 2\pi r_1 / v_1$:

$$\frac{4\pi^2 r_1}{P^2} = \frac{G m_2}{(r_1 + r_2)^2}$$

But we also define $R = r_1 + r_2 = r_1(1 + r_2/r_1)$. With the ratio of the masses instead of radii:

$$R = r_1 \left(1 + \frac{m_1}{m_2}\right) = \frac{r_1}{m_1} (m_1 + m_2)$$

Put this in:

$$\frac{4\pi^2 R^3}{G} = (m_1 + m_2) P^2$$

This is Kepler's third law! It's a bit more general than for just a system like the Earth and the Sun. We often write a instead of R .

The angular momentum is constant, and it can be useful. For each object $L_1 = m_1 v_1 r_1$ etc. So overall:

$$L = m_1 v_1 r_1 + m_2 v_2 r_2$$

But we know $m_1 r_1 = m_2 r_2$, so we can write:

$$L = m_1 r_1 (v_1 + v_2) = m_1 r_1 \left(\frac{2\pi r_1}{P} + \frac{2\pi r_2}{P} \right)$$

Substitute from before: $r_1 = m_2 R / (m_1 + m_2)$:

$$L = \frac{2\pi m_1 m_2 R^2}{(m_1 + m_2) P}$$

We substitute for P from Kepler's third law to get:

$$L = m_1 m_2 \sqrt{\frac{G R}{m_1 + m_2}}$$

III.6.1 How to Use This?

A basic result that we will use over and over again is $GM = a^3(2\pi/P)^2$. For circular systems, $v = \sqrt{GM/a}$. Using the center-of-mass we can write $a_1/a_2 = v_1/v_2 = m_2/m_1$, $a_1/a = v_1/v = m_2/M$.

III.6.2 $N > 2$

No general solution. Can be chaotic, not purely periodic, not closed. E.g., Jupiter perturbs the orbit of the Earth, asteroids, comets, slingshots.

How high does N get? Globular cluster: $N \sim 10^6$ stars. Galaxy cluster: $N \sim 10^3$ galaxies. Universe: $\sim 10^{11}$ galaxies. Need big computers to get approximate solutions.

Lecture IV Doppler Shifts and Waves

Kutner 2.2, 5.2

Light is a wave (and a particle). We talk about waves having wavelength λ and frequency ν : λ is a measure of how far before the wave repeats (in meters or such) and frequency is a measure of how often it repeats (in units of Hz which is 1/s). They are related by the speed of the wave, which for light is c :

$$c = \lambda\nu$$

But, the wavelength that a light is emitted at is not the same as it is absorbed at. It can change if there is relative motion toward or away from the light source. We can this change the Doppler shift, since it shifts the wavelength (or frequency). As long as the velocities are $\ll c$, then:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c}$$

So the shift is $\Delta\lambda$ (Δ usually means change). So if something is moving away from us ($v > 0$), then the wavelength gets longer. Since longer wavelengths are often associated with red, we call this a *red shift*. The opposite, when we go toward a lightsource, is a *blue shift*.

We can also use this in frequency, but the sign is opposite (higher frequency means smaller wavelength, so if the wavelength gets smaller ($\Delta\lambda < 0$) the frequency gets higher ($\Delta\nu > 0$):

$$\frac{\Delta\nu}{\nu_0} = \frac{\nu - \nu_0}{\nu_0} = -\frac{v}{c}$$

Lecture V Extra Solar Planets

Kutner 27.5

This is just how to find them — we will return to talking about the planets themselves later.

How do we find planets around other stars?

- Take pictures? Stars are much brighter, so this is very hard (although it has been done recently in some special cases).
- Main way: Newton

Take a planet in orbit with a star. Mostly the planet moves around the star, but since the mass of the star is finite, it moves a bit too around the center of mass: $v_* m_* = v_p m_p$, so $v_* = v_p (m_p / m_*)$. Even though $m_p \ll m_*$ (so $v_* \ll v_p$) it is still measurable, typically with velocities of a few m/s, via Doppler shifts. We have planets that are similar to Jupiter: $m_p \sim M_{\text{Jupiter}} \sim 10^{-3} M_{\odot}$.

Can also find via eclipses (transits). Here we see the dip in light when a planet goes in front of the star. The amount of light that is lost is $\sim (R_p / R_*)^2$, and since $(R_{\text{Jupiter}} / R_{\odot}) \sim 0.1$, the dip is $\sim 1\%$.

And we can (rarely) see the wobble of the star back and forth during the orbit. Here we use $m_* r_* = m_p r_p$, which gives a wobble of $\ll 1''$.

Lecture VI Tides

Kutner 23.5

Gravity to date has been point masses (or perfect spheres). If the sources are not points → we get tidal forces → which makes them not spheres. Tides are from **differential** forces across an object.

Consider two bits of stuff with mass m_1 and m_2 , separated by Δr ($m_1 = m_2$). They are a distance r away from a bigger thing with mass M : one is at r , the other at $r + \Delta r$. From gravity:

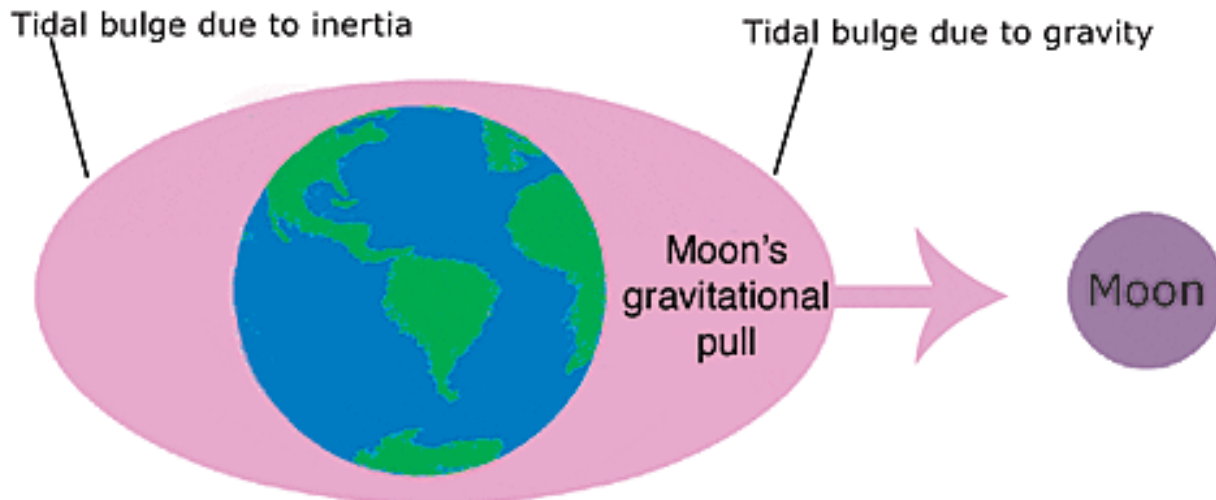
$$F_1 = GMm_1/r^2$$

and

$$F_2 = GMm_2/(r + \Delta r)^2 \approx F_1 + -2GMm/r^3 \Delta r$$

[the way to get this is magic or calculus]. It's the extra bit that gives rise to tidal forces, and the $1/r^3$ dependence is pretty general. In the Earth-Moon system, the total force is what keeps the orbit steady. But if you subtract off the forces on the center-of-mass you get the tides, which make a bulge that points at the moon.

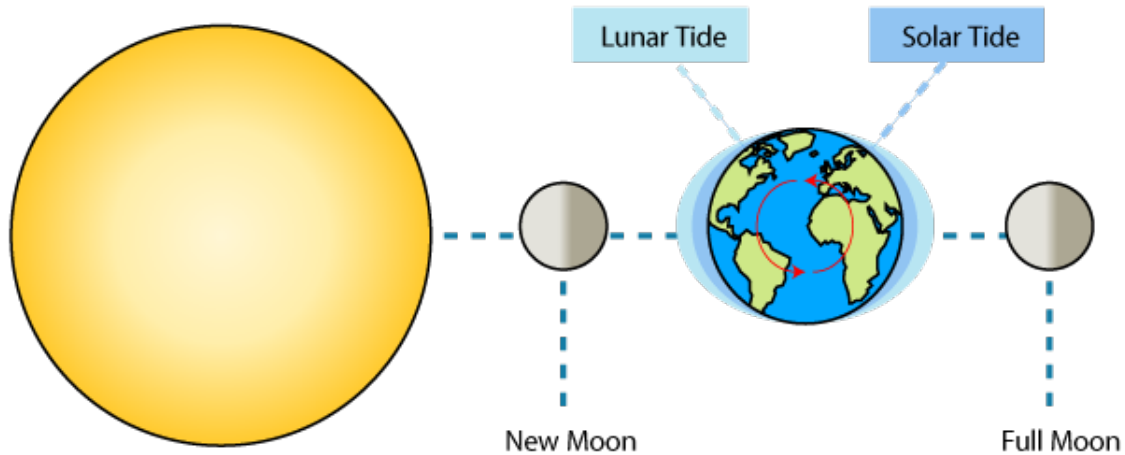
Another way to think about it: gravity balances centripetal acceleration $GM/r^2 = v^2/r = \omega^2 r$ to make a stable orbit. But that is only true at the center of mass: too close and gravity is stronger while centripetal acceleration is weaker, so gravity wins (bits that are too close get even closer). Too far and gravity is too weak while centripetal acceleration is stronger, so gravity loses (bits that are too far get farther). This makes bulges.



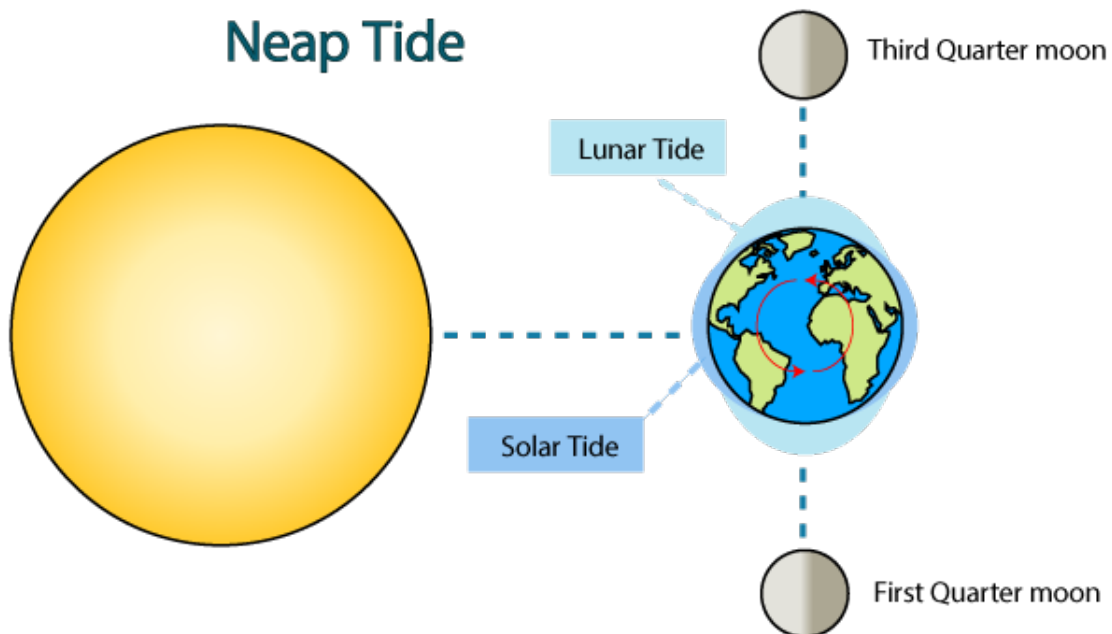
Tidal period = $2 \times$ forcing periods, from two bulges per rotation (toward and away). For the Earth, the height of tides from Sun \sim that from Moon. With rotation of the Earth, get mostly semi-diurnal (≈ 12.4 h) tides from $P_{\text{rot}} = 24$ h, which dominate in Atlantic. But some places have diurnal tides (complicated interactions between water and gravity).

From the orbit of the Moon get spring tides (when lunar lines up with solar) and neap tides (when they cancel out).

Spring Tide



Neap Tide



From the orbit of the earth get semi-annual tides, since the eccentricity of the Earth is not quite 0. Tides affect: ocean (which we know), but also atmosphere and rock. Do not affect Lake Michigan (why?).

How much is the acceleration due to the Moon's tides?

$$a_{\text{tide}} \sim \frac{2GM_{\text{moon}}}{r^3} R_{\oplus} \sim 2g_{\oplus} \frac{M_{\text{moon}}}{M_{\oplus}} \left(\frac{R_{\oplus}}{r} \right)^3$$

where we use $g_{\oplus} = GM_{\oplus}/R_{\oplus}^2$.

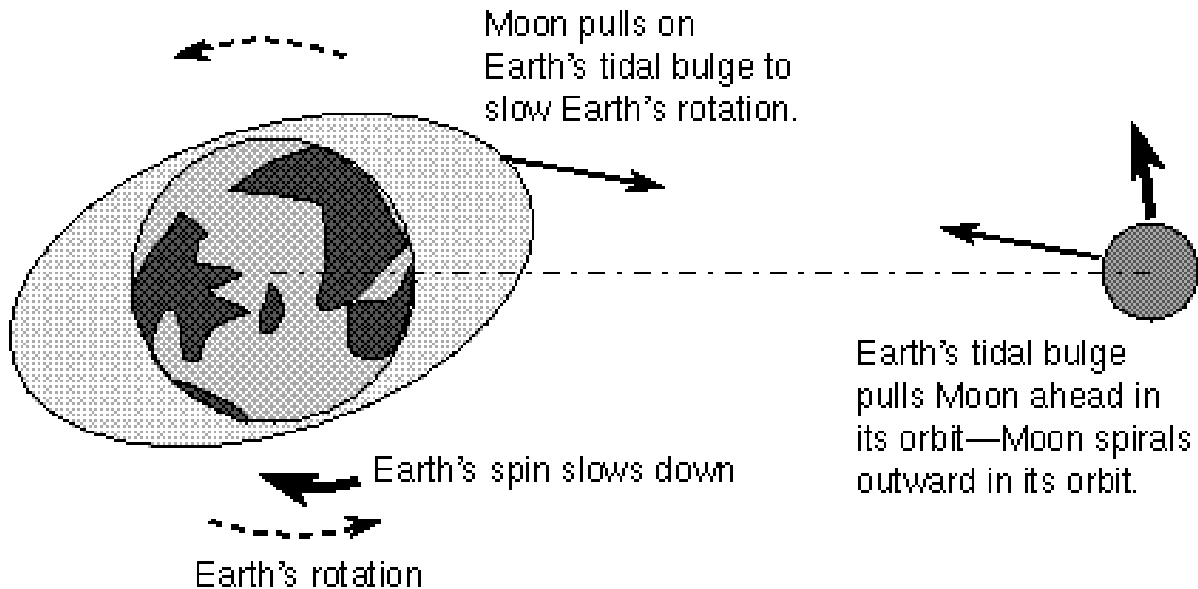
But the tidal bulge does not fly away due to this extra acceleration. Instead it makes a bit of extra gravity (from the extra mass) to cancel it out. Assume the bulge has height h , with extra gravity $g' \sim GM_{\text{bulge}}/R_{\oplus}^2$. We have $M_{\text{bulge}} \approx hR_{\oplus}^2\rho$, which gives $g' \sim GM_{\oplus}/R_{\oplus}^2(h/R_{\oplus}) \sim g_{\oplus}(h/R_{\oplus})$ (we have used $\rho R_{\oplus}^3 \sim M_{\oplus}$). Setting g' equal to a_{tide} , we find $h/R_{\oplus} \sim 2(M_{\text{moon}}/M_{\oplus})(R_{\oplus}/r)^3$. So we can estimate the order-of-magnitude of the tidal bulge on the Earth.

The size of this bulge from the moon on the Earth is about $10^{-7}R_{\oplus} \sim 60$ cm. And about 25 cm from the Sun. The Earth on the moon is about 2 m. This is of the right *order*, although the details are hard. For instance, in the Bay of Fundy, the tidal forcing period is the same as the time it takes for the water to slosh around. So you get a resonance, and the tides are really high (up to 9 m).

VI.2 Tidal Evolution

Water sloshes, loses energy (heat) — or generates electricity!

Note that the Earth spins faster (24h) than the Moon's orbit (28d). Friction drags the tidal bulge ahead of the moon.



This leads to a net torque that slows down rotation. Conserving L , the moon moves away ($L \propto \sqrt{a}$). This also makes orbits more circular, as tides are stronger for $e > 0$, and synchronized (like the moon is now, with rotation period of the moon equal to its orbital period).

We can measure: the moon moves away at ≈ 3 cm/yr. And the Earth day slows down at 0.0016 s/century. Note that the length of a day goes up, the length of a month goes up, but the number of days in a month goes down.

For Pluto+Charon, $P_{\text{orb}} = P_{\text{Pluto}} = P_{\text{Charon}} = 6.4 \text{ d}$, and $e = 0$. So this has already all happened here.

Weird cases exist, like Mars. Its moon Phobos orbits faster than Mars rotates. So tides pull it in! It will hit in about 50 Myr.

VI.3 Roche Limit

When do tides pull things apart? Simple answer: balance the acceleration of gravity holding an object m together (Gm/R^2 , where the size of the object is R) with the tidal forces coming from M_0 ($2GM_0R/r^3$, where the separation is r):

$$\frac{Gm}{R^2} \sim \frac{2GM_0}{r^3} R$$

There could also be other forces holding m together, so this will just be a limit. We take $\rho = m/(4\pi/3R^3)$ as the density of the thing being pulled apart and $\rho_0 = M_0/(4\pi/3R_0^3)$ as the density of the thing doing the pulling. Equating the accelerations we get roughly:

$$r \lesssim 2^{1/3} \left(\frac{\rho_0}{\rho} \right)^{1/3} R_0$$

Could this have given us Saturn's rings? They are inside such a radius (known as the *Roche Limit*), so large moons would have been pulled apart.

VI.4 Tides and Black Holes

Tides are how black holes kill you. It's not that gravity is too strong, it's that the difference in gravity between different parts of you is too strong.

Lecture VII Stars I

Kutner Chapter 9

What is a star?

- Big ball of gas
- Self-gravity \rightarrow need high pressure inside to support against collapse
- high pressure \rightarrow high T inside (basic gas physics)
- high $T \rightarrow$ emit light

A star's life is a long, losing battle with gravity.

Gravity pulls the parts of the stars in. Why do they not fall? Because pressure pushes them out. Which leads to high T , so the star loses energy. This means we need an energy source to keep the star shining. What is that? Gravity? Chemistry? Fission? Fusion?

VII.2 What supports against gravity?

Let us consider:

1. What if there were no support?
2. What provides the support?
3. How much support is necessary?

If the star is spherical, $g_r = GM(r)/r^2$ where $M(r)$ is the amount of mass contained within r . This is independent of $\rho(r)$, and the matter outside r does not matter.

VII.2.1 What if there were no support?

This is relevant for star formation, supernovae. We have things fall inward with radial velocity v_r and acceleration $a_r = -GM(r)/r^2$. The material falls toward the center taking a *free-fall timescale* t_{ff} :

$$|v_r| \sim \frac{r_0}{t_{\text{ff}}}, \quad a_r \sim \frac{v_r}{t_{\text{ff}}} \sim \frac{r_0}{t_{\text{ff}}^2} \sim \frac{GM_0}{r_0^2}$$

From this we can solve $t_{\text{ff}} \sim \sqrt{r_0^3/GM_0} \sim \sqrt{1/G\rho_0}$, where $\rho_0 \sim M_0/r_0^3$ is the mean density. We identify this free-fall timescale as the *dynamical time*. If you do out the math in detail (keeping factors of $4\pi/3$ etc.), you find:

$$t_{\text{ff}} = \sqrt{\frac{3\pi}{32} \frac{1}{\sqrt{G\rho_0}}}$$

So the time for collapse only depends on the density, not the size.

Object	r	M	ρ	t_{dyn}
Earth	$6 \times 10^6 \text{ m}$	$10^{-6} M_{\odot}$	5.5 g/cm^3	$\sim 10 \text{ min}$
Jupiter	$7 \times 10^7 \text{ m}$	$10^{-3} M_{\odot}$	1.3 g/cm^3	$\sim 10 \text{ min}$
Sun	$7 \times 10^8 \text{ m}$	$1 M_{\odot}$	1.4 g/cm^3	$\sim 10 \text{ min}$
White Dwarf	$7 \times 10^6 \text{ m}$	$1 M_{\odot}$	$1.4 \times 10^6 \text{ g/cm}^3$	3s
Neutron Star	10^4 m	$1.4 M_{\odot}$	$7 \times 10^{14} \text{ g/cm}^3$	0.1 ms

VII.2.2 Support comes from gas pressure

Pressure: resists compression. This comes from the kinetic energy of the gas particles. You can think of them each exerting a little force when they bounce off the walls of a box.

Pressure is force/area. In stars, most of the gas is an *ideal gas*, which means that the particles are all independent of each other. In the kinetic theory of ideal gases, the kinetic energy per particle is $3/2k_B T$ (where k_B is Boltzmann's constant). This is an average. It is both an average over time for 1 particle and an average over all particles at one time. From this we can derive the ideal gas law (in a slightly different form to what you might have seen in chemistry):

$$P = nk_B T$$

$n = N/V$ is the number density (units are m^{-3}), the number of particles N per volume V .

Are there non-ideal gases? Yes, we will discuss later. But these are gases where the particles are correlated (the wavefunctions overlap). They can be:

1. Fermi gases, with $P = P(n)$ only (electrons, protons, *degenerate gases*)
2. Bose gases, with $P = P(T)$ only (photons)

But with ideal gases, we have $P = nk_B T$. We can also write the number density n in terms of the mass density ρ (kg/m^3). Then we need to figure out how much the average particle weighs: $n = \rho/\mu m_H$. Here m_H is the mass of hydrogen, and μ is the mean molecular weight. If it's only hydrogen, then $\mu = 1$. If it's helium, then $\mu = 4$. What if it is *ionized* hydrogen? Then you have protons and electrons in equal numbers. The protons have mass $\approx m_H$, but the electrons have much less mass. So the average is $m_H/2$, which means $\mu = 0.5$. But writing the ideal gas law this way gives us:

$$P = \frac{\rho k_B T}{\mu m_H}$$

VII.2.3 How much support is needed?

Here we will derive *Hydrostatic equilibrium*. This supports fluid against gravity via a pressure *gradient*. We have the gravitational acceleration $mg = (\rho \Delta r A)g = P_{\text{bottom}}A - P_{\text{top}}A$. Cancelling

A, we find $P_{\text{top}} = P_{\text{bottom}} + \Delta P = P_{\text{bottom}} + (\Delta P/\Delta r)\Delta r$, or:

$$\frac{\Delta P}{\Delta r} = -\rho g$$

This is very general, and applies to stars as well as to atmospheres, oceans, etc. What it means is that P increases as you go down/inward.

What can we do with this? In general we need to know $\rho(r)$ and $g(r)$. But we can make some simplifications. We assume that $\rho \approx M/R^3$ and $g = GM(r)/r^2$. Then we get:

$$\frac{\Delta P}{\Delta r} = -\rho(r)\frac{GM(r)}{r^2}.$$

Furthermore, we will take an average over the whole star, the a difference between the inside ($P = P_c$ at $r = 0$) and the outside ($P = 0$ at $r = R$). So we get $\Delta P/\Delta r \approx (0 - P_c)/(R - 0) = -P_c/R$, with P_c the central pressure. Put this in and you get:

$$P_c \approx \frac{GM^2}{R^4}$$

For the Sun this is close: it gives 10^{14} N m^{-2} , which is a bit of an underestimate. For reference, 1 atmosphere is 10^5 N m^{-2} . Relations like these are very important for astronomy where the details are hard but we can make general scaling relations between different quantities.

To go further, we can say that the pressure is related to the temperature through the ideal gas law, $P \propto k_B T$. Putting in our approximate density we get $P = (M/R^3)k_B T/\mu m_H$, or $k_B T \approx GM\mu m_H/R$. Again, the details are hard, but the general expression that $T \propto M/R$ is very useful.

Lecture VIII Stellar Energy

Again, a star's life is a long protracted (but losing) battle with gravity. What can balance it? Fundamentally it is pressure, but what keeps the pressure going?

VIII.2 Virial Theorem

Due to Classius (1870). It concerns bound gravitational systems. In essence, the long-term average of kinetic energy is $1/2$ the average of the potential energy. These can each come from different places:

kinetic can be orbital (motion of blobs or stars) or thermal (random motion of particles)

For only orbital energy, that would be something like the Earth-Moon system. Only thermal would be the insides of stars. Or there are situations with a combination like elliptical galaxies and galaxy clusters.

We denote average by $\langle \dots \rangle$. So the Virial theorem states:

$$\langle K \rangle = -\frac{1}{2}\langle U \rangle$$

This is a bound system, so we have $\langle U \rangle < 0$. We can then look at the total energy $\langle E \rangle = \langle K \rangle + \langle U \rangle$, and if we substitute we find:

$$\langle E \rangle = \frac{1}{2}\langle U \rangle$$

VIII.3 What Powers the Sun and How Long Will It Last?

Kutner 9.1.2, 9.1.3.

We take the Solar luminosity to be 4×10^{26} W, and try to find a way to get that amount of energy out over a long time.

The first estimate was due to Lord Kelvin (1862, in Macmillan's Magazine). This estimate (known now at the Kelvin-Helmholtz time, t_{KH}) was shown to be < 100 Myr. But Darwin said (at the time) that fossils were at least 300 Myr old. So something weird was going on. Kelvin's estimate may have been wrong by a bit, but it couldn't be that bad. So there had to be some unknown energy source.

The lifespan of the Sun could be due to:

1. Chemical energy
2. Gravitational energy
3. Thermal energy (could it have just been a lot hotter in the past?)

4. Fission?

The answers for all of these are no. Kelvin's estimate concerned specifically gravitational. Chemical energy isn't enough, since we know about how much chemical energy a given reaction can release for a given amount of stuff. Same with fission.

VIII.3.1 Gravito-Thermal Collapse, or the Kelvin-Helmholtz Timescale

This ascribes the luminosity to the change in total energy: L is change in $E = K + U$.

If you do this you get a timescale of $t_{\text{KH}} \sim 10^7$ yr, which is $\gg t_{\text{ff}}$:

$$t_{\text{KH}} \sim \frac{E}{L}$$

But $E \sim GM_{\odot}^2/R_{\odot} \sim 10^{41}$ J = 10^{48} erg (1 J = 10^7 erg).

That is because as collapse occurs, $|U|$ increases so K increases too. That heats up the star, which slows down the collapse.

We can use the Virial theorem to get the central temperature T_c of the Sun. We assume that the center (the hottest/densest bit) dominates K :

$$K \sim \frac{3}{2} k_B T_c \frac{M}{\mu m_H}$$

And $K = -U/2$, with $U \sim -GM_{\odot}^2/R_{\odot}$. So we find $T_c \sim GM_{\odot} m_H / k_B R_{\odot} \sim 10^7$ K. This is pretty good (the real number is about 1.6×10^7 K).

Lecture IX To Make A Star

1. Support against gravity (P from HSE, E from Virial theorem)
2. Source of energy: nuclear

We can exclude all forms of energy besides nuclear fusion from powering the Sun. How does fusion work?

IX.2 Fusion

Kutner 9.3

What this boils down to is $E = mc^2$: if you can get rid of a bit of mass, you liberate a lot of energy.

atomic unit $u = 1.66054 \times 10^{-27} \text{ kg}$ (mass of $^{12}\text{C}/12$)

proton $m_p = 1.6726 \times 10^{-27} \text{ kg} = 1.007u = 938.8 \text{ MeV}/c^2$

neutron $m_n = 1.6749 \times 10^{-27} \text{ kg} = 1.0087u$

electron $m_e = 9.1 \times 10^{-31} \text{ kg} = 0.0055u$

hydrogen $m_H = 1.0078u = m_p + m_n - \text{electrostatic binding energy}/2$

He nucleus $m_\alpha = 4.002u = 2m_p + 2m_n - \Delta m$, with $\Delta m = 0.03u \sim 0.7\% \times (4m_H) \approx 28 \text{ MeV}/c^2$

So going from 4 protons to 1 He nucleus (α particle) releases 28 MeV. This is the energy released by fusion.

We can think of the binding energy as the energy released when you form something (a nucleus in this case), or as the energy that is required to break something up.

$$^1\text{H} : E_b = 0$$

$$^4\text{He} : E_b = 28 \text{ MeV} = 7.08 \text{ MeV/nucleon}$$

$$^{16}\text{O} : E_b = 7.97 \text{ MeV/nucleon}$$

$$^{56}\text{Fe} : E_b = 8.798 \text{ MeV/nucleon}$$

$$^{238}\text{U} : E_b = 7.3 \text{ MeV/nucleon}$$

^{56}Fe has the highest binding energy, so it's the most stable. Elements that are lighter or heavier are less stable. This means that reactions would naturally squeeze lighter elements together into Fe (fusion) and break heavier elements apart (fission).

IX.2.1 Basic Nuclear Physics

1. Binding Energy: A_ZX , with A the number of nucleons, and Z the number of protons. $E_b = (Zm_p + (A - Z)m_n - m_{\text{nuc}})c^2$
2. Strong force: binds nuclei together against Coulomb (electrostatic) repulsion (since protons are positively charged)
3. $A \lesssim 56$: strong force increases faster when A increases than Coulomb forces, so a larger A leads to nuclei that are more bound.
4. $A \gtrsim 56$: the opposite

So fusion builds nuclei up to Fe, while fission breaks them down.

IX.2.1.1 H \rightarrow Fe

This gives about 9 MeV/nucleon. Going from H to He gets 7 (or about 0.7% of mc^2). Going to O gets about 8 (0.8%). Going to Fe gets about 1% of mc^2 which is the most that fusion can do.

So for each proton you get $\approx 1\%mc^2 \sim 10^{-12}$ J (which means that 1 g of H could supply the annual energy of an american).

Fusion in the Sun: 10^{-12} J $\times M_\odot/m_p \sim 10^{45}$ J $\gg GM_\odot^2/R_\odot$. $t_{\text{nuc}} \sim E/L_\odot \sim 10^{11}$ yr, so the Sun could shine for that long.

The actual lifespan is about 10^{10} yr (and it's lived about half of that) for a few reasons:

- L_\odot increases later in life
- Not all H is burned
- It does not get hot enough to burn all the way to Fe

But it is clear that $t_{\text{nuc}} \gg t_{\text{KH}} \gg t_{\text{dyn}}$

Lecture X How to Power the Sun

The basics are $4 \times {}^1\text{H} \rightarrow {}^4\text{He} + 28 \text{ MeV}$, or releasing $\sim 7 \text{ MeV}/A$. But this has some problems.

X.1.2 Problem 1:

coulomb repulsion is strong. In order to have fusion you have to force together multiple hydrogen nuclei. These are protons, and are all positively charged. The strong force can only overcome the repulsion when the protons are *very* close together: $\sim 1 \text{ fm} = 10^{-15} \text{ m}$ (for comparison, an electron orbits at 10^{-11} m).

The *classical* (not quantum) solution to this is that protons get close just because of their motion. They are hot, so they zip around pretty quickly. Sometimes they will approach each other, and this may happen. Can we tell how much?

The Coulomb potential is: $U_C = \frac{1}{4\pi\epsilon_0} \frac{e_1 e_2}{r}$, and energy will be conserved when the protons approach. So if they are travelling fast far away, as they approach the potential barrier they slow down:

$$\frac{1}{2} m_p v_\infty^2 + U_C(\infty) = U_C(1 \text{ fm})$$

taking the limiting case that they have used all of their kinetic energy to get close enough. This gives us a requirement:

$$\frac{1}{2} m_p v_\infty^2 \geq \frac{e^2}{4\pi\epsilon_0(1 \text{ fm})}$$

where we can also relate $\frac{1}{2} m_p v_\infty^2 = \frac{3}{2} k_B T$. So we need

$$T \geq T_{\text{classical}} = \frac{e^2}{6k_B\pi\epsilon_0(1 \text{ fm})} \sim 10^{10} \text{ K}$$

This is pretty hot, given that we know $T_c \sim 10^7 \text{ K}$. So the center of the Sun is not hot enough to sustain nuclear fusion!?

In fact, Arthur Eddington proposed nuclear energy as a power source, but others thought stars were not hot enough. Eddington said: "I am aware that many critics consider the stars are not hot enough. The critics lay themselves open to an obvious retort; we tell them to go and find a hotter place."

In the end, Eddington was right!

X.1.3 Quantum Mechanics

The problem is that the temperature above was the *classical* result, but quantum mechanics are important here. In particular, we need to consider wave-particle duality and the Heisenberg uncertainty principle: the size of a particle depends on its momentum. Classically you could think of a particle at a certain place with velocity v (and hence momentum mv); but in a quantum

sense you need to consider that the particle's position is only known to a de Broglie wavelength $\lambda_B \sim h/p \sim h/mv$, where h is Planck's constant.

When two particles are within λ_B of each other, there is a finite probability that they will “tunnel” to within 1 fm of each other (“tunneling” through the potential barrier):

$$U_c = \frac{e^2}{4\pi\epsilon_0\lambda_B} \leq \frac{1}{2}m_p v_\infty^2 = \frac{1}{2} \frac{p^2}{m_p} \sim \frac{1}{2m_p} \left(\frac{h}{\lambda_B} \right)^2$$

Note that we have replaced 1 fm with λ_B . We then get a constraint on λ_B of:

$$\lambda_B < \frac{4\pi\epsilon_0 h^2}{2e^2 m_p} \sim 10^{-13} \text{ m} = 100 \text{ fm}$$

which is much bigger than the classical result. So we can be $100\times$ further away and still have fusion. This gives us a much gentler requirement for the temperature as well:

$$\frac{3}{2}k_B T = \frac{1}{2}m_p v_\infty^2 = \frac{1}{2m_p} \left(\frac{h}{\lambda_B} \right)^2$$

Which gives:

$$T \geq T_{\text{quantum}} \sim \frac{m_p e^4}{12\pi^2 \epsilon_0^2 k_B h^2} \sim 10^7 \text{ K}$$

which is OK!

So fusion is possible because of *quantum tunneling* at 10^7 K for the Sun. But, if $M < 0.08 M_\odot$, then it cannot even get this hot and fusion is impossible. Such objects are failed stars known as *brown dwarfs*.

Quantum tunneling being possible does not mean that it always happens. The probability is $\sim e^{-2\pi^2 U_c / k_B T}$. So for 10^7 K, the probability is only 10^{-8} for any two protons, and it goes up to 1 for 10^{10} K. But there are enough protons and the fusion rate increases quickly with temperature that you have *ignition* at 10^7 K.

X.1.4 How Often Do Protons Get Close Enough?

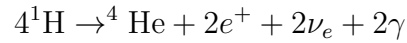
At the center of the Sun the density is $\rho \sim 100 \text{ g cm}^{-3}$, so the mean separation $l \sim n^{-1/3}$ is $\sim 10^{-11} \text{ m}$ which is $\gg \lambda_B$. To have fusion with $4 \times {}^1\text{H} \rightarrow {}^4\text{He}$ we need 4 protons to get very close together, which is hard.

This reaction is not one that needs 4 protons at once, but it is actually a sequence of 2-body reactions. At “low” temperatures it is the *proton-proton chain* (p-p chain), which at “higher temperatures” there is the *carbon-nitrogen-oxygen* (CNO) chain. This does not burn CNO, but uses them as catalysts.

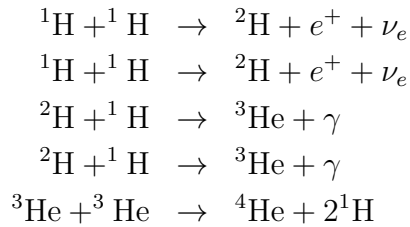
For later fusion reactions to build heavier elements, the higher charges on the nuclei (more protons) need higher temperatures to get close enough. For instance, to fuse He need 10^8 K. Others need close to 10^9 K. These will happen after the central H has been consumed.

X.1.4.1 p-p chain

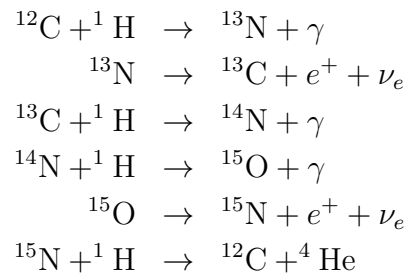
The main sequence in the Sun. The net reaction is:



Which is actually:

**X.1.4.2 CNO chain**

For more massive stars:



Notice that the CNO all stay the same throughout: anything that is produced is consumed and vice versa.

Lecture XI The Nucleus

We want to explain the binding energy in $m = Zm_p + (A - Z)m_n - E_b/c^2$, where the nucleus has Z protons and $A - Z$ neutrons, for a total number A nucleons.

XI.2 The Liquid Drop Model

$$E_B \approx a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z)$$

Let's look at each term:

$a_V A$: this is a volume term, since for constant density nucleons the volume will be $\propto A$. This covers the binding due to the strong force, which is $\propto A$

$a_S A^{2/3}$: this is a surface term. For a volume $\propto A$, the surface area will be $\propto A^{2/3}$. It works as a correction to the volume term since the nucleons near the outside will have fewer other nucleons to interact with.

$a_C \frac{Z(Z-1)}{A^{1/3}}$: this is the Coulomb term, showing the strength of electrostatic repulsion which is $\propto 1/r \sim 1/A^{1/3}$

$a_A \frac{(A-2Z)^2}{A}$: this is an asymmetry term, where nuclei with $A \approx 2Z$ are more bound

$\delta(A, Z)$: this is a pairing term

Overall this makes stable nuclei with $Z \approx A/2$, and says that the most bound nuclei are near $A = 60$. It is obviously a simplification, but it can be improved with the addition of a *shell model* (like for electrons) and using empirical data to set the various constants.

Lecture XII Radiation

Astronomy is based (so far) on observing light from objects. This means we see photons (electromagnetic radiation). Which give us information about temperature, density, chemical composition, etc.

We will discuss:

1. Diffusion and random walks
2. Blackbodies and temperature
3. Photospheres and energy transport

XII.2 Photons

Kutner 2.2, 2.4

Photons are light particles, but they also behave as waves. Each photon has energy $E = h\nu$ (with ν the frequency in Hz) or $E = hc\lambda$ (with λ the wavelength). This comes from the definition $c = \lambda\nu$.

We divide up the electromagnetic spectrum into wavelength regions:

γ -rays : $\lambda < 0.01$ nm

X-rays : 1 nm \rightarrow 10 nm

ultraviolet (UV) : 10 nm \rightarrow 400 nm

optical (visual) : 400 nm \rightarrow 700 nm

infrared (IR) : 700 nm \rightarrow 1 mm

radio : > 1 mm

XII.3 Diffusion & Random Walks

Heat (energy) is produced at the centers of stars through nuclear reactions. It escapes ultimately as photons. How long does that take? A naive answer is $\sim R_{\odot}/c = 2$ s.

But that is wrong (although it is true for neutrinos). It actually takes $\sim 10^7$ yrs. Why? Because a star is a very crowded place, and photons (even though they move fast) cannot move very far before they wack into something else and end up going in another direction. They easily bounce (scatter) off of ions, electrons, and atoms, and even other photons.

Each bounce tends to make the photon lose energy, but more photons are then produced, conserving energy. In the center the photons start out as X-ray photons, but by the time they get to the surface of the star they are optical photons. They get there via a *random walk*.

Assume that a photon will move (on average) a distance l_{mfp} before it hits something and changes direction. That distance is the *mean free path*. It travels a distance d after N collisions. We can determine what $d(N)$ is. Assume each one moves \vec{l}_i for $i = 1 \dots N$, with $|\vec{l}_i| = l_{\text{mfp}}$. So the total distance is the vector sum:

$$\vec{d} = \sum_{i=1}^N \vec{l}_i$$

We want the magnitude of this, $|\vec{d}| = \sqrt{\vec{d} \cdot \vec{d}}$. But

$$\vec{d} \cdot \vec{d} = \sum_i^N \vec{l}_i \cdot \vec{l}_i + \sum_{i \neq j} \vec{l}_i \cdot \vec{l}_j$$

The second term there will go to 0 on average, since the directions are different. So $|\vec{d}|^2 = N|\vec{l}|^2 = Nl_{\text{mfp}}^2$, or $d = \sqrt{N}l_{\text{mfp}}$. This is in fact a general result with applicability to a wide range of areas.

From this we can determine how long does it take for a photon to diffuse out of the star. To go a distance d , it takes:

$$N \frac{l_{\text{mfp}}}{c} = \frac{d^2}{l_{\text{mfp}} c} \quad \begin{array}{l} l_{\text{mfp}} > d \\ l_{\text{mfp}} < d \end{array}$$

This is also often referred to as a “drunkard’s walk”.

XII.4 What Happens When A Photon Hits Something?

- Photon A generates an oscillating electromagnetic field
- Matter (ion, electron, atom) is shaken by that field (absorbing the photon)
- But this shaking is itself a fluctuating field, so it makes a new EM field, releasing photon B

There are 3 basic types of interactions:

1. Scattering: A and B have the same energy (or frequency) but different directions. So the matter gains momentum but no energy
2. Absorption: no photon B is emitted. Matter absorbs energy and generally something happens to it
3. Emission: matter emits B (and it can be with or without A)

We take each obstacle as having cross-sectional area σ (in units of m^2). And we have n of them per m^3 (number density n). So what is l_{mfp} ?

1. A dimensional analysis: l_{mfp} [m] could be $1/\sigma n$, $\sigma^2 n$, $n^{-1/3}$, ...

2. Physical: shoot a bullet into a box of total area A and depth l_{mfp} . It has N balloons in it each with area σ . The bullet is likely to hit a balloon if $N\sigma = A$. $N = n \times \text{volume} = nl_{\text{mfp}}A$. Equating $nl_{\text{mfp}}A\sigma = A$ gives $l_{\text{mfp}} = 1/n\sigma$

3. How big are the balloons (what is σ)?

electrons 10^{-28} m^2

H atoms 10^{-20} m^2

So for $n \sim 10^{30} \text{ m}^{-3}$ (which you get for $\rho = 10^3 \text{ kg m}^{-3}$ like for water), we find $l_{\text{mfp}} \sim 10^{-11} \text{ m}$ to 10^{-2} m , both of which are $\ll R_{\odot}$. Therefore there are many bounces:

electrons : $l_{\text{mfp}} \sim 10^{-2} \text{ m}$, so $t \sim 5000 \text{ yr}$ (10^{22} bounces)

H atoms : $l_{\text{mfp}} \sim 10^{-11} \text{ m}$, so $t \sim 5 \times 10^{12} \text{ yr}$ (10^{40} bounces)

The actual answer is about 10^7 yrs, taking into account the changing structure and density of the Sun.

XII.5 Temperature of Radiation

Kutner 2.3

Temperature is defined for ideal objects (“blackbodies”) that radiate a *universal* spectrum: the emission depends only on temperature.

To do so, it must absorb all of the light that hits it (hence *black*). But it can appear to have a color when it is hot (like an oven).

Blackbody is a specific shape **sketch**. The peak is at $\lambda = 0.0029/T \text{ m}$, which determine the color. The total energy put out per square meter (the flux) is $F = \sigma T^4 \text{ W m}^{-2}$. These are the *Wien displacement law* and *Stefan-Boltzmann law* (σ is sb constant). The important thing is that only T matters.

Wikipedia page, animations.

F is energy per time per area. If the object is a sphere (like a star) it has total area $A = 4\pi R^2$, so total energy per time (luminosity L) is $L = 4\pi R^2 \sigma T^4$. This is a very useful expression.

What are “good” blackbodies? Nothing is perfect, but some things are close:

- 3 K cosmic microwave background
- surface or interior of a star
- human skin
- candle
- lava

What are “bad” blackbodies? These don’t have a nice smooth distribution, but instead concentrate the light at specific wavelengths:

- neon light
- fluorescents

But even the Sun isn’t perfect.

XII.5.1 Planck Function

The detailed function that describes how much light at each wavelength:

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{h\nu/k_B T} - 1} \text{ J/s/m}^2/\lambda$$

Note that h appears, so this has to be a quantum effect. B_λ is energy per time per area *per wavelength*: if you have a bigger range of wavelengths (i.e., red and green) then you get more energy.

XII.5.1.1 Limits

sketch

$$\frac{hc}{\lambda} \frac{1}{k_B T} \gg 1$$

(long wavelength, low frequency) is the Rayleigh-Jeans limit:

$$B_\lambda(T) \approx \frac{2ck_B T}{\lambda}$$

Note that there is no more h : this limit can be derived classically.

$$\frac{hc}{\lambda} \frac{1}{k_B T} \ll 1$$

(short wavelength, high frequency) is the Wien limit:

$$B_\lambda(T) \propto e^{-hc/\lambda k_B T} / \lambda^5$$

If you add up $B_\lambda(T)$ for all λ , you recover the Stefan-Boltzmann law.

XII.6 Photosphere

What is the surface of a star? What is the temperature of a star?

After all, it’s a flaming ball of gas. It’s hotter on the inside (Sun is 10^7 K), so why do we see it as cooler on the outside? What defines the temperature that we measure (about 5800 K)?

Photosphere is defined as the “surface”, it is where $T(\text{depth}) = T_{\text{effective}}$. We define $T_{\text{effective}}$ such that $L = 4\pi R^2 \sigma T_{\text{eff}}^4$. This is the layer from which photons escape (stop bouncing or scattering). They end up one mean free path from the surface, and from there they are free!

This is where the star leaves an imprint on the photons that escape. Everything that happens deeper down gets washed away from multiple scatterings.

Lecture XIII Photons & Spectra

Kutner Chapter 3

Spectra: disperse light through “prism”, spread it out so we can see each wavelength separately.

Stars generally have *absorption line*: most of the wavelengths are bright, but a few specific wavelengths are dark. To understand this, need Kirchoff’s laws:

- Hot background, cold foreground: absorption lines
- Cold background, hot foreground: emission lines

What matters is what is in front. What is in front of the star? It’s is that it is hotter on the inside than the outside. So the spectrum of a star is (mostly) a blackbody with some wavelengths absorbed. These wavelengths were identified before we knew what caused them.

Fraunhofer lines: lines in Sun from things like Na, Ca. But there are also lines from H, He that are very important.

Cecilia Payne was one of the first people to identify the spectral lines in the Sun (and other stars). She showed that the elements in the Sun were very different from those on Earth: here we have almost no free H, but that is the majority of what’s in the Sun.

XIII.2 Energy Levels for H

proton + electron in Bohr model (approaching proper quantum mechanics, but not quite): “planetary” orbits. Instead of Gravity, Coulomb force:

$$U = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

and we use the Virial theorem again, so $E = K + U = -U/2$. This would have infinite choices for r : anything is OK. The Bohr model says that r can only have particular values that are *quantized*. What is necessary is that, if you take a de Broglie wavelength, the orbit starts and stops in the same part of a wave. You can also write this as:

$$J = m_e v r = n \hbar$$

is the angular momentum. $\hbar = h/2\pi$, so this is quantum mechanical. If you do this, you get discrete energy levels for $n = 1, 2, 3, \dots$:

$$\frac{-1}{4\pi\epsilon_0} \frac{e^2}{2r} = -\frac{1}{2} m_e v^2 = -\frac{1}{2} \frac{(n\hbar)^2}{m_e r}$$

This can only be true at certain values of r :

$$r = r_n = 4\pi\epsilon_0 \frac{\hbar^2 n^2}{m_e e^2} \approx 0.5 \text{ \AA} n^2$$

The energy levels associated with this are:

$$E_n = \frac{1}{n^2} \frac{-m_e e^4}{2(4\pi\epsilon_0\hbar)^2} = \frac{-13.6 \text{ eV}}{n^2}$$

The constant 13.6 eV is the ionization energy of H, known as a Rydberg. Why does this ionize? Start at $n = 1$. How much energy to get to infinitely far away? This would take us to $r = \infty$, so $n = \infty$. The difference in energy levels is how much energy it would take:

$$\Delta E = E_1 - E_\infty$$

But since $E_\infty = 1/\infty = 0$, this is just E_1 or 13.6 eV.

<http://astro.unl.edu/classaction/animations/light/hydrogenatom.html>

XIII.3 Photon & Matter: Spectral Lines

Spectral lines are associated with transitions between energy levels. See in both absorption and emission.

For example, to excite an atom from $n = 1$ to $n = 2$ takes $\Delta E = hc/\lambda = E_2 - E_1 = (-3.4 \text{ eV} - (-13.6 \text{ eV})) = 10.2 \text{ eV}$. This gives a wavelength of $\lambda = 1216 \text{ \AA}$. Lyman α . **Sketch Ly, Balmer, Pa, Brackett.** Ly $\alpha = 1216 \text{ \AA}$, H $\alpha = 6563 \text{ \AA}$, P $\alpha = 18,700 \text{ \AA}$, Br $\alpha = 40,500 \text{ \AA}$.

Emission lines: hot gas on cool background (neon light).

Absorption lines: cool gas in front of hot background.

Some astronomical objects are primarily spectral line emitters. e.g., planetary nebulae and HII regions: clouds of hot gas, where most of the emission is just what we've described.

Lecture XIV Main Sequence

Kutner 3.5, 9

sketch L vs. T

temperature from color of star. luminosity from how bright it appears, combined with some knowledge of distance. From this we get *color-magnitude diagram*. Also *Hertzsprung-Russel diagram*. **main sequence** is where they sit for most of the time. Stars do not move along it: they end up at one point determined by their mass.

Stars were originally classified based on spectral lines. We have now been able to re-order that sequence in terms of temperature:

O : O5 is 40,000 K. H ionized, see lines of H, He

B : similar but cooler

A : A0 is 10,000 K, Vega (standard comparison star)

F : start seeing lines of H, “metals”

G : G2 is 5,800 K, Sun

K : start seeing molecules (star is cool, so they can be stable)

M : M0 is 4,000 K

Within each class goes from 0 (hottest) to 9 (coolest).

Taking stars together, we observe:

$$L \propto M^4$$

$$R \propto M^{0.76}$$

Simply put, more massive stars are bigger and (a lot) brighter. They also end up being hotter (via Stefan-Boltzmann law), $T_{\text{eff}} \propto M^{0.6}$.

XIV.2 More massive is bigger and brighter

only mass matters along the main sequence: stars sit there doing almost nothing for most of their lives.

XIV.2.1 Why bigger?

T_c roughly constant (ignition). Virial theorem gives us $T_c \propto M/R$, so then we would have $R \propto M$. Which is close, but actually T_c goes up a bit, so instead we have $R \propto M^{0.76}$.

XIV.2.2 Why brighter?

We can understand $T_c \propto M/R$, and $P_c \propto GM\rho/R$. The other piece we need is to look at how the photons get out of the star. Remember that they have to bounce around a lot. We can write:

$$\frac{T_c}{R} \sim \frac{L\rho}{T_c^3 R^2}$$

which is valid if the amount of time that it takes a photon to get out does not depend on T (true in hot stars). This then gives us: $L \propto M^3$, which is close to what is observed.

XIV.2.3 More massive has shorter life?

lifetime \propto fuel/rate of consumption $\sim M/L$. Since $L \propto M^3$ or M^4 , lifetime $\propto M^{-2}$ or M^{-3} .

At the low-mass end, energy is not transported by photons but by bubbles, so this reasoning (and these scalings) break.

XIV.3 What Limits What Could Be A Star?

XIV.3.1 Minimum Mass

No fusion possible (not hot enough): $M < 0.08M_\odot$ (brown dwarf)

XIV.3.2 Maximum Mass

Star gets very hot inside, and T_c increases as M increases. The pressure due to the blackbody radiation inside increases a lot: $P_{\text{rad}} \propto T_c^4$. When this pressure becomes too big, it will dominate over the normal gas pressure ($P \propto \rho k_B T$), and when it does the star becomes unstable: it will blow itself apart. For $M \sim 100 M_\odot$, unstable, very hard to even form. Even for $\sim 50 M_\odot$, very violent, short-lived.

Lecture XV Life After the Main Sequence

Remember: stars do not move along the MS. They move onto it when they form, sit there for a long time, and then. . .

Main sequence is about where we find 80% of the stars. H in the core is burned into He. This is accompanied by a slow increase in L :

- $P = \rho k_B T / \mu m_H$, where μ is mean weight. 0.5 for H, higher for He.
- As H goes to He, μ goes up, so P would go down if T didn't go up.
- As T goes up, fusion region increases, L increases

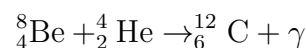
Then what? End of MS is when H is done in core. After that: giants (bigger and brighter). Depends on mass.

XV.2 Low Mass

$< 8 M_\odot$

no more energy from core, so contracts, gets hotter (Virial theorem). Layers above core contract too, H burns in "shell" around He core. This makes "red giant": $R \sim 100R_\odot$, lasts 10^8 yr. As this happens the star is a lot brighter too **draw HR**.

Eventually core contracts, $T_c \sim 10^8$ K, starts burning He via triple-alpha process:



Unless the second reaction happens, Be will decay (break apart) so needs high density. Higher temperature because He has charge of +2, so Coulomb repulsion is 4 times as much as H.

When enough C, starts burning to O (T higher still).

When He burning starts, star becomes bigger still: Asymptotic Giant Branch **Draw onion diagram, C/O core, He shell, H burning, H envelope**. $\sim 600R_\odot$ for 10^6 yr, $10^5 L_\odot$.

As this happens the star puffs off a lot of gas: larger R means lower GM/R on the surface, so stuff can escape. Goes from $10^{-14} M_\odot/\text{yr}$ (Sun) to $10^{-7} M_\odot/\text{yr}$ (RG) to $10^{-5} M_\odot/\text{yr}$ (AGB). This material sticks near the star for a time, forming a *planetary nebula* (nothing to do with planets), but enriches the surrounding area with "metals" formed in star. What's left at the center is C/O *white dwarf* (later).

Moves around on HR diagram, $L = 4\pi R^2 \sigma T^4$.

XV.3 High Mass

Burning keeps going to where Fe is produced. After that, cannot get energy out. Star collapses (*core collapse supernova*). **show onion**. Gravitational potential energy released (10^{46} J), mostly as neutrinos. Some compact remnant (possibly) left behind.

Lecture XVI Clusters: Testing Stellar Evolution

Most stars are not formed alone, but rather in groups (small) or clusters (large).

- Open cluster: young(ish), 10^3 stars, in a size of $\sim\text{pc}$, $\sim 10^3$ known, in the Galactic disk
- Globular cluster: old, 10^5 stars, still $\sim\text{pc}$, ~ 200 around the MW in a spherical cloud (halo). Often very *metal poor*: these stars were formed when the material had not been enriched by previous stellar burning (originally matter was almost all H, He)

In both, the stars are all born together from the same stuff at the same distance. So this removes a lot of the uncertainties that make astronomy hard. They are great testing grounds.

XVI.2 Make HR diagram out of a single cluster

sketch. Can determine distance by finding how bright the stars at T_{\odot} appear to be.

<http://astro.unl.edu/naap/distance/animations/clusterFittingExplorer.html>

Can determine age by looking at the highest-mass star that is still on the MS **Sketch**

Look for objects that are not on the MS: *binaries, dwarfs, etc.*

calibrate models of stellar evolution.

Lecture XVII White Dwarfs

Kutner 10.4

leftover remnant from the core of a low-mass star ($< 8M_{\odot}$).

core gets hotter & denser, as heavier elements need higher T to burn. As each phase of burning ends, collapse a bit, T up, P up. Can this keep going? *NO*

After get to O, P will no longer depend on T , so you cannot keep getting more burning. Why?

XVII.2 Degeneracy

electrons are *Fermions*: 2 cannot be in the same *state*. State = position, momentum, spin (Pauli exclusion principle). From the uncertainty principle:

$$(\Delta x)(\Delta p) \sim \hbar$$

p is momentum, $m_e v$. Density is number per volume, or 1/volume per particle. So $n \sim 1/\Delta x^3$. When the particles get squeezed too close they start to overlap, get to where

$$p = p_F \sim \hbar/\Delta x$$

(Fermi momentum). Which will be $p_F \sim n_e^{1/3}$. We can then use the momentum to get the kinetic energy:

$$E_F = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e (p_F/m_e)^2 = \frac{1}{2} \frac{p_F^2}{m_e}$$

(note that this is only true if $v \ll c$). So $E_F \propto p_F^2 \propto n_e^{2/3}$. This is the total energy per particle. Pressure has the same units as energy per volume (energy density), so we can multiply energy per particle by density to get pressure:

$$P_F \propto n_e E_F \propto n_e^{5/3} \propto \rho^{5/3}$$

There is no T ! Unlike ideal gas law ($P = n_e k_B T$), this is *not ideal*. Requires quantum mechanics. Ideal gas law $P = P(\rho, T)$, but here $P = P(\rho)$ only.

XVII.3 Build a Degenerate Star

(This is a white dwarf).

HSE gives:

$$\frac{P}{R} \sim \frac{GM}{R^2} \rho$$

So

$$P \sim \frac{GM}{R} \rho \sim \rho^{5/3}$$

We can then get $M/R \sim \rho^{2/3}$, but $\rho \sim M/R^3$, so we have:

$$M^{1/3} \sim 1/R \quad R \sim M^{-1/3}$$

This is weird. Unlike a star ($M \sim R$) or a normal rock ($R \sim M^{+1/3}$, since $\rho \sim \text{constant}$) or something, as M increases, it gets smaller!

XVII.3.1 When Does This Matter?

When electron spacing $\Delta x \sim \lambda_B$ (de Broglie wavelength from before). $\lambda \sim h/p \sim h/m_e v$, and $m_e v^2 \sim k_B T$, so

$$\lambda_B \sim 10^{-12} \text{ m} \left(\frac{10^9 \text{ K}}{T} \right)^{1/2}$$

For $1M_\odot$ and $R = R_\oplus$, get $\Delta x \sim 10^{-12}$ m. Since $T < 10^9$ K (C burning), λ is small enough to be degenerate, and $R_{\text{WD}} \sim R_\oplus$.

So a White Dwarf has the mass of the Sun squeezed into something the size of the Earth.

XVII.3.2 How Bright?

$L = 4\pi R_\oplus^2 \sigma T^4$, so even if T is higher than the Sun, $R_\oplus \ll R_\odot$ and the WD will be very faint.

XVII.3.3 Can A WD Be Any Size?

Can you keep piling mass on, or is there a limit?

$$\rho^{2/3} \sim \frac{M}{R} \sim \frac{M}{M^{-1/3}} \sim M^{4/3}$$

or $\rho \sim M^2$ in the center. So as you make it more massive, the density increases a lot. And as the density increases, so does the pressure and E_F . What happens when $E_F \sim m_e c^2$ (i.e., $v_e \sim c$)? Things get *unstable*. Before,

$$E_F \sim m_e v^2 \sim \frac{p_F^2}{m_e}$$

But now, including *special relativity*, $v \sim c$

$$E_F \sim p_F c \propto n_e^{1/3}$$

$$P \propto n_e E_F \sim n_e^{4/3}$$

Put that into $P \sim GM\rho/R \sim \rho^{4/3}$ and you find

$$\rho^{1/3} \sim \frac{M}{R}$$

But $\rho \sim M/R^3$, so $\rho^{1/3} \sim M^{1/3}/R$:

$$\frac{M^{1/3}}{R} \sim \frac{M}{R}$$

This cannot be true! When $v \sim c$ the velocity cannot increase fast enough to supply enough pressure, and the WD becomes unstable. This happens at the *Chandrasekhar Mass* $1.4M_{\odot}$: if it gets to this limit, it will collapse.

Lecture XVIII Neutron Stars

XVIII.2 Higher Masses: What Happens?

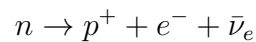
Kutner 11

Once we get to Fe in the core, we cannot get energy out. The star is still shining (so it's losing energy) but no longer creating it. So the core starts to cool. In order to support the rest of the star, the pressure needs to be the same, so the density goes up and up to keep the pressure constant.

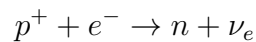
If the mass of the Fe core is less than the Chandrasekhar mass, then electron degeneracy pressure can support the star. But once it gets past there that is not enough. At this point, it is roughly 5×10^9 K, 5000 km in radius.

As it gets to the Chandrasekhar mass, electrons cannot support the star. It starts to collapse. The collapse liberates some gravitational energy, but the Fe *absorbs* that energy, liberating protons. Protons take up extra space, and they are increasingly squeezed by the rest of the star.

Remember β decay (nuclear decay):



This happens spontaneously for various nuclei. The inverse can also happen, although it isn't spontaneous:



When things get too dense, the inverse reaction is energetically favorable. This makes a bunch of neutrons, removing electron support. Neutrinos also leak out, removing energy.

So the core collapses down in a free-fall timescale of $\sim 1/\sqrt{G\rho} \sim 10$ s or less. This reaction happens at a density of $\sim 10^9$ kg m⁻³. Much of the star is blown off in a *supernova explosion*: the gravitational energy of 10^{46} J is released mostly as neutrinos, with a small amount creating a blast wave of material moving at 10,000 km/s.

XVIII.3 Core Collapse

It gets squeezed down to ~ 10 km. This is a core-collapse SN (there are other kinds), and happens for $M < 25M_\odot$ or so. A lot of the gravitational energy is released:

- 10^{46} J total
- 10^{44} J is in the KE of the ejected material
- 10^{43} J is in photons ($10^{10}L_\odot \sim L_{\text{galaxy}}$ for 10 days)

For example, SN1054 which is 2 kpc away was seen during the day quite easily.

99% of the energy (or more) goes out as neutrinos ($10^{19} L_{\odot}$). About 20 were detected from a SN 50 kpc away.

A lot of the blasted away material goes into the nearby interstellar space, enriching it with metals that were made in the star. This is how elements heavier than C/O get into the universe.

XVIII.4 What Remains

Kutner 11.2

A neutron star is like a WD, except instead of electron degeneracy, the reaction $p + e \rightarrow n$ makes neutrons, and neutron degeneracy pressure supports it. For $1.4M_{\odot}$ the size is about 10 km. This gives $\rho \sim 10^{17-18} \text{ kg m}^{-3}$, compare to a nucleus is 2×10^{17} .

$$\rho_{\text{WD}} \sim \frac{m_H}{(h/m_e c)^3} \sim 10^8 \text{ kg m}^{-3}$$

$$\rho_{\text{NS}} \sim \frac{m_H}{(h/m_p c)^3} \sim 6 \times 10^{17} \text{ kg m}^{-3}$$

We are confining one nucleon (proton or neutron) in a box. For the WD the size of the box is the de Broglie wavelength of the electron. For the NS it's the de Broglie wavelength of the neutron. Since the wavelength is $\sim 1/m$, the neutron's box is much smaller.

Consider a nucleus. The distance between neutrons is $r_0 \sim 10^{-15} \text{ m}$. If you take a Solar mass in neutrons, that means $A \sim M_{\odot}/m_H \sim 10^{57}$ neutrons, so the size is $R_{\text{NS}} \sim r_0 A^{1/3} \sim 10 \text{ km}$.

The properties of this object: surface gravity $g \sim 10^{12} \text{ m s}^{-2}$, escape speed $\approx 0.6c$.

The interior is very complicated: still under investigation. Likely superconducting (no electrical resistance) superfluid (no friction).

We have an upper limit to neutron star mass: keep $v_n < c$ gives a limit of $2 - 3M_{\odot}$ (details are hard).

XVIII.4.1 Spin

Angular momentum $J = I\omega$, $\omega = 2\pi/P$. Conserved. What made the NS?

$$\frac{R_{\text{core}}}{R_{\text{NS}}} \sim \frac{m_e}{m_n} \left(\frac{Z}{A} \right)^{5/3} \sim 500$$

going from the core (supported by electron degeneracy) to the NS. Conserving J :

$$I_{\text{core}}\omega_{\text{core}} = I_{\text{NS}}\omega_{\text{NS}}$$

and $I \approx MR^2$. So

$$\omega_{\text{NS}} \sim \omega_{\text{core}} \left(\frac{R_{\text{core}}}{R_{\text{NS}}} \right)^2$$

since the mass is the same. Or, going to period:

$$P_{\text{NS}} \sim 4 \times 10^{-6} P_{\text{core}}$$

Much faster! Core can rotate as fast as 30 min, so P_{NS} can be as little as 5 ms.

XVIII.4.2 Magnetic Field

Instead of angular momentum, conserve magnetic flux $\Phi = BR^2$. Same arguments give $B_{\text{NS}} \sim 250,000 B_{\text{core}}$. What might the initial field be? We measure fields of 10 T in white dwarfs, so the field in a NS could be up to 10^7 T in NS. In fact we see NSs with fields up to $1000\times$ this.

XVIII.4.3 A Limit To Rotation?

How fast can a NS rotate? 2 limits:

- Keep the equator $< c$
- centripetal force $<$ force of gravity

First one:

$$\frac{2\pi R}{P} = v_{\text{equator}} < c$$

would give a limit of $2\pi R/c = 0.2$ ms.

The second:

$$\frac{v^2}{R} = \frac{GM}{R^2} = \frac{4\pi^2 R}{P^2}$$

would give a limit $P < \sqrt{4\pi^2 R^3/GM} = 0.4$ ms. (this is actually Kepler's third law, $P^2 \propto R^3$).

XVIII.5 Pulsars

1967 Jocelyn Bell, looking for “twinkling” radio sources. Found something that “blipped” every 1.337 s. It was from the sky: it came every day at the same *sidereal time* (not LGM). It was too fast and too regular to be anything big (white dwarf, binary star, etc.)

More of these were soon found. Some were in supernovae remnants: the clouds of gas flying out at 10,000 km/s from where supernova explosions occurred.

Crab Nebula: glowing cloud of gas. Needs $10^5 L_{\odot}$ to power it. Found pulsating radio source inside with $P = 33$ ms. But they found that while regular, P was getting longer very slowly, change at $\dot{P} = dP/dt = 4 \times 10^{-13} \text{ s s}^{-1}$.

What would rotational energy be of neutron star spinning at $\Omega = 2\pi/P$? $(1/2)I\Omega^2$, $I \approx 10^{38} \text{ kg m}^2 = (2/5)MR^2$ is moment of inertia. What if the rate of spinning is slowing? Then it is losing kinetic energy at a rate $\sim I\Omega\dot{\Omega}$. If we do this, we find this is $5 \times 10^{31} \text{ W}$ is about

$10^5 L_\odot$: just right! This showed that the spin-down of a neutron star is what powers the Crab nebula (Tommy Gold).

Overall, pulsars were found to be rotating neutron stars. We see blips when the “lighthouse” beam crosses the Earth **show animation**. The majority of the energy from the spin-down is invisible: the radio blips are a tiny fraction of the energy.

It is the strong magnetic field that makes this happen.

XVIII.5.1 Spin-Down

(please pardon the calculus)

Light cylinder: where v to go around is c . We take the magnetic field to be a dipole, $B(r) = B_0(r/R)^{-3}$. A changing magnet releases electromagnetic power per unit area S (*Poynting flux*) $\sim cB^2/\mu_0$. We can roughly relate the spin-down energy loss $I\Omega\dot{\Omega}$ to the Poynting flux through the light cylinder:

$$4\pi R_{\text{LC}}^2 S_{\text{LC}} \approx I\Omega\dot{\Omega}$$

with $\Omega = 2\pi/P$, $\dot{\Omega} = -2\pi\dot{P}/P^2$. $R_{\text{LC}} = cP/2\pi$, so $S_{\text{LC}} = (c/\mu_0)B_{\text{LC}}^2 = (c/\mu_0)B_0^2 R^6/R_{\text{LC}}^2$. So we have:

$$4\pi R_{\text{LC}}^2 \frac{c}{\mu_0} B_0^2 \frac{R^6}{R_{\text{LC}}^2} = 4\pi R^6 \frac{c}{\mu_0} B_0^2 \left(\frac{cP}{2\pi}\right)^{-4} \sim \frac{R^6}{\mu_0} \frac{B_0^2}{c^3} P^{-4} \sim I \frac{\dot{P}}{P^2}$$

This gives:

$$B_0^2 \sim \frac{c^3 \mu_0 I}{R^6} P \dot{P}$$

So from the spin period and the rate at which it is slowing down, we can determine what the magnetic field is!

We can then use this (assuming $B = \text{constant}$) to get $P(t)$. We find that the age is $\tau \approx P/2\dot{P}$, so we also get the age of the system from P and \dot{P} . Do this for the Crab pulsar get 1250 years, which is very close to the true age of about 950 years (since people saw the supernova).

P - \dot{P} diagram: HR diagram for pulsars. **draw**. Move through the diagram from upper left to lower right until you die from low voltage (don't actually die, just shut off). This takes 10^{7-8} yrs to get to $P = 10$ s from a typical starting point of 10 ms. Usually born with 10^8 T, but there is a range.

XVIII.5.2 What happens after death?

Not much, unless in a binary star system. If in a binary: after the pulsar dies (remember, this still happens quickly compared to a main-sequence lifetime), the second star will evolve. It will leave the MS and puff up into a RG. When this happens, the outer bits of that star may get captured by the gravity of the NS **draw**. This is called *Roche-lobe overflow* (remember Roche from the tidal forces?). It leads to *accretion* onto the NS. That dumps angular momentum and mass onto the NS.

In the early 80's, they found pulsar with $P = 1.6$ ms, $P/2\dot{P} = 200$ Myr. So it couldn't have been born with that low a period since it is way too old. How could it get there? It was *recycled* into a

millisecond pulsar draw. For these, B is much weaker ($\sim 10^5$ T) although we do not really know why.

XVIII.5.3 Hulse-Taylor Binary

The second star in a binary can also eventually become a NS. The H-T binary is one such system: two NSs in a 8-hr orbit. But, with General Relativity we predict that such a system will lose energy, angular momentum due to *gravitational radiation* (like a moving charge emits EM radiation). The change in the period of the binary was observed from very precise measurements: 1993 Nobel prize in Physics.

Will merge in 300 Myr: explosion and burst of GW.

Lecture XIX Gravitational Redshift & Black Holes

Kutner 8.3, 8.4

a photon needs to climb out of a potential well. When that happens it loses energy. A rocket climbing out of a well would slow down, when it loses energy, but photons must go at c , so change frequency instead.

draw

If we start at R_1 and go to $R_2 = \infty$: starts with $m = E/c^2 = h\nu_1/c^2$, so the total energy of the system is $h\nu_1 - GmM/R_1$ and it ends with the same total energy

$$h\nu_1 - G \frac{h\nu_1 M}{c^2 R_1} = h\nu_2 - \frac{h\nu_2}{c^2 \infty}$$

which gives:

$$h(\nu_2 - \nu_1) = h\Delta\nu = \frac{Gh\nu_1 M}{c^2 R_1}$$

or $\Delta\nu/\nu = GM/Rc^2$ (and a shift in wavelength is similar). This is sort of like the Doppler shift, except it comes just from gravity, not velocity.

XIX.1.4 WD

for a white dwarf, $\Delta\lambda/\lambda \approx 74 (M/M_\odot)^{4/3} GM_\odot/R_\odot c^2$. It has been measured. For 40 Eri B (first WD) is it 6×10^{-5} .

XIX.1.5 NS

for a neutron star, this can be 20%! Can be quite large, although yet to be measured (I'm trying...).

XIX.1.6 Earth

1960 at Harvard. Shot γ -ray up a 22.6 m tower. Found a change $\Delta\nu/\nu = -gh/c^2 = -2.5 \times 10^{-15}$.

XIX.2 More Mass?

What happens if we take a NS and add mass? $> 3M_\odot$ (or so): like a WD, the particles get to $v \sim c$ and it cannot support itself. Collapses. And nothing can stop it. Gravitational redshift goes up. Escape velocity goes up. $v_{\text{esc}} = \sqrt{2GM/R}$ cannot be $> c$: eventually we get to where this is $= c$ at

$$R = R_{\text{Schwarzschild}} = \frac{2GM}{c^2}$$

This is a black hole! (note that this derivation is not correct, but it gives the right answer).

At this point, the gravitational redshift becomes:

$$\frac{\Delta\lambda}{\lambda} = \left(1 - \frac{R_{\text{Sch}}}{R}\right)^{-1/2} - 1$$

which goes to $1/0 = \infty$ at $R = R_{\text{Sch}}$. (and this reduces to the form we already did for $R \gg R_{\text{Sch}}$.)

XIX.3 BH

A BH has mass squeezed into R_{Sch} : event horizon. Cannot get anything out. For $1M_{\odot}$, $R_{\text{Sch}} = 3 \text{ km} \ll R_{\text{NS}}$.

How do we identify black holes? We look for things moving really fast in a really small volume, and we look for $M > M_{\text{NS}}$

draw

Material moves in *accretion disk* around BH, gradually spiralling in. We can deduce that it has a velocity (from redshift/blueshift) such that it would be inside a NS (from Kepler's laws).

Accretion can happen with NS too. But there it can have any orbit. For a BH, stable orbits can only happen for $R > 3R_{\text{Sch}}$. Inside that, the material is doomed. Must fall in.

As matter falls in, releases gravitational energy. Gets hot! Makes jet!

draw

Happens for small BHs: X-ray binary, *microquasar*.

Happens for big BH. Most galaxies seem to have one. $M_{\text{BH}} = 10^{6-9} M_{\odot}$, *active galactic nucleus* or AGN. Can outshine rest of galaxy (*quasar*). Brightness depends on how much stuff is falling in. MW: $2 \times 10^6 M_{\odot}$.

Event horizon does not kill. tidal forces do.

Eventually, GR (Einstein) says that no orbit will stay stable. For massive objects moving quickly (i.e., in short orbits) this will happen in $< t_{\text{Universe}}$. Like the Hulse-Taylor binary in 200 Myr.

What will happen? They will merge & (often) explode. Details not know, but for instance NS-NS binary will probably make a gamma-ray burst.

When this happens, the moving masses will also distort space-time, sending out waves. We did detect this on Earth with Laser Interferometer Gravitational Observatory: astronomy with gravitational waves, not photons.

Lecture XX Supernovae

Kutner 11.1

explosions, new stars. a few known historically (e.g., Crab in 1054). Here we talk about core collapse (from massive stars).

10^{46} J released, 1% into KE, 0.01%–0.1% into photons. Collapse makes fusion well past Fe.

Lightcurve **draw**: decline set by radioactive decay of Ni (6 days) and Co (77 days) made from shock slamming into Fe.

XX.2 Chemistry

Rest of elements past Fe mostly from SNe. Reactions: r-process (rapid). Heavy element + many neutrons \rightarrow very heavy, unstable nucleus. These then decay to something stable.

Also s-process (slow) in post-MS evolution of stars.

XX.3 Shock Wave

Hot gas out at 10,000 km/s. 10^{44} J at $T = 10^7$ K into ISM (10^4 K, $n \sim 10^6$ m $^{-3}$). Blast wave:

- Starts really fast: free expansion, supersonic
- Until sweeps up mass \sim mass in shell (10's to 100's of years)
- Enters Sedov-Taylor phase (first derived for nuclear explosions): $E \sim (\rho_{\text{ISM}}/t^2)(r/c)^5$

Lecture XXI Interstellar Medium

Kutner 14

What is between stars? Gas (99%) and dust (1%). Like a star, mostly H, then He.

XXI.2 Dust

Obvious as dark patches in the sky **show**: “holes” w/o stars.

Light is blocked by dust. Work by Trumpler (1930).

Clusters of stars each have main sequence: should line up to get distances. Trumpler: d = distance, D = diameter. Angular diameter $\theta = D/d$:

$$\theta^2 = \frac{D^2}{d^2} \propto \frac{1}{d^2}$$

Apparent brightness is the flux $F = L/4\pi d^2 \propto 1/d^2$.

If L , D are typical values then should see $\theta^2 \propto F$

draw

systematic departure for distant objects (small F , small θ^2), with F less than expected. Moreover, distant clusters were also redder than expected. Both from dust.

Like a sunset. Red light transmitted, blue light scattered/reflected.

Dust is little balls of C, Si. Makes things dimmer and redder.

XXI.3 Gas

Most of mass of ISM, few % mass of stars.

Gas can be warm or cold, dense or diffuse, atoms or molecules. Average is $n \sim 1 \text{ cm}^{-3} = 10^6 \text{ m}^{-3}$. For comparison, best vacuum on Earth is 10^9 m^{-3} .

component	volume	T	n	state	see via?
molecular clouds	< 1%	10-20	10^{8-12}	H ₂	molecules, emit & abs.
CNM	1-5%	70	3×10^7	H	HI 21 cm abs
WNM	10%-20%	10^4	10^6	H	HI emit
WIM	20%-50%	10^4	10^6	H II	H α
H II regions	< 1%	10^4	10^{8-10}	H II	H α

The last surrounds hot stars.

Remember: hot light and cold cloud: absorption. Hot cloud: emission.

XXI.3.1 HI 21 cm

diagnostic for gas. Spin flip **draw**. $\nu = 1420$ MHz or $\lambda = 21$ cm. First seen in 1950's.

XXI.3.2 Pressure Equilibrium

Different parts are “roughly” at same $P = nk_B T$ (equilibrium).

Molecular clouds: small and dense, where stars are born

H II regions surround hot stars (OB, WD)

the rest takes up space in between.

measure T from Doppler width of lines. Measure n from brightness of lines.

XXI.3.3 Using HI

Doppler shift gives v_{radial} . In Milky Way, get distance, velocity of gas, can map spiral arms.

XXI.3.4 Physics That Happens

Heat ISM via:

- cosmic rays (protons)
- light from stars
- shocks (SNe)
- stellar winds

Cool via

- emission of lines
- free-free (emission of continuous radiation from hot, ionized gas)

XXI.3.5 H II Regions

HI ($T \sim 10^2$ K) around H II around star.

Star is $> 10^4$ K so enough photons have $h\nu > 13.6$ eV. Can then ionize.

Size of H II: N_* = number of photons per second with $E > 13.6$ eV (beyond Lyman limit).

Assume each photon is absorbed by 1 atom.

But, for each e, p there is a chance that recombine $e + p \rightarrow H$. This balances ionization.

$$\mathcal{R} = \frac{\# \text{recombination}}{\text{volume} \times \text{time}}$$

$\mathcal{R}V = N_* = \mathcal{R}(\frac{4}{3}\pi r^3)$. What is \mathcal{R} ? Depends on rate that e hits p . $\mathcal{R} \propto n_e n_p \propto n_e^2$. So

$$r = \left(\frac{3N_*}{4\pi\alpha n_e^2} \right)^{1/3}$$

Stromgren sphere. $\alpha = \alpha(T) \sim 3 \times 10^{19} \text{ m}^3 \text{ s}^{-1}$. $n_e \sim 10^8 \text{ m}^{-3}$, $N_*(O5) = 3 \times 10^{49} \text{ s}^{-1}$. Gives $r \sim \text{few pc}$.

Can have multiple stars (bigger) or WD (smaller). See via Balmer lines ($H\alpha$ etc.) in emission, also lines from N, O, He.

Lecture XXII Star Formation

See other notes

Lecture XXIII Solar System

Kutner 22–26

Solar system: terrestrial (like Earth) planets are Mercury, Venus, Earth, Mars; rocky, with very thin atmospheres. Gas giants Jupiter, Saturn, Uranus, Neptune; have cores, but outer layers are mostly gas. And then there are other objects.

Study planets now, get clues to how they formed.

see other notes.

Lecture XXIV Terrestrial Planets & Atmospheres

Kutner 23, 24.

XXIV.2 Atmospheres

The Earth's atmosphere is 78% N₂, 21% O₂, and 1% others (Ar, CO₂). How did it get that way? Did it happen naturally during the formation of the Earth?

No: at 1 AU, conditions during the formation of the Earth were too hot for those elements to condense:

- Mass too low to attract the gas on its own (unlike gas giant)
- Hot early on, so gases would escape

Answer: gradual accumulation + *life*.

First atmosphere:

- H and He
- Much escapes

Second:

- CO₂/NH₃ which outgassed from volcanos
- H₂O, some from comets (icy balls)
- Pressure ~ 100 bar (Earth now is ~ 1 bar)
- < 100° C

Third:

- CO₂ absorbed
- H₂O liquid
- O₂ produced
- 1 bar
- 15° C

Sink of CO₂: rock (with water, it dissolves), water, life
sources: volcanos (+humans)

Sink of H₂O: tectonic plates subduct, carry water into the mantle
sources: volcanic outgassing, comets

It is a sensitive balance, where too much CO₂ can drive off water

Venus: similar to Earth at some point in the past. Why different now?

- Closer to Sun, so warmer, less water?
- Too much CO₂ to start?

Non-linear system, so even if starting points were very similar small differences can be amplified (in unexpected ways)

O₂: not natural, unstable. Produced over billions of years by life: CO₂ + H₂O → O₂ + sugar.

Show greenhouse, energy balance.

XXIV.3 Interiors of Planets

Differentiated (in layers)

- Dense metallic core (solid interior, liquid outer)
- Lighter rocky mantle
- Lighter rocky crust

Heat sources:

- Accretion (things falling)
- Differentiation (settling into layers, releases potential energy)
- Radioactive (U, Th)
- Tidal

Heat flow:

- Conduction in core
- Convection in mantle (lava lamp w/ ~ 100 Myr, rock creep)
- Conduction through surface

Relative sizes of the components change a lot between the different planets.

Exteriors shaped by:

- Impacts (rare now, more common before, so # can be used to “date” surface)
- Volcanos (Earth, Venus now; Mars before)
- Tectonics (Earth now)
- Erosion by winds, liquids (Earth, Mars, ?)

Continental Drift/Plate Tectonics **show**

Evidence:

- Similar fossils on different continents
- Continents fit together
- Spreading of mid-ocean ridges

All explained by motions of plates on plastic mantle.

Venus shows evidence of tectonics but not plates: is this because less water makes the rock more plastic, so it will not be in pieces?

XXIV.4 CO₂ Cycle

Where is all of the CO₂ on Earth compared to Venus?

Sink: dissolves in water, forms carbonate rocks, rain erodes rock and the pieces fall into the ocean.

Source: volcanos

So the sink depends on the temperature, but the source does not.

Earth:

- If it were too cool
 - Ocean dissolves less CO₂
 - Fewer carbonates
 - more CO₂
 - more greenhouse effect, so it would be warmer
- If it were too warm
 - dissolve more CO₂

- more carbonates
- less CO₂ in atmosphere
- less greenhouse, so it would be cooler

this acts as a thermostat to regulate the temperature (within some range).

XXIV.4.1 Snowball Earth

(not entirely solved problem)

Try to explain recurrent freezings of (much of) Earth. These were triggered by some even (volcanic, impact):

- drives temperature down
- extra ice, so shinier (*albedo* goes up)
- temperature goes down
- this makes more ice, T goes down further
- eventually, \sim total ice (evidence for glaciers at equator)
- once this happens, oceans are covered, so CO₂ sinks disappear, builds up
- greenhouse, rapid warming, melt, albedo down

Most recent episode \sim 650 Myr ago? Questions regarding snowball vs. slushball, details, extend, frequency.

Lecture XXV Formation of Solar System

Kutner 27

Protoplanetary disk: disk around proto-Sun 4.6 Gyr ago (**show**).

Common around other stars (first 10–100 Myr of star's life)

Formation: details still unknown, but basic concepts:

- Minimum mass solar nebula
- planetismals
- frost line

ways we test the theory:

- giant planets vs. rocky
- isotope dating

XXV.2 Minimum Mass Solar Nebula

Sun collapsed out of gas cloud. Angular momentum made much of it form disk. Had same elements as the Sun: 70% H, 28% He, CNO etc. 1% (also seen in meteorites). Early on this was substantial, but since then it has either been formed into planets or been blown away.

Current total of all objects (except Sun): $1.5 M_J = 0.0015 M_\odot$. Was a lot more in the past. How can we reconstruct what might have been? Can we set a *lower limit*?

Put back all light elements (H, He) to make planets have the same abundances as the Sun (only adding):

- Mercury: $\times 350$
- Earth: $\times 235$
- Jupiter: $\times 5$
- Saturn: $\times 8$
- Uranus: $\times 15$

Add this all together \rightarrow MMSN = $10M_J = 0.01M_\odot$.

- This would have been densest near the Sun

- at least 85% was lost before forming planets
- Similar in mass wrt Sun as many observed disks

Problem: this disk cannot collapse on its own. Any bulge that forms will be quickly torn apart by tidal forces from the Sun (tidal force \gg self-gravity)

XXV.2.1 Cannot Collapse

Ball with mass M_{gas} , size d , a from Sun:

$$F_{\text{tide}} \sim \frac{GM_{\odot}M_{\text{gas}}}{a^2} \left(\frac{d}{a}\right)$$

And self-gravity:

$$F_{\text{self}} = \frac{GM_{\text{gas}}^2}{d^2} = G\rho dM_{\text{gas}}$$

($\rho \approx M_{\text{gas}}/d^3$). To make something collapse, need $F_{\text{self}} > F_{\text{tide}}$

$$G\rho dM_{\text{gas}} > \frac{GM_{\text{gas}}M_{\odot}}{a^3}d$$

which needs $\rho > M_{\odot}/a^3 \sim 2 \times 10^{-4} \text{ kg m}^{-3} (1 \text{ AU}/a)^3$. This is known as the *Toomre criterion*. Compare with:

$$\rho_{\text{MMSN}} \sim 10^{-5} \text{ kg m}^{-3} (1 \text{ AU}/a)^{5/2}$$

which is too low by at least a factor of several–20. So how did planets form?

XXV.2.2 Planetesimal Hypothesis

dust

(grains are small & sticky, can stick together with a force that is $>$ gravity)

- Disk cools, forms dust, $m \sim 10^{-15} \text{ kg}$, size $\sim \mu\text{m}$
- These stick together to become pebbles ($\sim\text{g}$, cm)
- These collide to form *planetesimal* ($\sim 10^{12} \text{ kg}$, km)

The remnants of this process might form the asteroid belt (in part).

- A few of these dominate everything nearby, *planetary embryos* ($\sim 10^{19} \text{ kg}$, 100 km)
- These collide, *planetary cores* ($\sim 10^{21} \text{ kg}$, 1000 km, proto-Earth)
- The cores accrete gas, make gaseous planets ($\sim 10^{24} \text{ kg}$, proto-Jupiter).

XXV.2.3 Hill Sphere

What does *dominate* mean? Define Hill Radius R_H : where P_{orb} around embryo = P_{orb} around Sun. Get:

$$R_H = a \left(\frac{M}{M_{\odot}} \right)^{1/3}$$

which is $1.4R_{\oplus}$ for 10 km comet at 5 AU. Stuff within that region will be bound to the embryo.

XXV.2.4 Frost Line

(or Snow line)

sketch disk, with frost line

Where is equilibrium temperature $T < 150$ K or so: make ices. water, ammonia, methane solid rather than gas (vacuum lowers melting point, no liquid phase)

Outside frost line, planets can grab a lot of H in the form of ices (mix with C, H, N). These will be accreted onto the rocky cores.

Inside, planets largely rocky, made of *refractory* elements (high boiling point).

Outside, volatiles. So the mass of the planet jumps a lot since you can get many of the light elements that dominate the MMSN. Masses go up by > 5 . With even more mass, can get gasses too. Somewhere $\sim 2 - 3$ AU.

This explains:

- giants have much gas, ice
- terrestrial do not

From this we expect a rocky core inside the giant planets.

A related concept is the *Habitable Zone*. It is where liquid water can exist (needs pressure > 0). Here it is 0.7–3 AU (Venus to Mars). But this is rough, since greenhouse effect and the history of the planet can change the answer.

XXV.3 Date Of Formation

Look at pristine materials: asteroids, comets, moon, Mars. We want things that have not melted (and driven off any elements) since they formed.

We see that e.g., meteorites that land on Earth have abundances much like the Sun. From these we measure ratios of isotopes with natural decay, like $^{238}\text{U} \rightarrow ^{206}\text{Pb}$ by way of Th, etc. Takes $t_{1/2} = 4.47$ Gyr. From this, get:

- Age = 4567.2 ± 0.6 Myr

- From other isotopes, expect was a supernova a few MYr before formation (^{60}Fe , 1.5 Myr). Was star formation triggered by SN shock?
- Earth formed within 10 Myr
- Moon \sim 30 Myr after the solar system
- oldest fossils, 3.5 Gyr

XXV.4 Earth's Moon

size is uniquely large compared to all other planets, 1% of Earth mass.

We can tell that areas of the surface are older or younger based on numbers of craters

Moon rocks: little water or other volatiles. Low in Fe (core $<$ 200 km?), but similar isotopes relative to Earth.

Possibilities?

- Capture (was floating around system)
- Accretion disk (might account for Jupiter's moon, like mini solar nebula)
- Impact (explains lack of Fe, volatiles)

Need $M \sim$ Mars and $v \sim$ 10 km/s to melt Earth. This gives

$$\frac{1}{2}mv^2 \approx k_B T \frac{M_{\oplus}}{\mu m_H}$$

Some mass was lost, but the rest became the Moon.

Lecture XXVI Extrasolar Planets

Kutner 27

Planets outside the solar system, first found in 1995 (except for pulsar system in 1992)

XXVI.2 How To Find?

show app? Can we see directly? How bright compared to star (contrast)?

Reflected light:

$$\frac{\pi R_p^2}{4\pi a^2} \approx 10^{-8} \left(\frac{a}{1 \text{ AU}} \right)^{-2}$$

Thermal light (planets are warm, like Jupiter):

$$\frac{\pi R_p^2}{\pi R_s^2} \left(\frac{T_p}{T_s} \right)^4 \approx 10^{-6}$$

Angular separation: $0.1''(a/1 \text{ AU})$ at 10 pc away. This is very hard, but has been done in a few occasions.

Main method: radial velocities (relative motion of star tugged by planet)

$$\Delta v = 30 \text{ m/s} \left(\frac{a}{1 \text{ AU}} \right)^{-1/2} \left(\frac{M_p}{M_J} \right)$$

sketch. Can see multiple planets. But inclination unknown.

Astrometry: see motion of star itself, not the velocity. Very hard.

$$\Delta\theta \sim 0.1 \text{ mas} \left(\frac{a}{\text{AU}} \right) \left(\frac{M_p}{M_J} \right) \left(\frac{d}{10 \text{ pc}} \right)^{-1}$$

This depends on d , unlike the previous two methods.

A lot of progress lately on *transits*: see dip in light when planet goes in front of star (Kepler!).

Depth

$$\frac{\pi R_p^2}{\pi R_s^2} \sim 0.01$$

or less, but only if orbit is edge-on. Chance of transit (**sketch**):

$$\sim \frac{R_s}{a} \sim 0.05 \left(\frac{0.1 \text{ AU}}{a} \right)$$

Works best for big, nearby planets.

All search methods have biases: certain systems show up more easily. Hard to understand what the real underlying population is.

These are from protoplanetary disks. We see disks around 5% of stars younger than 10 Myr. We see planets around roughly 5% of stars with $M \sim M_J$.

Occasionally we see systems with both disk and planet: the planet clears a space around it (like Saturn's rings) and accretes material, growing.

XXVI.3 Properties

Look at exoplanets.eu

XXVI.3.1 Mass

large range, high-M easier to detect, see up to $10M_J$. $N \propto 1/M$ roughly.

XXVI.3.2 Period

short end: very close, orbits down to a few days (makes it easier to detect). We call these *hot jupiters*

long end: limited by how long a search goes on and sensitivity of searches.

XXVI.3.3 Eccentricity

at low a , most have $e = 0$ (tides make things circular)

at high a , e can be anything.

XXVI.3.4 Hot Jupiters

$$T_p = T_S \sqrt{\frac{R_S}{2a}} (1 - A)^{1/4} \approx 1100 \text{ K} \left(\frac{0.1 \text{ AU}}{a} \right)^{1/2}$$

Compare to 120 K for Jupiter. How can they be so hot? Wouldn't they boil away? Could they form there?

See large planets with orbits all the way down to 0.01 AU. Mercury is at 0.4 AU.

XXVI.3.5 Metallicity

Find more around stars with high metallicity. More metals = more grains and ices = bigger cores?

- < 1% when metals are < 30% of Sun
- > 20% when metals are $3 \times$ Sun

sketch

XXVI.3.6 Radii

Depth, length of transit can get radius of planet too (otherwise just mass) Many are large than Jupiter **show**. Large range, with a fractof of ~ 4 in ρ .

Are they bloated because they are hot? This is hard. Did they just never shrink when they formed?

XXVI.3.7 Detecting Atmospheres

Transit: planet blocks star light, blocks all wavelengths

If there is an atmosphere around the star, certain wavelengths will get extra absorbed right near when the main part of the planet transits

XXVI.3.8 Migration

Expect that we form giants far out (past frost line). But we see many close in (hot Jupiters). Why?

Even Jupiter might have formed ~ 0.5 AU further out. Slowly spirals in (*migration*) from interactions between disk and planet. Hard to get details right:

- need it to happen fast enough that close is common
- need it to happen slow enough that they stay there

Lecture XXVII Telescopes

Kutner 4

Why do we use telescopes? How big a telescope do you need?

photons come at some rate: source has a flux density F_λ in $\text{J/s/m}^2/\text{\AA}$. To get rate of photons:

$$F_\lambda \times \frac{\text{telescope area}}{\text{photon energy}} \times \text{filter width} \times \text{efficiency}$$

gives N (photons/s). Number detected is $N \times \text{time}$:

$$n = F_\lambda \frac{A}{hc/\lambda} \Delta\lambda \Delta t \eta$$

e.g., magnitude 20 (faint for eyes, not for telescopes) star: $F_\lambda = 3.6 \times 10^{-20} \text{W m}^{-2} \text{\AA}^{-1}$ (by eye can see mag= 6, so this is $10^{-(20-6)/2.5} = 2.5 \times 10^{-6}$ times as bright). Use:

- $\lambda = 500 \text{ nm}$, so $hc\lambda = 4 \times 10^{-19} \text{ J}$
- diameter $D = 10 \text{ m}$, so area $A = 78 \text{ m}^2$
- efficiency $\eta = 20\%$
- width $\Delta\lambda = 1000 \text{ \AA}$

gives 1.4×10^3 photons/s. How long do we need to observe for? How many seconds are enough?

XXVII.2 Poisson Statistics

for counting

expect r events/s, wait t seconds. So we *expect* $rt = n$ events (cars, raindrops, people, etc.). How many actually come? Probability that we see m when expect n :

$$\frac{e^{-n} n^m}{m!}$$

- $P(0) = e^{-n}$
- $P(1) = n e^{-n}$
- $P(2) = n^2 e^{-n} / 2$

sketch.

Expected number is $m = n$, but we often get more or less. What is important here is width: we don't always see exactly as many as we expect, but we want to know how close we will come

on average. In general, 68% of the time we see somewhere in $n \pm \sqrt{n}$ (this is 1- σ , central limit theorem, $n \gg 1$). 95% of the time we see $n \pm 2\sqrt{n}$. 99.7% of the time we see $n \pm 3\sqrt{n}$.

So if we want to be *sure*...

We want to see enough photons that we can be sure that there is something real there. Usually we say $> 3\sigma$ confidence, so only wrong 1 time in 1000. $n/\sqrt{n} = 3$, so $n > 9$. That means we need 9 photons per exposure, so we can have $\Delta t = 6$ ms (as I said, this is very easy for a telescope).

But objects can be a lot fainter (27th mag), can have background noise, can have lower spectral width (spectra).

XXVII.3 Other Wavelengths

Optical observing: light behaves mostly like a particle, can do from the ground. Does this change?

Consider:

$$\delta \approx \Delta\nu \left(\frac{F_\nu}{h\nu} \right) \Delta\tau A_c$$

with $\Delta\tau$ the coherence time, and A_c the coherence area. Based on the uncertainty principle $\Delta\tau\Delta\nu > 1$ and $A_c \approx \lambda^2 = (c/\nu)^2$, so we get:

$$\delta \approx \frac{F_\nu}{h\nu} \left(\frac{c}{\nu} \right)^2 = c^2 \frac{F_\nu}{h\nu^3}$$

From a blackbody,

$$F_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

So

$$\delta \sim \frac{2}{e^{h\nu/kT} - 1}$$

For the Sun at $T = 6000$ K, at optical wavelengths $\lambda = 500$ nm we find $\delta = 0.02$. This is $\ll 1$, so it behaves like a particle.

At radio wavelengths $\lambda = 1$ m, $\delta = 8 \times 10^5 \gg 1$ so it behaves like a wave. $\delta = 1$ at a wavelength of $2 \mu\text{m}$.

XXVII.4 Radio Telescopes

$\lambda > 1$ mm or so, can know phase of wave. Atmosphere is transparent up to wavelengths of 10 m (beyond that is blocked by the ionosphere).

Do we need dark skies? No: need free from interference:

- 90–100 MHz: $\lambda = 3$ m, FM band
- 1 GHz (30 cm): WiFi, phones, microwaves, etc.

RFI. Better in valley than on mountain.

Telescopes can be anything from “light buckets” to coat hangars. Need a lot of area.

The surface needs to be smooth to $< \lambda/4$. Optical: 100 nm (polished glass, heavy & expensive).
Radio: 1 cm (chicken wire).

So we can make things much bigger, which is good since sources are faint.

D up to 300 m (Arecibo). To detect, no longer look at individual photons, but treat like a wave.

XXVII.5 Seeing

Optical: resolution $> 1''$ by *seeing*: turbulence in the atmosphere. To do better: go to top of mountain, go to space (expensive), correct for turbulence (hard). Last is *adaptive optics*. Use laser guide stars: make a (fake) perfect star, then see how it gets distorted. Can compute how to undo that.

XXVII.6 Resolution

Diffraction limit. We can only see things that are bigger than λ/D . In the optical this is limited in any case to $> 1''$. compare to Arecibo: $49''$ at 6 cm, so much worse. And we can't make a dish (much) bigger. But luckily we don't need to:

use multiple dishes (interferometer). Then resolution is λ/B where $B \gg D$. Need to combine as a wave (maintain phase). Area (signal-to-noise) limited by D (or area = $N \times D^2$), but resolution from B .

- Very Large Array: 27 dishes, $D = 25$ m each. B up to 30 km, so total A is 1/9 Arecibo by θ down to $1''$ or better
- Very Long Baseline Array: 10 dishes, 25 m, B up to continent (8000 km using islands). $\theta = 1$ mas

Do interferometers work at other wavelengths? Yes, but it's hard.

- IR: Keck, VLT, some others. Up to 4 telescopes, for specialized projects, within a few 10's of m. Need to know spacing between telescopes to $\lambda/4$ which is very hard.
- Others: eventually
- lion at 10 km: $100''$
- movie screen at the Moon: $0.01''$

XXVII.7 Infrared

targets: warm things. dusty things (blocks optical light, re-emits as IR). Proto-stars, star-forming galaxies, gas clouds.

$\lambda < 5 \mu\text{m}$ from ground. After that atmosphere absorbs, need to go to space. Better from high mountain in any case.

Issues: Wien displacement, peak of BB in the IR from anything warm (300 K is $10 \mu\text{m}$). So telescopes, people, sky all make “noise.” Solution is to make the whole telescope cold, ideally in space.

XXVII.8 UV/X-ray

Targets: hot things. WD, BH, NS.

atmosphere blocks so we mostly need to go to space. X-rays are even harder: cannot make a mirror to focus the light. Have to use *grazing incidence* or other tricks.

XXVII.9 Astronomy Without Photons

neutrinos, cosmic rays, gravitational waves. All hard, in infancy.

ν and CR: indirect detection. e.g., ICECUBE. 1 km^3 of ice at south pole. strings of light detectors. When ν passes through, mostly goes on without interacting. Occasionally hits proton, generates e or μ . When it does they will be traveling faster than local c (not c in vacuum). This makes Cerenkov light, or a blue flash (sort of like a shock wave).

Lecture XXVIII Content & Structure of the Milky Way

Kutner 16

XXVIII.2 What is a galaxy?

A collection of stars ($\sim 10^6$ to $\sim 10^{12}$, roughly), gas, dust and dark matter, held together by gravity. Bigger than a star cluster, smaller than a cluster of galaxies. (This question isn't entirely settled; see <http://arxiv.org/abs/1101.3309v1>) There are hundreds of billions of galaxies, but we'll start with an overview of our galaxy, the Milky Way.

XXVIII.3 Early observations

On a dark night, the Milky Way can be seen as a luminous band of light across the sky. This was called *galaktikos kuklos* in ancient Greek, meaning “milky circle;” this is the origin of the word “galaxy.” Galileo observed the band to be made up of many individual stars. A hypothesis that explains the Milky Way is that the sun is embedded in a thin disk of stars; when we look perpendicular to the disk we see few stars and the sky is dark, while we see many stars when we look into the plane of the disk. This disk of stars is a major component of the galaxy.

XXVIII.4 Globular cluster distribution

Between 1915 and 1919, Harlow Shapley arrived at a better estimate of our place in the Milky Way using the distribution of globular clusters (compact, spherical clusters of old stars whose distances can be estimated because they contain variable stars whose periods depend on their absolute brightness—more on that later). Shapley noticed that globular clusters aren't distributed uniformly across the sky; instead they're concentrated in one half of the sky, centered on constellation Sagittarius. He concluded that the globular clusters were all orbiting the center of the galaxy, which lies in the direction of Sagittarius. He also measured the distances to the globular clusters to estimate the size of the galaxy; got it a bit wrong, since he thought that RR Lyrae stars were brighter than they are, but had the order of magnitude right. We are about 8 kpc from the center of the Milky Way.

Still use both of these methods, with better understandings of dust absorption and distances.

XXVIII.5 Components of the Galaxy

The Galaxy has three basic components: disk, halo and bulge.

XXVIII.5.1 The Disk of the Milky Way

- Most luminous component of Galaxy

- Radius ~ 25 kpc, stars to 15–20 kpc
- Gas disk of galaxy, seen in emission from neutral hydrogen, extends further, to $R \sim 25$ kpc from the Galactic center
- Thickness of disk is small compared to its radius: most stars are less than 0.5 kpc from the midplane of the disk
- Studying stellar properties as a function of z , the distance from the midplane, shows that the disk can be further divided into two components: a **thin disk** containing stars of all ages including stars that are currently forming, and a **thick disk** made up of stars older than ~ 5 Gyr.
- Near the Sun, the distribution of stars in both the thin and thick disks falls exponentially with distance z from the midplane.
 - Thin disk: $n(z) = n_{\text{thin}} \exp(-|z|/h_{\text{thin}})$, where the scale height $h_{\text{thin}} \approx 350$ pc and n_{thin} is the number density of thin disk stars at $z = 0$. (Scale height: the height at which the density drops by a factor of $e^{-1} \simeq 0.37$.)
 - Thick disk: $n(z) = n_{\text{thick}} \exp(-|z|/h_{\text{thick}})$, where the scale height $h_{\text{thick}} \approx 1$ kpc.
 - In the midplane, $n_{\text{thin}} \approx 10n_{\text{thick}}$.
 - Sun is member of the thin disk and is about 30 pc above the midplane.
- The disk shows spiral structure, with star formation concentrated in the spiral arms. We'll talk about this more when we discuss spiral galaxies.
- The disk also contains gas and dust. Gas mostly neutral hydrogen, and $M_{\text{dust}}/M_{\text{gas}} \simeq 0.007$.

XXVIII.5.2 Bulge

- Galaxy has a central bulge, about 1 kpc in radius, extending above and below the disk.
- Scale height ranges from 100 to 500 pc, depending on ages of stars measured. Younger stars have smaller scale heights.
- Tiny nucleus in the center, bright at radio wavelengths. More on that later.
- Difficult to observe because of large amounts of dust extinction.
- Contains a mixture of stars of different ages

XXVIII.5.3 Halo

- Roughly spherical distribution of stars
- Radius ~ 100 kpc

- Same luminosity as bulge, but volume $\sim 10^6$ times larger
- Old, low metallicity stars
 - Metallicity Z : fraction by mass of elements heavier than helium
 - Universe is about 74% H, 24% He, 2% other elements
 - Big Bang produced H, He, a bit of Li—all heavier elements formed in stars
 - Stars form out of gas, create heavy elements, and return the elements to the gas when they die, so metallicity increases with each generation of star formation: good estimate of the age of a stellar population
 - Population I stars are young and metal-rich ($Z \geq 0.01$), population II stars are relatively old and low in metals ($Z \leq 0.001$).
 - Thin disk is Population I, halo is Population II, thick disk is intermediate.
- Globular clusters
 - Spherical clusters of old stars located in halo
 - At least 150
 - Ages range from 11 to a little over 13 Gyr old (age of universe 13.7 Gyr).
- Dark matter. About 95% of the mass of the Galaxy. More on that later.

XXVIII.5.4 Mass-to-Light Ratios

A simple way to get information about the types of stars responsible for the generation of light. Consider the thin disk: $M \simeq 6.5 \times 10^{10} M_{\odot}$ (stars and gas), and (blue) luminosity $L_B = 1.8 \times 10^{10} L_{\odot}$. Divide these (always in solar units!) to get the mass-to-light ratio $M/L_B \approx 3 M_{\odot}/L_{\odot}$ (the units M_{\odot}/L_{\odot} are usually implied).

Recall (or learn...) that a star's luminosity depends strongly on its mass:

$$\frac{L}{L_{\odot}} = \left(\frac{M}{M_{\odot}} \right)^{\alpha} \quad (1)$$

where $\alpha \simeq 4$ for $M \gtrsim 0.5 M_{\odot}$ and $\alpha \simeq 2.3$ for $M \lesssim 0.5 M_{\odot}$. Solve for M and substitute our observed M/L to find an average mass

$$\langle M \rangle = (M/L)^{1/(1-\alpha)} M_{\odot} \simeq 0.7 M_{\odot} \text{ for } M/L = 3 \quad (2)$$

assuming $\alpha = 4$, which indicates that the disk is dominated by stars slightly less massive than the sun. Makes sense, since low mass stars are the most common and M dwarf stars are the most common type of star observed near the Sun.

What would an M/L much, much higher than this mean? What's the M/L of the whole galaxy (total mass $\sim 10^{12} M_{\odot}$, total $L \sim 2 \times 10^{10} L_{\odot}$)? What would a lower M/L mean?

Lecture XXIX Kinematics of the Milky Way

XXIX.2 Differential Galactic rotation

Now we'll look at rotational motion in other parts of the Galaxy. The basic principles of Galactic rotation were determined by Jan Oort in 1927. Different types of rotation:

- Rigid body rotation: rotation as a single solid body, like a wheel, with a constant angular speed and $\Theta \propto R$, so stars on larger orbits would move faster.
- Keplerian rotation: all mass concentrated at the center ($M = \text{constant}$), $\Theta \propto R^{-1/2}$ and stars farther from the center move more slowly.
- Constant orbital speed: $\Theta = \text{constant}$, $M \propto R$. This is a reasonable approximation for most of the Galaxy.

XXIX.3 Measuring Galactic rotation

XXIX.3.1 21-cm H emission

It's hard to see stars to large distance (especially toward the Galactic center) because of dust extinction, so a better way to map the structure of the Milky Way is by looking at gas. We use radio telescopes to look at 21-cm emission from neutral hydrogen (digression: electron spin flip of neutral H), and by looking at the shifts in wavelength of the 21 cm line we measure the radial velocities of gas clouds along a line of sight. We have to use the more general expressions for v_r and v_t for this, since the gas clouds won't necessarily be near the Sun.

Toward the inner part of the Galaxy, radial velocity will be a maximum at the point along the line of sight closest to the Galactic center, and $d = R_0 \cos l$. The line of sight is tangent to the orbit at this point, so the velocity is purely radial and we are measuring the actual rotational velocity at this point. This is called the tangent point, and this is the **tangent point method** of determining Galactic rotation. We measure lots of lines of sight, and assume that clouds with the highest velocities are at the tangent point. For clouds with lower velocities, there is some ambiguity since we don't know the distance to the cloud; can try to estimate it from the angular size, but not all clouds are the same size. Also harder for outer part, since there is no unique orbit with a maximum radial velocity. Really do need to know the distance to the cloud then.

XXIX.3.2 The rotation curve and dark matter

A plot of rotational velocity vs radius is called a rotation curve. Measurements of velocities of objects at $R > R_0$ (Cepheid variables for which we can measure distances... explain Cepheids?) show that the rotational speed $\Theta(R)$ doesn't significantly decrease with distance beyond R_0 . This was a big surprise.... if most of the mass were concentrated in the center, stars would follow

Keplerian orbits and velocity would decrease with increasing radius as $\Theta \propto R^{-1/2}$. Instead the rotation curve is flat to the edge of the measurements. This implies that there is significant mass beyond R_0 —the dark matter halo.

Other spiral galaxies also have rotation curves like this. We'll talk about what this implies about the distribution of mass in the outer parts of galaxies when we talk about spiral galaxies later.

Note also that near the center of the galaxy, the rotation speed rises rapidly with radius. This is consistent with rigid body rotation, and implies that the mass is roughly spherically distributed and the density is nearly constant. Which brings us to the Galactic center.

Lecture XXX The Galactic Center

Kutner 16.6

- Hard to observe because of dust. 30 magnitudes of extinction (10^{-12} of light gets through) in optical, so must observe either at IR wavelengths or longer (radio), or in X-rays or γ -rays.
- Observing in IR shows that number density of stars $n_* \sim 10^7 \text{ pc}^{-3}$. Compare $n \sim 0.1 \text{ pc}^{-3}$ near the Sun.
- Density of stars rises toward the center as $r^{-1.8}$ to a radius between 0.1 and 1 pc; this is roughly the distribution expected from dynamical consideration of the rapidly rising, “rigid body” portion of the rotation curve.
- Strong radio source called **Sagittarius A** at the center of the Galaxy
 - About 8 pc across, with more extended structure to ~ 100 pc
 - Synchrotron emission, produced by relativistic electrons accelerated by a magnetic field
 - Also emission from molecular and ionized gas
 - Complex structure, evidence for recent violent event
- Within Sagittarius A is a highly compact radio source called **Sagittarius A*** (Sgr A*)
 - Angular size measured from radio interferometry: $d \sim 0.8$ milliarcsec ~ 6 AU.
 - Sagittarius A* is also an X-ray source, and varies on a timescale < 1 hour, which means $d < ct$ where t is 1 light-hour; so $d \lesssim 7$ AU.
 - Luminosity (Sgr A* alone) $\lesssim 3 \times 10^4 L_\odot$ (variable...)
- Sagittarius A* is probably an accreting supermassive black hole
 - We can measure the orbits of stars near Sgr A*: e.g. S2 has semimajor axis $a = 920$ AU, period $P = 14.5$ yr. From Kepler’s 3rd law, using this and other stars near Sgr A*, we calculate the enclosed mass:

$$M_{\text{SgrA}^*} = 3.7 \pm 0.2 \times 10^6 M_\odot \quad (3)$$

- All stars are on Keplerian orbits, therefore mass is a single central object
- One star comes within 45 AU of Sgr A*, so size must be smaller than that
- Must be a black hole — only way to get that much mass in that small a volume
- We can calculate the Schwarzschild radius (radius at which the escape velocity is equal to the speed of light; the event horizon of the black hole):

$$R_{\text{SgrA}^*} = \frac{2GM_{\text{BH}}}{c^2} = 0.08 \text{ AU} = 16 R_\odot \quad (4)$$

This is below the ~ 2 AU resolution limit of current observations, but the Event Horizon Telescope is trying to do better in the next few years.

- Where does all the energetic radiation come from? Accretion onto the black hole. Let's check. Virial theorem: total energy of a system is equal to 1/2 the (time-averaged) potential energy, $E_{\text{tot}} = 1/2 U$. Consider a particle of mass M spiraling onto a black hole from an initial radius r_i to a final radius r_f . According to the virial theorem, the energy radiated is half the change in potential energy,

$$E = \frac{1}{2} \left(\frac{GM_{\text{BH}}M}{r_f} - \frac{GM_{\text{BH}}M}{r_i} \right). \quad (5)$$

Assume $r_i \gg r_f$ and $r_f = r_S$, the Schwarzschild radius. Then

$$E = \frac{1}{2} \frac{GM_{\text{BH}}M}{r_S}. \quad (6)$$

Assume luminosity is $L \sim E/t$ and mass accretion rate is $\dot{M} = M/t$, and substitute the expression for r_S :

$$L \sim \frac{E}{t} = \frac{1}{2} \left(\frac{GM_{\text{BH}}}{r_S} \right) (\dot{M}) = \frac{1}{4} \dot{M} c^2. \quad (7)$$

The observed accretion rate is 10^{-3} to $10^{-2} M_{\odot} \text{ yr}^{-1}$. If we assume $\dot{M} \approx 10^{-3} M_{\odot} \text{ yr}^{-1}$ we find $L \sim 3 \times 10^9 L_{\odot}$, much higher than the observed luminosity of Sgr A* of $3 \times 10^4 L_{\odot}$. So first, accretion can easily provide enough energy to account for the observed luminosity, and second, the energy release from accretion isn't as efficient as we've assumed in this order of magnitude calculation.

Lecture XXXI Extra-Galactic Astronomy

Kutner 20.1, 20.2.1

XXXI.2 Units and scales

Galaxies and cosmology deal with sizes and masses that are both very large and very small.

Distance units: AU, distance from Earth to Sun, 1.5×10^{11} m. Not so useful beyond the solar system, so we use parsec (pc), distance at which 1 AU subtends an angle of 1 arcsec: $1 \text{ pc} = 3.1 \times 10^{16} \text{ m} = 3.26 \text{ light years}$. We are 1.3 pc from Proxima Centauri (nearest star) and 8000 pc = 8 kpc from center of Galaxy. For intergalactic distances, we use megaparsec (Mpc): $1 \text{ Mpc} = 3.1 \times 10^{22} \text{ m}$. We are 0.7 Mpc from M31 (Andromeda galaxy) and 15 Mpc from the Virgo Cluster (nearest big cluster of galaxies).

Mass: standard unit of mass is solar mass, M_{\odot} : $1 M_{\odot} = 2.0 \times 10^{30} \text{ kg}$. Mass of Milky Way $\approx 10^{12} M_{\odot}$. Sun also provides standard unit of power or luminosity: $1 L_{\odot} = 3.8 \times 10^{26} \text{ W}$. Total luminosity of Milky Way, $L_{\text{gal}} = 3.6 \times 10^{10} L_{\odot}$. (Approximately how many stars are in the galaxy?)

Time: 1 year = 3.2×10^7 s. In a cosmological context we use Gyr, $1 \text{ Gyr} = 10^9 \text{ yr} = 3.2 \times 10^{16} \text{ s}$. The Universe is 13.7 Gyr old.

XXXI.3 Olbers' Paradox: Why is the sky dark at night?

Named after Heinrich Olbers, who wrote a paper on the subject in 1826, but first proposed by Thomas Digges in 1576.

Let's suppose the universe is infinite and static, as Isaac Newton believed; a universe that isn't infinite will collapse inward due to its own self-gravity. Now let's compute how bright we expect the night sky to be in this infinite universe, where every line of sight ends at a star. Let n be the average number density of stars in the universe, and let L be the average stellar luminosity. The flux received here at Earth from a star of luminosity L at a distance r is given by an inverse square law:

$$f(r) = \frac{L}{4\pi r^2} \quad (8)$$

Now consider a thin spherical shell of stars, with radius r and thickness Δr , centered on the Earth. The intensity of radiation from the shell of stars (that is, the power per unit area per steradian of the sky) will be

$$\Delta J(r) = \frac{L}{4\pi r^2} \cdot n \cdot r^2 \Delta r = \frac{nL}{4\pi} \Delta r. \quad (9)$$

The total intensity of starlight from a shell thus depends only on its thickness, not on its distance from us, since the $1/r^2$ for the flux cancels the r^2 for the area. So if the Universe extends infinitely

far away, we will end up with infinitely many shells, and the total light on Earth will be infinitely bright.

We have shown that the night sky is infinitely bright. Why is this wrong? Discuss.

- Stars have finite size, so we don't actually have an unobstructed line of sight to all stars. But still, each line of sight ends at a star, so sky should have the surface brightness of a typical star.
- Interstellar matter that absorbs starlight? No, because the matter would be heated by starlight until it has the same temperature as the surface of a star, and then it would emit as much light as it absorbs and glow as brightly as the stars.
- Assumed that number density and mean luminosity of stars are constant throughout the universe; distant stars might be less numerous or less luminous than nearby stars.
- Assumed that universe is infinitely large. If universe has size r_{\max} , then the total intensity of starlight we see in the night sky will be $J \sim nLr_{\max}/(4\pi)$. Note that this result will also be found if the universe is infinite in space, but is devoid of stars beyond a distance r_{\max} .
- Assumed that the universe is infinitely old. When we see stars farther away, we're also seeing stars farther back in time. If universe has finite age t_0 , intensity of starlight will be at most $J \sim nLct_0/(4\pi)$. Also applies if stars have only existed for time t_0 .
- Assumed that flux of light from a distant source is given by inverse square law; i.e. we have assumed that the universe obeys laws of Euclidean geometry and that the source of light is stationary with respect to the observer. Einstein showed that universe may not be Euclidean, and if the universe is expanding or contracting then the light will be red or blueshifted to lower or higher energies.
- Primary resolution: universe has a finite age, and the light from stars beyond some distance—called the horizon distance—hasn't had time to reach us yet. First person to suggest this was Edgar Allen Poe in 1848: “Were the succession of stars endless, then the background of the sky would present us an [sic] uniform density ...since there could be absolutely no point, in all that background, at which would not exist a star. The only mode, therefore, in which, under such a state of affairs, we could comprehend the voids which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all.”

XXXI.4 Basic observations

The cosmological principle:

- The Universe is homogenous. There are no preferred locations: the universe looks the same anywhere.

- The Universe is isotropic. There is no preferred direction: the universe looks the same in all directions.

This is only true on large scales; obviously not true on the scale of a person or a planet or even a galaxy or cluster of galaxies. On scales of ~ 100 Mpc, Universe is homogenous and isotropic; this is roughly the scale of superclusters of galaxies and the voids between them.

Also called the Copernican principle, after Copernicus, who determined that the Earth is not the center of the Universe. There is no center.

- Galaxies show a redshift proportional to their distance.

Consider light at a particular wavelength observed from a distant galaxy: λ_{obs} is the observed wavelength of some absorption or emission feature in the galaxy's spectrum. λ_{em} is the wavelength at which that feature is measured on Earth. In general, $\lambda_{\text{obs}} \neq \lambda_{\text{em}}$; the galaxy has a **redshift** z given by

$$z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}. \quad (10)$$

Most galaxies have redshifts—this is because the Universe is expanding.

In 1929 Edwin Hubble plotted galaxy redshifts against their distance (redshifts are easy to measure, but distances are hard), and showed that the redshift of a galaxy was linearly proportional to its distance. This is now known as Hubble's law:

$$z = \frac{H_0}{c} r, \quad (11)$$

where H_0 is a constant now called the **Hubble constant**.

If we interpret redshifts as Doppler shifts (not strictly true, but we'll talk about that later), $z = v/c$ and Hubble's law takes the form

$$v = H_0 r, \quad (12)$$

where v is the radial velocity of the galaxy. Therefore we can get the Hubble constant from dividing velocity by distance, and it has the units $\text{km s}^{-1} \text{Mpc}^{-1}$ (note that the actual units are inverse time). Hubble's original estimate was $H_0 = 500 \text{ km s}^{-1} \text{Mpc}^{-1}$, but he was severely underestimating the distances to galaxies. The Hubble constant has been measured with precision only in the last ~ 10 years; the best current value is $H_0 = 70.4_{-1.4}^{+1.3} \text{ km s}^{-1} \text{Mpc}^{-1}$ (2010, from WMAP seven-year results, with priors from other estimates).

We can also use the Hubble constant to define a time. If galaxies are moving apart from each other, they must have been together at some point in the past. Consider two galaxies separated by a distance r and moving at a constant velocity v with respect to each other. The time elapsed since the galaxies were in contact is

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1}, \quad (13)$$

independent of r . The time H_0^{-1} is called the **Hubble time**, and is an approximate timescale for the age of the Universe (it is only equal to the age of the Universe if galaxy velocities were the same at all times in the past). $H_0^{-1} = 13.8$ Gyr.

Let's use this to return to Olbers' paradox. If the universe is of finite age, $t_0 \sim H_0^{-1}$, then the night sky can be dark, even if the universe is infinitely large, because light from distant galaxies has not yet had time to reach us. Galaxy surveys tell us that the luminosity density of galaxies in the local universe is

$$nL \approx 2 \times 10^8 L_\odot \text{ Mpc}^{-3} \quad (14)$$

This luminosity density is equivalent to a single 40 watt light bulb within a sphere 1 AU in radius. If the horizon distance is $r_H \sim c/H_0$, then the total flux of light we receive from all the stars from all the galaxies within the horizon will be

$$F_{\text{gal}} \sim nL \frac{c}{H_0} \sim 9 \times 10^{11} L_\odot \text{ Mpc}^{-2} \sim 2 \times 10^{-11} L_\odot \text{ AU}^{-2}. \quad (15)$$

By the cosmological principle, this is the total flux of starlight you would expect at any randomly located spot in the universe. Comparing this to the flux we receive from the Sun,

$$F_{\text{sun}} = \frac{1 L_\odot}{4\pi \text{AU}^2} \approx 0.08 L_\odot \text{ AU}^{-2}, \quad (16)$$

we find that $F_{\text{gal}}/F_{\text{sun}} \sim 3 \times 10^{-10}$. Thus, the total flux of starlight at a randomly selected location in the universe is less than a billionth the flux of light we receive from the Sun here on Earth. For the entire universe to be as well-lit as the Earth, it would have to be over a billion times older than it is, and you'd have to keep the stars shining during all that time.

Lecture XXXII Types of Galaxies and the Hubble Sequence

Kutner 17.1

XXXII.2 Are there other galaxies outside the Milky Way?

Yes.

XXXII.3 The Hubble sequence

Once it was known that there were other galaxies out there, we needed to classify them. Goals of a galaxy classification scheme:

- Impose order
- Reveal correlations between properties or evolution
- Classification should be complete—include every galaxy
- Classification should be economical—don't include irrelevant details (how do we know which details are irrelevant?)

In 1926, Edwin Hubble suggested a classification scheme which is still used today. This is a morphological classification scheme which divides galaxies into categories based on their overall appearance. Three general categories, with subdivisions:

- Ellipticals (E)
- Spiral, divided into normal spirals (S) and barred spirals (Sb)
- Irregular

Galaxies intermediate between spirals and ellipticals are called lenticular (S0 or Sb0).

The types of galaxies were arranged in a diagram shaped like a tuning fork. A galaxy's classification according to this scheme is called its Hubble type.

Hubble originally thought that this might be an evolutionary sequence, and therefore called elliptical galaxies *early types* and spiral galaxies *late types* (with Sc later than Sa). This isn't true, but astronomers still use the early and late terms to describe these types of galaxies.

XXXII.3.1 Elliptical galaxies

Elliptical galaxies are classified according to their observed **ellipticity**. We measure the major axis a and minor axis b , and define the ellipticity

$$\epsilon \equiv 1 - b/a. \quad (17)$$

We classify the galaxy as En , where $n = 10(1 - b/a)$, rounded to the nearest integer.

Wide variety of elliptical galaxies: giant ellipticals are the biggest galaxies in the universe, smallest dwarfs comparable in size to globular clusters. Mass range $\sim 10^7$ to $\sim 10^{13} M_{\odot}$.

XXXII.3.2 Spiral galaxies

Features used to classify spiral galaxies:

- Bulge-to-disk ratio, B/D , the ratio of the luminosities of the bulge and the disk. Largest $L_{\text{bulge}}/L_{\text{disk}} \sim 0.3$.
- Smoothness of the distribution of stars. Reflects current star formation, since bright spots are regions where hot, bright young stars have just formed.
- Pitch angle of spiral arms: how tightly wound the spiral arms are

Galaxies with the largest bulge-to-disk ratios, smoothest stellar distributions, and most tightly wound spiral arms are Sa (or Sba). Sc (and Sbc) galaxies have smaller B/D ($L_{\text{bulge}}/L_{\text{disk}} \sim 0.05$, loosely wound spiral arms, and the spiral arms are clumpy, can be resolved into stars and HII regions (HII, ionized hydrogen—compare HI. Hot young stars ionize the gas around them, and these are called HII regions.)

Spiral galaxies are large, with masses $\sim 10^9$ to $\sim 10^{12} M_{\odot}$, and don't vary as much as elliptical galaxies. M31 (Andromeda) is Sb. Milky Way is probably SBbc.

XXXII.3.3 Irregular galaxies

Remaining galaxies were called irregular, placed off to the side of the diagram. These are usually not very large, but can vary widely.

XXXII.4 General trends

- Spiral galaxies are generally blue, ellipticals red
- Spirals have more dust and gas than ellipticals
- Most current star formation occurs in spirals
- These things are related!

Lecture XXXIII Spiral Galaxies I

Goal: understand the 3-d structure of galaxies—how do we get to this from what we observe?

XXXIII.2 Kinematics of spiral galaxies

Measuring the surface brightness profiles of galaxies tells us about the distribution of luminous matter, but not about the total distribution of mass. In order to measure this, we need to study the kinematics of galaxies.

We saw that the rotation curve of the Milky Way flattens at large radii, with a constant rotational velocity of $\sim 220 \text{ km s}^{-1}$. In the 1970s and 80s Vera Rubin showed that spiral galaxies generally have flat rotation curves (surprise!), and that therefore there is a lot of mass beyond the luminous matter we can see.

What do flat rotation curves tell us about the mass distribution? Consider a particle of mass m , in a circular orbit at radius r around a spherically symmetric mass distribution M . The gravitational force on the particle is

$$F_{\text{grav}} = \frac{GMm}{r^2} \quad (18)$$

and the centripetal force is

$$F_c = \frac{mv_c^2}{r} \quad (19)$$

where v_c is the (constant with radius!) circular velocity of the particle. These forces are equal:

$$v_c^2 = \frac{GM}{r}. \quad (20)$$

We can use this to determine the dark matter density profile in the outer parts of the galaxy.

Lecture XXXIV Spiral Galaxies II

Kutner 17.3

Optical images of spiral galaxies are dominated by the arms, especially in blue light, because massive, hot (O and B) stars are found in the spiral arms. These stars live for only ~ 10 Myr, so their presence indicates active star formation. The bulk of disk of the galaxy is dominated by older, redder stars. Spiral arms also have gas and dust.

XXXIV.2 Trailing and leading spiral arms

There are two possibilities for how the spiral arms can be oriented with respect to the rotation of the galaxy. The tips of **trailing arms** point in the opposite direction from the direction of rotation, and **leading arms** are the opposite. Intuitively, it looks as if the arms should be trailing, and this is the case in most galaxies in which it can be measured. A few galaxies have a combination of leading and trailing arms, however; this is probably a result of an encounter with another galaxy.

XXXIV.3 The winding problem

An obvious suggestion is that spiral structure is due to galactic rotation. Differential rotation, with stars closer to the center of the galaxy moving faster, will naturally generate spiral arms. Problem: after a few orbits, the arms will be too tightly wound to be observed (recall Sun has orbited Milky Way ~ 20 times). This is called the **winding problem**. This also shows that the spiral arms can't be fixed components of the same stars and gas.

XXXIV.4 The origin of spiral structure: density waves

Leading theory for the origin of spiral structure is the **Lin-Shu density wave theory**. This says that spiral structure is due to long-lived **quasistatic density waves**. These are regions in the galactic disk where the mass density is ~ 10 to 20% higher than average. Stars and gas clouds move through these regions of enhanced density as they orbit around the center of the galaxy. This is like cars moving through a traffic jam—the density increases in the traffic jam, and the cars slowly move through it, but the traffic jam itself doesn't move (much).

This theory explains observations:

- Star formation is concentrated in spiral arms: as gas in the galaxy passes through the density wave it's compressed, and this increase in density makes it more likely to collapse and form stars.
- Distribution of star formation: collapse of gas into stars takes some time, so star formation will be observed somewhat downstream from the leading edge of the spiral arm.
- O and B stars concentrated in spiral arms, red stars distributed throughout disk: The most massive O and B stars don't live very long, so they don't have time to pass entirely through

the spiral arm and become generally distributed throughout the galaxy. Longer-lived, redder stars do.

Lecture XXXV Elliptical Galaxies

According to the Hubble sequence, elliptical galaxies are classified only by their degree of observed ellipticity. This turns out to be not particularly useful; unlike the properties used to classify spirals, the observed ellipticity shows very little correlation with other properties of the galaxies.

Elliptical galaxies have a much wider range in mass than spirals. A lot of different types:

- **Cd galaxies.** These are the biggest and most massive galaxies in the universe. Rare, found near the centers of large galaxy clusters. Can be nearly 1 Mpc across, have masses between 10^{13} and $10^{14} M_{\odot}$. Very high mass-to-light ratios, implying lots of dark matter.
- **Normal ellipticals.** Centrally condensed, high central surface brightness. Mass range 10^8 to $10^{13} M_{\odot}$, sizes from < 1 kpc to almost 200 kpc. M/L ranges from 7 to > 100 .
- **Dwarf elliptical (dE).** Lower surface brightness than normal ellipticals, mass range 10^7 to $10^9 M_{\odot}$, sizes ~ 1 –10 kpc.
- **Dwarf spheroidal (dSph).** Very low luminosity, low surface brightness; detected only near the Milky Way. Masses $\sim 10^7$ – $10^8 M_{\odot}$, diameters 0.1 to 0.5 kpc.
- **Blue compact dwarf galaxies (BCD).** Small and unusually blue, indicating rapid star formation. Masses $\sim 10^9 M_{\odot}$, sizes < 3 kpc. Large gas masses and low M/L , consistent with their high star formation rates.

XXXV.2 Dynamics of elliptical galaxies

Unlike spiral galaxies, elliptical galaxies generally aren't coherently rotating—the stars are on **random orbits**. To measure the velocities of a system like this, we use the **velocity dispersion** σ^2 . This is just the standard deviation (**draw**) squared.

This is measured from the widths of absorption or emission lines in the spectrum of a galaxy—the light from each star will be Doppler shifted due to the velocity of that star, so for a wider range of stellar velocities, the total width of a line produced by stars will be broader.

How do we estimate the mass of a system like this?

Virial theorem

$$-2K = U \quad (21)$$

For a galaxy of N identical stars of mass m and with no coherent motions, average velocity is 0, so the kinetic energy is

$$K = \frac{M_{\text{tot}}\sigma^2}{2}$$

Potential energy?

$$U = -\frac{3}{5} \frac{GM_{\text{tot}}^2}{R}$$

The $3/5$ factor is a little tricky, but the rest is easy.

So

$$-2 \left[\frac{M_{\text{tot}} \sigma^2}{2} \right] = \frac{3}{5} \left(\frac{GM_{\text{tot}}^2}{R} \right) \quad (22)$$

for a spherical, constant density galaxy (can work out more accurate versions for any $\rho(r)$).

Rearranging:

$$M_{\text{tot}} = \frac{5R}{G} \left[\frac{\sigma^2}{3} \right] \quad (23)$$

Bigger mass \Rightarrow Larger velocity dispersion.

The mass obtained this way is called the **virial mass**.

Lecture XXXVI Galaxy Clusters

Kutner 18

Galaxies are not distributed randomly throughout the universe; they are usually found in associations called **groups** or **clusters**. In both cases, the galaxies are gravitationally bound to each other and orbit the system's center of mass.

XXXVI.2 Classification of clusters

XXXVI.2.1 Galaxy groups

- Usually have less than 50 members
- About 1.4 Mpc across
- Galaxies of the group have velocity dispersion $\sim 150 \text{ km s}^{-1}$ (this now refers to the velocity of galaxies relative to each other, **not** to the velocity of stars within galaxies)
- Mass of an average group $\sim 2 \times 10^{13} M_{\odot}$, from the virial theorem
- Typical mass-to-light ratio $\sim 260 M_{\odot}/L_{\odot} \Rightarrow$ lots of dark matter

XXXVI.2.2 Galaxy clusters

- Contain between ~ 50 to thousands of galaxies
 - Few galaxies \Rightarrow **poor** cluster
 - Lots of galaxies \Rightarrow **rich** cluster
- Higher velocity dispersion than in a group. Characteristic velocity dispersion is 800 km s^{-1} , may exceed 1000 km s^{-1} for very rich clusters
- Typical virial mass $\sim 10^{15} M_{\odot}$
- Typical mass-to-light ratio $\sim 400 M_{\odot}/L_{\odot} \Rightarrow$ even more dark matter than in groups
- Further classified as **regular** (spherical and centrally condensed) and **irregular**

XXXVI.3 The Local Group

About 35 galaxies are known to lie within ~ 1 Mpc of the Milky Way — this collection of galaxies is called the Local Group. Most prominent members are the three spiral galaxies, the Milky Way, Andromeda (M31), and M33. The next most luminous are the Large and Small Magellanic Clouds,

irregular galaxies near the Milky Way; the LMC is 48 kpc away, and the SMC is about 60 kpc away (can see them from the Southern hemisphere). They are two of the 13 irregular galaxies in the Local Group. The remaining galaxies are dwarf ellipticals or dwarf spheroidals, very small and very faint (which means they're hard to see; there may be some we haven't found yet). Also notable is the Magellanic Stream, a long ribbon of gas tidally stripped from the Magellanic Clouds.

Most of the galaxies in the Local Group are clustered around the Milky Way and Andromeda (see Figure 1), which are on opposite sides of the Local Group about 770 kpc apart. The center of mass of the group is between them.

Andromeda and the Milky Way are approaching each other with a velocity of 119 km s^{-1} (the gravitational attraction between them is strong enough to overcome the Hubble flow, i.e. the expansion of the universe). We don't know whether or not they will actually collide because we can't measure the transverse velocity of Andromeda; indirect constraints suggest it's $\sim 100 \text{ km s}^{-1}$ or less. Neglecting the transverse velocity, we can calculate that they will collide in about $t_c = d/v = 6.3$ billion years (more careful estimates suggest that at least the dark matter halos of the galaxies will collide). This is an overestimate because the galaxies will accelerate as they approach.

There are other groups of galaxies within 10 Mpc of the Local Group—about 20 small groups of galaxies closer to us than the Virgo cluster. Most galaxies live in small groups and poor clusters; probably at most 20% of galaxies live in rich clusters like the Virgo cluster.

XXXVI.4 The Virgo Cluster and the Coma Cluster

The nearest rich clusters are the Virgo cluster and the Coma cluster.

XXXVI.4.1 The Virgo cluster

- Covers $10^\circ \times 10^\circ$ region of the sky. Huge!
- Center about 16 Mpc away.
- Contains about 250 large galaxies and > 2000 smaller ones, within a region about 3 Mpc across.
- Mix of galaxy types: 4 brightest are giant ellipticals, spirals dominate overall, but ellipticals become increasingly common near the center of the cluster. Why?
- M87, giant E1 elliptical, brightest galaxy in the Virgo cluster. Mass $\sim 3 \times 10^{13} M_\odot$, $M/L \simeq 750 M_\odot/L_\odot$. Over 99% dark matter! Not a normal elliptical galaxy.

XXXVI.4.2 The Coma Cluster

Another rich cluster, about five times farther away than Virgo. Provided first evidence of dark matter—in 1933 Fritz Zwicky measured the radial velocities of galaxies in the Coma cluster, calculated the velocity dispersion, which we now know is $\sigma = 977 \text{ km s}^{-1}$, and used the virial

theorem to estimate the cluster's mass. With radius 3 Mpc:

$$M \approx \frac{5\sigma^2 R}{G} = 3.3 \times 10^{15} M_{\odot} \quad (24)$$

Luminosity is about $5 \times 10^{12} L_{\odot}$, so $M/L \approx 660 M_{\odot}/L_{\odot}$. Zwicky understood that this was strange, and wrote that the mass of the cluster considerably exceeds the sum of the masses of the individual galaxies. This was the first recognition of dark matter.

(More Zwicky: with Walter Baade, proposed the existence of neutron stars in 1934, only a year after the discovery of the neutron, and proposed that supernovae were transitions of normal stars into neutron stars. Also proposed that galaxy clusters could be gravitational lenses. And other, crazier stuff.)

XXXVI.5 Hot, Intracluster Gas

Some of the missing mass in galaxy clusters was discovered when the first X-ray satellites were launched (1977). It was observed that many clusters emit X-rays, from much of their volume. This revealed the **intracluster medium**: a diffuse, irregular collection of stars, and **hot intracluster gas**.

The X-rays are produced by **thermal bremsstrahlung emission** (braking radiation, also called free-free emission). This is emission which occurs when a free electron passes near an ion, emits a photon, and slows down (the ion is necessary for conservation of energy and momentum). Bremsstrahlung radiation has a characteristic, easily identifiable spectrum, and the luminosity density (luminosity per unit volume) depends on the electron density and the temperature:

$$\mathcal{L}_{\text{vol}} = 1.42 \times 10^{-40} n_e^2 T^{1/2} \text{ W m}^{-3}. \quad (25)$$

(Important parts: $\mathcal{L}_{\text{vol}} \propto n_e^2 T^{1/2}$) The temperature can be measured from the X-ray spectrum of the gas; for the Coma cluster it is 8.8×10^7 K. The total X-ray luminosity of the cluster is

$$L_x = \frac{4}{3} \pi R^3 \mathcal{L}_{\text{vol}}. \quad (26)$$

We can measure R , L_x and T , so these two equations can be combined to solve for n_e . For the Coma cluster, $n_e = 300 \text{ m}^{-3}$. Very diffuse!

We can then compute the total mass of the gas:

$$M_{\text{gas}} = \frac{4}{3} \pi R^3 n_e m_H \quad (27)$$

since there is one proton for every electron. For the Coma cluster, $M_{\text{gas}} = 1.05 \times 10^{14} M_{\odot}$. This is a lot of gas, but much less than the total cluster mass $3.3 \times 10^{15} M_{\odot}$ we computed earlier.

Summary: visible light (1%), gas (9%), dark matter (90%)

(next lecture: types and colors of galaxies in clusters, galaxy interactions in clusters, galaxy interactions generally)

Lecture XXXVII Active Galaxies I

Kutner 19

A few percent of galaxies are peculiar in that they produce huge amounts of energy in excess of the normal stellar light. These are called **active galaxies** or **AGN (Active Galactic Nuclei)**.

Distinctive characteristics of active galaxies:

- Large amounts of nonstellar emission, some of it nonthermal in origin. Active galaxies produce more X-ray and radio emission than would be produced by their stars.
- Much of the light is concentrated in a small, central region called an active galactic nucleus, AGN
- Light from AGNs is variable on short timescales, at virtually all wavelengths. Timescale for variability depends on luminosity and wavelength, with most rapid variability seen at short wavelengths and low luminosities. X-rays in low luminosity AGNs can vary on timescales of minutes
- Some active galaxies have jets detectable at X-ray, visible and radio wavelengths. The jets contain ionized gas flowing outward at relativistic speeds
- The UV, visible and IR spectra of AGNs are dominated by strong emission lines

Not all active galaxies have all of these features.

Accumulated evidence indicates that the activity in AGN comes from accretion onto massive black holes. Most bright galaxies have black holes in their centers, but not all bright galaxies are active galaxies. To be an active galaxy, the central black hole must be accreting gas rapidly enough to produce luminosity as bright or brighter than the galaxy's stars. We will start by discussing the different types of active galaxies. The situation initially seems very complicated, but we'll see that most of the observations can be explained by a unifying scenario.

XXXVII.2 Variability and physical size

Flux from AGN can vary significantly on very short timescales

- Luminosity of broad lines and continuum can vary by a factor of ~ 2 on day to month timescales
- Variations in broad emission lines typically lag behind those of continuum by ~ 1 month
- Variations of a few percent in visible, X-rays. X-ray flux can vary on timescales of minutes

AGN with rapid time variability are often called **BL Lac objects** (after prototype BL Lacertae) or **blazars**. Changes in flux can be up to $\sim 30\%$ in a day, up to $\sim 100\%$ over longer periods. Nearly devoid of emission lines (continuum dominates flux)

The timescale of variability puts limits on the size of the source. A source with size R will take a time of at least $\Delta t = R/c$ to change its luminosity, since information can't travel from one side of the source to the other faster than c . If we observe $\Delta t = 1$ hour,

$$R \simeq c\Delta t = 7.2 \text{ AU}. \quad (28)$$

This is very small for something so energetic!

XXXVII.3 Accretion by supermassive black holes

XXXVII.3.1 Energetics

The release of gravitational potential energy through mass accretion is a very efficient way to generate energy. Consider a mass m falling a large distance $r \gg r_{\text{Sch}}$ toward the Schwarzschild radius of a black hole. Recall

$$r_{\text{Sch}} = \frac{2GM_{\text{bh}}}{c^2}. \quad (29)$$

The loss of gravitational potential energy will be

$$\Delta E = -\frac{GM_{\text{bh}}m}{r} + \frac{GM_{\text{bh}}m}{r_{\text{Sch}}} \approx \frac{GM_{\text{bh}}m}{r_{\text{Sch}}} \approx \frac{1}{2}mc^2. \quad (30)$$

If the mass doesn't stop before it reaches the Schwarzschild radius it will pass the event horizon with speed $v \sim c/\sqrt{2}$, and its kinetic energy will increase the mass of the black hole. If instead it's decelerated by an accretion disk, its kinetic energy will be converted into thermal energy and then radiation. This process isn't perfectly efficient, and we write the energy carried away by photons as

$$\Delta E_{\text{phot}} = \eta mc^2 \quad (31)$$

where η is a dimensionless number called the efficiency of the black hole. We expect $\eta \leq 1/2$ from Equation 3 above. In practice, we think $\eta \approx 0.1$, which means that a gram of matter falling toward the black hole gives 9 trillion joules of radiation energy (!).

As gas falls into the black hole at rate \dot{M} , the **accretion luminosity** of the AGN is

$$L = \eta \dot{M} c^2. \quad (32)$$

We can therefore estimate an AGN's accretion rate \dot{M} from its luminosity:

$$\dot{M} = \frac{L}{\eta c^2} = 0.018 M_{\odot} \text{ yr}^{-1} \left(\frac{L}{10^{37} \text{ W}} \right) \left(\frac{\eta}{0.1} \right)^{-1}. \quad (33)$$

We don't expect \dot{M} to be constant with time, which probably accounts for some of the variability of AGN.

Lecture XXXVIII The Extragalactic Distance Scale

Kutner 18

In order to study the structure of the universe, we need to be able to measure how far away things are. We also need to measure distances in order to measure the Hubble constant: $v = H_0 d$, so we need to measure both v (easy, for things that are far enough away so that peculiar velocities don't matter) and d (hard).

There are many methods of measuring distances, making up what's called the **extragalactic distance scale** or **cosmological distance ladder**. For objects within ~ 1 kpc we can measure **trigonometric parallaxes**, but beyond that we need other methods. There are many; we'll cover some of the more important here.

XXXVIII.2 Extragalactic distance indicators

Most astronomical distance indicators are what are called "standard candles." A standard candle is an object with a fixed luminosity, or with some other property we can measure that will tell us what its luminosity is. If we measure the apparent brightness (the flux) and know the absolute brightness (the luminosity) we can determine the distance.

Also used but less common are "standard rulers," objects with a fixed physical size; if we can measure the angular size and know the physical size, we can determine the distance.

XXXVIII.2.1 Cepheid variables

Variable stars whose period is directly related to their luminosity, discovered by Henrietta Swan Leavitt in the early 20th century. Stars pass through the **instability strip** on the HR diagram during late phases of stellar evolution (**show**). Luminosity directly related to period: if you know the period you know the luminosity. So this works as a standard candle.

Most distant known Cepheids are 29 Mpc away.

XXXVIII.2.2 Type Ia supernovae lightcurves

The most important extragalactic distance indicator, because it reaches to the largest distances. Review: what's a Type Ia supernova? Standard model (in textbook) is that a CO white dwarf accretes material from a companion which pushes it over the Chandrasekhar limit and causes it to explode. Recent work suggests that there are some problems with this scenario; almost certainly have to do with explosion of a CO white dwarf, but it could be a merger of two or some other scenario.

Whatever the cause, the useful fact for measuring distances is that the brightnesses and light curves (a plot of luminosity vs. time) of Type Ia SNe are very similar. They have average absolute magnitudes at maximum light of $\langle M_B \rangle \simeq \langle M_V \rangle \simeq -19.3 \pm 0.03$. Note: small scatter, and very bright;

this is as bright as an entire galaxy.

What we actually measure is the brightness of the supernova at various points in time, and there is a well-defined inverse correlation between the maximum luminosity and the rate of decline of the light curve: brighter supernova take longer to decline. We can use this to determine the intrinsic peak luminosity.

Type Ia supernovae can be used to measure distances to > 1000 Mpc ($z \sim 0.25$) with an uncertainty of $\sim 5\%$. We will return to these, since they are very important to the measurement of dark energy and the acceleration of the expansion of the universe.

XXXVIII.2.3 Summary

This has not been a comprehensive discussion! There are more methods in Chapter 18 of the text, and even more that aren't in the book. Main message: the different methods work best for different types of galaxies and to different distances, and have different uncertainties associated with them. We need to use as many different methods as possible to calibrate the various methods and understand their uncertainties.

Lecture XXXIX The Expansion of the Universe

XXXIX.2 The expansion of the universe

In 1929 Hubble published the famous paper “A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae,” showing that galaxies had recessional velocities proportional to their distance, $v = H_0 d$ and that therefore the universe is expanding. What does this actually mean?

Suppose the Earth doubles in size in an hour. Right now, from Milwaukee, it’s 500 miles to Pittsburgh, 1000 miles to Dallas, and 2000 miles to Seattle. After the expansion, it will be 1000 miles to Pittsburgh, 2000 miles to Dallas, and 4000 miles to Seattle, so the expansion velocities we will observe will be 500 miles an hour for Pittsburgh, 1000 miles an hour for Dallas, and 2000 miles an hour for Seattle. The velocity of expansion is directly proportional to the distance; this is a result of expansion that is isotropic and homogeneous. Note that we would observe the same thing no matter where we were, because every point is moving away from every other point. The expansion has no center.

Galaxy redshifts are often described as Doppler shifts, but this isn’t strictly correct. The redshift is due not to the galaxy moving through space, but because of the expansion of space itself. It is a **cosmological redshift**, not a Doppler shift; the redshift is produced by the expansion of the universe, as the wavelength of light is stretched along with space. The motion of galaxies due to the expansion of the universe is called the **Hubble flow**. Galaxies also have **peculiar velocities**, which are their velocities through space, independent of the Hubble flow. Also important to note that gravitationally bound structures (galaxies, clusters of galaxies) do not participate in the expansion.

XXXIX.2.1 The Hubble constant: review

The Hubble constant is measured using as many different independent distance indicators as possible, and through other tests of cosmological parameters we’ll talk about later. For most of the 20th century, the Hubble constant was only known to be between 50 and 100 $\text{km s}^{-1} \text{Mpc}^{-1}$ (and there were two distinct camps with very strong opinions about whether it was 50 or 100).

In conventional units, the Hubble constant is inverse time:

$$H_0 = 2.30 \times 10^{-18} \text{ s}^{-1}, \quad (34)$$

To estimate how long ago the Big Bang occurred, we assume (incorrectly) that the recessional velocities of galaxies is constant. We call the time since the Big Bang t_H ; this is the time required for a galaxy to travel to a distance d at speed v . So

$$d = vt_H = H_0 dt_H \quad (35)$$

and the **Hubble time** t_H is

$$t_H = \frac{1}{H_0} = 4.35 \times 10^{17} \text{ s} = 1.38 \times 10^{10} \text{ yr}. \quad (36)$$

So the universe is about 13.8 Gyr old. This is a pretty good estimate.

Lecture XL Newtonian Cosmology

Now we will describe the evolution of the universe mathematically. This will take some time, but it isn't too complicated when we consider only gravitational forces; the evolution of the universe is essentially a battle between expansion and gravity.

The universe we consider is homogeneous and isotropic, with constant density ρ (the textbook calls this a universe of pressureless dust, and it's a simple, special case). Consider, as we often do, a spherical shell of radius r and mass m . The shell is expanding due to the expansion of the universe, with velocity v .

As the universe expands, the mass inside the shell $M(< r)$ is **constant**—the expansion is uniform, so mass that starts within the shell will always be within the shell. So $\rho r^3 = \text{constant}$.

Because the mass is uniformly distributed, mass outside the shell doesn't matter.

The total energy of the shell as it expands is the sum of its kinetic and potential energies:

$$\frac{1}{2}mv^2 - \frac{GM(< r)m}{r} = E \quad (37)$$

Now let's parameterize the total energy of the shell:

$$E = -\epsilon \frac{1}{2}mc^2, \quad (38)$$

where ϵ is just a dimensionless number which we'll rewrite later. So

$$\frac{1}{2}mv^2 - \frac{GM(< r)m}{r} = -\epsilon \frac{1}{2}mc^2. \quad (39)$$

Next we substitute $M(< r) = (4\pi/3)r^3\rho$, cancel the shell mass m , and multiply by 2:

$$v^2 - \frac{8\pi}{3}G\rho r^2 = -\epsilon c^2. \quad (40)$$

Now let's return to our dimensionless number ϵ , and rewrite it as

$$\epsilon \equiv k\varpi^2. \quad (41)$$

More on ϖ later. k contains information about whether the shell is **gravitationally bound or unbound**.

- Bound: $k > 0$. Total energy $E < 0$, so shell can recollapse. We call this **closed**.
- Unbound: $k < 0$. Total energy $E > 0$, so kinetic energy wins and shell keeps expanding. We call this **open**.
- $k = 0$: **Critical** or **flat**. Perfect balance between expansion and gravitational resistance.

So, we have

$$v^2 - \frac{8\pi}{3}G\rho r^2 = -kc^2\varpi^2. \quad (42)$$

Now we write the radius r in terms of a global **scale factor** $R(t)$, which describes the overall expansion of the universe:

$$r(t) = R(t)\varpi \quad (43)$$

$R(t)$ is called the “scale factor” of the universe, and ϖ is called the **comoving coordinate**. The comoving coordinate describes the distance between objects, but it stays constant as the universe expands; this is a useful way to talk about distances when the whole universe is expanding.

For instance, at time t_1 , two galaxies are separated by a distance $r(t_1) = R(t_1)\varpi$, and at t_2 they are separated by a distance $r(t_2) = R(t_2)\varpi$. Their comoving separation stays the same, but their physical separation r increases due to the increasing scale factor $R(t)$.

XL.2 Cosmological redshift

The evolution of the scale factor affects more than galaxies. The expansion of the universe also changes the wavelength of light; light stretches as the universe expands.

Suppose light is emitted with wavelength $\lambda_1 = \lambda_{\text{em}}$ at some time $t_1 \ll t_0$, when the universe was smaller than its current size—say $R_1 = 1/3$. At some later time t_2 the universe has doubled in size to $R_2 = 2/3$, changing the wavelength of light along with it, so the wavelength of the light is also twice as big, $\lambda_2 = 2\lambda_{\text{em}}$. We detect the light at time t_0 , when $R_0 = 1 = 3R_1$, and the wavelength of the light is now $\lambda_3 = 3\lambda_{\text{em}}$. So, recalling the definition of redshift,

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} \equiv (1 + z) = \frac{R(t_0)}{R(t_1)}. \quad (44)$$

For this example, the redshift $z = 2$.

So the redshift and scale factor are directly related, and when we talk about the history of the universe we usually just parameterize it in terms of the redshift we would measure today for photons which were emitted at some earlier time $t < t_0$ when the universe was more compact.

$$(1 + z) = \frac{R(t_0)}{R(t_1)}, \quad (45)$$

so

$$R(t) = \frac{1}{(1 + z)}, \quad (46)$$

independent of any of the cosmological parameters like H_0 or Ω_0 . So, for example, at a redshift $z = 3$, the universe was 1/4 of its current size.

Now we rewrite the expansion velocity

$$v = \varpi V \quad (47)$$

where V is the rate at which R is changing. Substitute this and $r = R\varpi$:

$$\varpi^2 V^2 - \frac{8\pi}{3} G\rho R^2 \varpi^2 = -kc^2 \varpi^2. \quad (48)$$

We cancel ϖ :

$$\boxed{V^2 - \frac{8\pi}{3} G\rho R^2 = -kc^2} \quad (49)$$

This is an expression for the evolution of only the scale factor R — no more references to specific shells or coordinates!

Now we'll do a few more things. Divide out R^2 from the left side:

$$\left[\left(\frac{V}{R} \right)^2 - \frac{8\pi}{3} G\rho \right] R^2 = -kc^2 \quad (50)$$

We can choose the normalization of $R(t)$ in any way we like. For convenience, let's set the scale factor today equal to 1: $R(t_0) = 1$ (t_0 = time today—0 subscript refers to the value of something at the present time, e.g. H_0 is the value of the Hubble constant (which isn't constant) today, and ρ_0 is the density of the universe today). So

$$\left[\frac{V(t_0)}{R(t_0)} \right]^2 - \frac{8\pi}{3} G\rho_0 = -kc^2 \quad (51)$$

at the present time.

Now let's look at the first term. According to the Hubble law,

$$v(t) = H(t)r(t) = H(t)R(t)\varpi. \quad (52)$$

Also,

$$v(t) = \varpi V(t) \quad (53)$$

Equating these two expressions for v , we find an expression for the Hubble constant at any time t :

$$H(t) = \frac{V(t)}{R}. \quad (54)$$

So in general,

$$H(t) = \frac{V(t)}{R(t)}, \quad (55)$$

and at the present day

$$H_0 = \frac{V(t_0)}{R(t_0)}. \quad (56)$$

We recognize this as the left term in Equation 51 and substitute it in:

$$\boxed{H_0^2 - \frac{8\pi}{3} G\rho_0 = -kc^2} \quad (57)$$

In order from left to right, the three terms in this equation describe the kinetic, potential and total energies.

Now let's return to that "critical" universe with $k = 0$, where expansion and gravity are perfectly balanced. We can calculate the density we need to make this happen. This is the **critical density** ρ_c . If $k = 0$,

$$H_0^2 - \frac{8\pi}{3}G\rho_c = 0 \quad (58)$$

and

$$\boxed{\rho_c = \frac{3H_0^2}{8\pi G}} \quad (59)$$

This is about $9.5 \times 10^{-27} \text{ kg m}^{-3}$. For comparison, our best estimate of the density of baryonic¹ matter is $4.17 \times 10^{-28} \text{ kg m}^{-3}$; this is about 4% of the critical density.

If $\rho_0 < \rho_c$, there isn't enough mass to reverse the expansion and the universe is open. If $\rho_0 > \rho_c$, gravity wins and the universe is closed.

We parameterize the density of the universe in terms of a dimensionless number Ω :

$$\Omega_0 \equiv \frac{\rho_0}{\rho_c} \quad (60)$$

- $\Omega_0 = 1$: flat universe with $k = 0$
- $\Omega_0 < 1$: open universe with $k < 0$
- $\Omega_0 > 1$: closed universe with $k > 0$

We can then also rewrite Equation 23 as

$$H_0^2(\Omega_0 - 1) = kc^2 \quad (61)$$

at the present day.

¹Baryons are particles made of three quarks. Protons and neutrons are the most common. Baryons make up nearly all of the visible matter in the universe. (Electrons are leptons—not composed of quarks.) Most dark matter is probably non-baryonic.

Lecture XLI The Cosmic Microwave Background

XLI.2 Cooling of the universe after the Big Bang

A key point of the Big Bang theory is that the early universe was very dense and hot. We expect that this hot, dense universe would have been in thermodynamic equilibrium, and that therefore the radiation field had a blackbody spectrum. We can compute the cooling of this radiation as the universe expands.

The energy density of blackbody radiation is

$$u = aT^4, \quad (62)$$

where a is the radiation constant $a \equiv 4\sigma/c$. By looking at how a blackbody changes under expansion, we find:

$$R^4 aT^4 = aT_0^4 \quad (63)$$

and the current temperature of the blackbody radiation is related to the temperature at an earlier time by

$$RT = T_0. \quad (64)$$

When the universe was half as large it was twice as hot. Recalling that

$$R = \frac{1}{1+z} \quad (65)$$

we can also write

$$T = (1+z)T_0 \quad (66)$$

for the dependence of the temperature of the radiation on redshift.

We can make an order of magnitude estimate of the current temperature of the blackbody radiation by considering the conditions needed to produce helium in the early universe. The early universe was hot and dense enough for nuclear reactions to take place, and the heaviest element that was formed in these reactions was He (and a very small amount of Li). This fusion requires approximately $T \simeq 10^9$ K and $\rho_b \simeq 10^{-2}$ kg m⁻³, where the b subscript refers to the baryon density. (If the temperature were higher the deuterium nuclei needed for the reaction would photodisassociate, and if the temperature were lower it would be too difficult to overcome the Coulomb barrier. The density is needed to produce the observed amount of He.) We can therefore estimate the value of the scale factor at the time of helium formation:

$$R \simeq \left(\frac{\rho_{b,0}}{\rho_b} \right)^{1/3} = 3.5 \times 10^{-9} \quad (67)$$

We can then combine this with the temperature required, $T(R) = 10^9$ K, to determine the current temperature of the radiation:

$$T_0 = RT(R) \simeq 3.5 \text{ K} \quad (68)$$

We will see that this simple calculation gives a very good estimate of the actual temperature of the radiation. Also note that this was predicted in 1948, well before it was discovered.

XLI.3 Discovery and measurement of the cosmic microwave background radiation

- Discovered in 1963 by Arno Penzias and Robert Wilson, working at Bell Labs
- Couldn't get rid of background hiss in their signal... even after removing the pigeons from the antenna
- Knew that a 3 K blackbody would produce the signal, but didn't know of any possible source until they heard about recent work at Princeton (Robert Dicke and Jim Peebles) calculating the temperature of relic radiation from the Big Bang
- Penzias and Wilson published the discovery with the title "A Measurement of Excess Antenna Temperature at 4080 Megacycles per Second," a very modest title for an extremely important discovery!
- The spectrum of the CMB was measured by the COBE satellite in 1991, and proved to be a spectacular confirmation of the prediction—a nearly perfect blackbody with temperature 2.725 K
- The CMB radiation fills the universe and is isotropic. An observer moving with respect to the Hubble flow (the general expansion of the universe) will see a Doppler shift in the CMB. This change in wavelength can be expressed as a change in temperature using Wien's law. The Sun's peculiar velocity produces a **dipole anisotropy** in the CMB: the temperature depends on the peculiar velocity of the Sun in the direction we're looking. This allows us to determine the peculiar velocity of the Sun with respect to the Hubble flow—it's $370.6 \pm 0.4 \text{ km s}^{-1}$. We can decompose this into motions of the Sun around the Galaxy, the Milky Way within the Local Group, and the Local Group with respect to the Hubble flow.
- Once we remove the dipole, the CMB is incredibly isotropic. However, it does have small variations in temperature: on scales of 1° or less, the temperature departs from the average value by about one part in 10^5 . These anisotropies produce the famous map of the CMB you've probably seen (from the WMAP satellite), and measurements of these anisotropies are the primary source of our precise measurements of cosmological parameters. More on that later.

XLI.4 The origin of the CMB

What are we actually seeing when we look at the CMB radiation?

After the Big Bang, the universe was hot and dense, and filled with free electrons, free protons, and photons. The photons scattered off the free electrons, and could only travel short distances between scatterings. The frequent scatterings kept the electrons and photons in thermal equilibrium (they had the same temperature). As the universe expanded, the electrons became farther apart and the photons could travel longer between scatterings. This process, of the expansion diluting the density of a particle until it no longer interacts with other particles, is called **decoupling**, and it occurred for

other particles such as neutrinos—early in the history of the universe, neutrinos stopped interacting with other particles because the distances between them became too great.

In the case of photons, something else happened: the universe became cool enough for the free electrons and free protons to combine into neutral atoms, so the photons no longer had electrons to interact with, and matter and radiation began to evolve independently. This completed the decoupling of radiation and matter. The formation of neutral atoms is called **recombination**, even though that doesn't actually make sense since the electrons and protons were never combined to begin with. Decoupling is a better word.

Once there were no free electrons, the opacity of the universe was much lower, and the photons could stream freely without scattering. The CMB photons we see were last scattered during recombination.

For this reason we define the **surface of last scattering**. This is a spherical “surface,” centered on the Earth, from which all the CMB photons come. Because the universe was opaque before recombination, this is the farthest redshift we can possibly observe. The surface of last scattering actually has a thickness Δz , because recombination didn't happen all at once.

The redshift at which recombination occurred, and the temperature of the universe at that time, can be calculated, and measured from the WMAP CMB observations. The result is

$$z_{\text{dec}} = 1089 \pm 1 \quad (69)$$

and

$$T_{\text{dec}} = T_0(1 + z_{\text{dec}}) = 2970 \text{ K}. \quad (70)$$

This corresponds to an age of $t_{\text{dec}} = 379,000$ years.

Lecture XLII Relativistic Cosmology

We've been talking about different scenarios for the evolution of the universe: open, flat or closed, depending on the density of mass and energy. This is very closely related to the geometry of the universe, which is described by general relativity. To understand the curvature of the universe, we need to consider some of the principles of non-Euclidean geometry.

XLII.2 Euclidean, Elliptic and Hyperbolic Geometries

In about 300 BC, Euclid worked out 5 postulates from which all the rules of geometry could be derived—these rules lay out the basic behaviors of straight lines, right angles, etc.

5th postulate: Given a line and a point not on the line, there is exactly one parallel line which passes through the point.

In the 18th century, mathematicians realized it was possible to make fully consistent definitions of geometry using Euclid's first 4 postulates, but modifying the 5th. These correspond to **curved spaces**.

These examples are curved 2-dimensional surfaces embedded in 3-d spaces—it's not too hard to understand the geometry of something when you can leave the space to look at it. But we can't do that with the universe—we can't pop off into a 4th spatial dimension to look at the curvature. Instead we have to try to understand it using measurements entirely within the space itself. These are called “inner properties.”

The inner properties of a curved space are closely related to how distances are measured within that space.

- On a flat Euclidean plane, a circle has circumference $C = 2\pi r$.
- On a sphere of radius R , the radius of the circle $r = R\theta$, but the circumference is

$$C = 2\pi(R \sin \theta) \quad (71)$$

$$= 2\pi r \left(\frac{\sin \theta}{\theta} \right) \quad (72)$$

$$< 2\pi r \quad (73)$$

So, an observer on the surface of a sphere could measure the radius and circumference of a circle, and deduce that they were living in a positively curved world.

- Similarly, in a negatively curved space, $C > 2\pi r$.

In our Newtonian derivation of the Friedman equation, k referred to the total energy of the universe, but when this equation is derived from general relativity, k is the curvature constant defined above.

Lecture XLIII The Cosmological Constant

Consider again the now familiar equation

$$\left[\left(\frac{V}{R} \right)^2 - \frac{8\pi}{3} G\rho \right] R^2 = -kc^2. \quad (74)$$

We derived this equation from Newtonian physics, by considering the kinetic and potential energy of the universe, but this equation is also a solution to Einstein's field equations for an isotropic, homogeneous universe. In 1922 the Russian mathematician Aleksandr Friedmann solved the field equations and obtained this equation for a non-static universe. We've been calling it the Friedmann equation all along, but strictly speaking this refers to the equation as derived from general relativity. As we've seen, the constant k refers to the curvature of the universe.

XLIII.2 The cosmological constant

Einstein developed his field equations before Hubble's discovery of the expanding universe, and he believed that the universe was static. In their original form, his field equations couldn't produce a static universe, so Einstein added an additional term (a constant of integration) in order to make the universe static, $V = 0$. This term is the cosmological constant Λ , and with this addition the general solution to Einstein's field equations is

$$\left[\left(\frac{V}{R} \right)^2 - \frac{8\pi}{3} G\rho - \frac{1}{3} \Lambda c^2 \right] R^2 = -kc^2. \quad (75)$$

In our original Newtonian derivation, this would result from adding an additional potential energy term

$$U_\Lambda \equiv -\frac{1}{6} \Lambda m c^2 r^2 \quad (76)$$

to our equation for the energy balance of the universe. The result of this new potential is a force

$$F_\Lambda = \frac{1}{3} \Lambda m c^2 r \quad (77)$$

which is radially outward for $\Lambda > 0$: a repulsive force on the mass shell countering gravity, which allowed Einstein to balance the universe in an (unstable) equilibrium.

XLIII.3 Effects of the cosmological constant

After Hubble's discovery of the expanding universe, Einstein called the inclusion of this term the "biggest blunder" of his life. However, recent results have indicated that the universe is actually dominated by some sort of energy which behaves like the cosmological constant. We call this **dark energy**, and we'll now look briefly at its effect on the dynamics of the universe.

We write the Friedmann equation in a form that makes it clear that we're now dealing with a three-component universe of mass, relativistic particles and dark energy:

$$\left[\left(\frac{V}{R} \right)^2 - \frac{8\pi}{3} G (\rho_m + \rho_{\text{rel}}) - \frac{1}{3} \Lambda c^2 \right] R^2 = -k c^2. \quad (78)$$

We also introduce the **equation of state**, which relates the density ρ and pressure P of any component of the universe:

$$P = w \rho c^2. \quad (79)$$

For matter with no pressure, as we have considered so far, $w = 0$. For photons or other relativistic particles, $w = 1/3$.

The Friedmann equation and another equation called the fluid equation (which won't be presented here) can be combined to produce an equation describing the acceleration a (the rate of change of the velocity) of the universe:

$$a = \left[-\frac{4\pi G}{3} \left[\rho_m + \rho_{\text{rel}} + \frac{3(P_m + P_{\text{rel}})}{c^2} \right] + \frac{1}{3} \Lambda c^2 \right] R \quad (80)$$

We now define the equivalent mass density of dark energy

$$\rho_\Lambda \equiv \frac{\Lambda c^2}{8\pi G} = \text{constant} = \rho_{\Lambda,0} \quad (81)$$

so that the Friedmann equation becomes

$$\left[\left(\frac{V}{R} \right)^2 - \frac{8\pi}{3} G (\rho_m + \rho_{\text{rel}} + \rho_\Lambda) \right] R^2 = -k c^2. \quad (82)$$

Note that because ρ_Λ remains constant as the universe expands, more and more dark energy must appear to fill the larger volume.

The pressure due to dark energy is given by its equation of state

$$P_\Lambda = -\rho_\Lambda c^2 \quad (83)$$

(this can be derived from the fluid equation). In the general equation of state $P = w \rho c^2$, $w = -1$ for dark energy. The pressure due to the cosmological constant is *negative*, while the equivalent mass density is positive. We can now substitute expressions for ρ_Λ and P_Λ into the acceleration equation (Eq 80):

$$a = \left[-\frac{4\pi G}{3} \left[\rho_m + \rho_{\text{rel}} + \rho_\Lambda + \frac{3(P_m + P_{\text{rel}} + P_\Lambda)}{c^2} \right] \right] R \quad (84)$$

From this equation and the various equations of state we can see how each component of the universe affects its acceleration. First consider a universe containing only matter, for which $w = 0$ and therefore $P_m = 0$. We set all the terms containing relativistic particles and Λ to zero to find

$$a = -\frac{4\pi G}{3} \rho_m R. \quad (85)$$

As expected, a positive matter density leads to a negative acceleration, as the gravitational attraction of the matter in the universe causes the expansion to slow down.

For relativistic particles such as photons, the equation of state is $P_{\text{rel}} = \frac{1}{3}\rho_{\text{rel}}c^2$. Setting the matter and Λ terms to zero and substituting this equation of state into the acceleration equation, we find that for a universe containing only relativistic particles,

$$a = -\frac{8\pi G}{3}\rho_{\text{rel}}R. \quad (86)$$

Radiation or other relativistic particles also cause the expansion of the universe to decelerate.

Finally we consider a universe containing only the cosmological constant, for which $P_{\Lambda} = -\rho_{\Lambda}c^2$. Setting the matter and relativistic particle terms to zero and substituting this into the acceleration equation gives

$$a = \frac{8\pi G}{3}\rho_{\Lambda}R. \quad (87)$$

Because a positive density of dark energy has negative pressure, dark energy leads to *positive* acceleration of the universe, unlike the other components.

The Friedmann equation can also be written in terms of the Hubble constant and the density parameter Ω (we showed this for the single component universe a while back) as

$$H^2[1 - (\Omega_m + \Omega_{\text{rel}} + \Omega_{\Lambda})]R^2 = -kc^2 \quad (88)$$

where

$$\Omega_m = \frac{\rho_m}{\rho_c} = \frac{8\pi G\rho_m}{3H^2} \quad (89)$$

$$\Omega_{\text{rel}} = \frac{\rho_{\text{rel}}}{\rho_c} = \frac{8\pi G\rho_{\text{rel}}}{3H^2} \quad (90)$$

$$\Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_c} = \frac{\Lambda c^2}{3H^2}. \quad (91)$$

We define the **total density parameter**

$$\Omega \equiv \Omega_m + \Omega_{\text{rel}} + \Omega_{\Lambda}. \quad (92)$$

Note that Ω without a subscript refers to the total density parameter for all of the components of the model under consideration. The Friedmann equation is then

$$H^2(1 - \Omega)R^2 = -kc^2. \quad (93)$$

A flat universe with $k = 0$ requires $\Omega(t) = 1$.

The Planck (2015) measurements of the cosmic microwave background in combination with other methods give us our best values for the three components of the universe today:

$$\Omega_{m,0} = 0.3089 \pm 0.0062 \quad (94)$$

$$\Omega_{\text{rel},0} = 9.161 \times 10^{-5} \quad (95)$$

$$\Omega_{\Lambda,0} = 0.6911 \pm 0.0062 \quad (96)$$

Adding these together, the total density parameter $\Omega_0 = 1.000 \pm 0.0088$. Within our ability to measure it, the universe is flat and currently dominated by dark energy.

We can also define the **deceleration parameter** $q(t)$:

$$q(t) \equiv -\frac{R(t)[a(t)]}{[V(t)^2]} \quad (97)$$

The name and the minus sign, which gives a positive value for a decelerating universe, reflect the once-common belief that the universe had to be decelerating. The deceleration parameter can also be written in terms of the density parameters for the different components of the universe:

$$q(t) = \frac{1}{2}\Omega_m(t) + \Omega_{\text{rel}}(t) - \Omega_\Lambda(t). \quad (98)$$

With current values,

$$q_0 = -0.54, \quad (99)$$

telling us (because of the minus sign) that the universe is currently accelerating.

XLIII.4 The Λ era

We've already discussed the dependence of the density of radiation and matter on the scale factor: $\rho_m \propto R^{-3}$ and $\rho_{\text{rel}} \propto R^{-4}$. Because the radiation density decreases more quickly as the universe expands, the universe was dominated by radiation at early times but then became matter-dominated at a redshift of $z_{r,m} = 3270$. We now have another component to consider, ρ_Λ , which is *constant*. This means that at some point, as the matter density decreases as the universe expands, the universe will become dominated by the cosmological constant. As it turns out, this has already happened. The scale factor at the equality of matter and dark energy is

$$R_{m,\Lambda} = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3} = 0.764, \quad (100)$$

which corresponds to a redshift $z_{m,\Lambda} = 0.308$.

We can also use the acceleration equation to find when the acceleration of the universe changed from negative to positive. The result is

$$R_{\text{accel}} = \left(\frac{\Omega_{m,0}}{2\Omega_{\Lambda,0}}\right)^{1/3} = 0.607, \quad (101)$$

corresponding to a redshift $z_{\text{accel}} = 0.648$. So the universe began to accelerate *before* the equality of matter and dark energy; this is because dark energy has pressure as well as equivalent mass density which affects the dynamics of the universe, and (non-relativistic) matter does not. Note also that unlike all of the other cosmological times we've calculated, these are relatively recent. We routinely observe objects at these redshifts, and can use them to understand the dynamics of the universe.

XLIII.5 What is dark energy?

We don't really know, but the leading candidate is **vacuum energy**. In classical physics there is no such thing, but quantum physics allows for energy in a vacuum. The Heisenberg uncertainty principle allows particle-antiparticle pairs to spontaneously appear and then annihilate in an otherwise empty vacuum. The total energy and lifetime of these particles must satisfy the relation

$$\Delta E \Delta t \geq h. \quad (102)$$

There is an energy density associated with these particle-antiparticle pairs, and this energy density is a quantum phenomenon that doesn't care at all about the expansion of the universe or the passage of time. However, calculating the actual value of this energy density is an exercise in quantum field theory that hasn't yet been completed. One suggestion is that the natural value for the vacuum energy density is the Planck energy density,

$$u_{\text{vac}} = \frac{E_P}{l_P^3}. \quad (103)$$

However, the Planck energy is large (by particle physics standards; $E_P = \sqrt{\hbar c^5/G} = 1.96 \times 10^9$ J) while the Planck length is very small ($l_P = 1.6 \times 10^{-35}$ m), and the resulting energy density is 124 orders of magnitude larger than the current critical density of the universe (!). So the question of what dark energy is and why it has the value it does is very much still open.