Astronomy 211 Formulas

Distance $d[\text{pc}] = 1/\text{parallax}[\text{arcsec}]$

magnitudes $m = C - 2.5\log_{10} F$; $m_1 - m_2 = -2.5\log_{10}(F_1/F_2)$, $M - m = 5 - 5\log_{10}(d/\text{pc})$

Doppler shift $\Delta\lambda/\lambda = -\Delta\nu/\nu = v_{\text{rad}}/c$

Gravity and Tides accel$= GM/r^2$, accel$_{\text{tide}} \approx (GM/r^2)(2R/r)$; escape speed $v_{\text{esc}} = \sqrt{2GM/R}$

Kepler $GM = \Omega^2 a^3 = \left(\frac{2\pi}{P}\right)^2 a^3$; $\frac{a_1}{a} = \frac{v_1}{v} = \frac{m_2}{M}$; reduced mass $\mu = m_1 m_2/(m_1 + m_2)$, total mass $M = m_1 + m_2$; for circular orbit $v = \sqrt{GM/a}$, $L = \mu\sqrt{GMa}$; for Solar units (years, AU, $M_\odot$): $P^2 = a^3/M$

Virial Theorem $E_{\text{kin}} = -\frac{1}{2}E_{\text{pot}}$; $E_{\text{tot}} = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2}E_{\text{pot}}$, [where $E_{\text{pot,binary}} = -\frac{Gm_1m_2}{a}$ and $E_{\text{pot,star}} \approx -\frac{GM^2}{R}$]

Ideal Gas $P = nk_BT = \frac{\rho}{\mu m_H}k_BT$; typical KE per particle is $\frac{3}{2}k_BT$; energy density $u = \frac{3}{2}nk_BT = \frac{3}{2}P$

Degenerate Gas $\Delta x\Delta p \sim \hbar$; $E_{\text{F}} = \frac{1}{2}\frac{p_{\text{F}}^2}{m_e} \propto n_e^{2/3}$; $P \propto n_e E_{\text{F}} \propto n_e^{5/3} \propto (\rho/\mu)^{5/3} \rightarrow R \propto M^{-1/3}$ [non-relativistic]

Photon Propagation $l_{\text{mfp}} = \frac{1}{n\sigma} = \frac{1}{\kappa\rho}$; $t_{\text{randomwalk}} = \frac{R}{l_{\text{mfp}}}\frac{R}{c}$

Blackbody $L = 4\pi R^2\sigma T_{\text{eff}}^4$, $F = \sigma T_{\text{eff}}^4$; $\lambda_{\text{peak}} = 0.29\,\text{cm}/T$

Light $c = \lambda\nu$, $E = h\nu = hc/\lambda$, $p = E/c$, energy density $u = aT^4$, pressure $P = (a/3)T^4$

Hydrostatic Equilibrium $\frac{\Delta P}{\Delta r} = \rho\frac{GM}{r^2} = -g\rho \rightarrow P \propto M^2/R^4$

Stars $T_c \propto M/R$, $\rho_c \propto M/R^3$, $P_c \propto M^2/R^4$

Timescales $\tau_{\text{free-fall}} \sim \sqrt{1/G\rho}$; $\tau_{\text{Kelvin-Helmholtz}} \sim \frac{GM^2/R}{L}$

Hydrogen Fusion $E = \Delta mc^2 \approx 0.7\%c^2$

Hydrogen Atom $E_n = -13.6\,\text{eV}/n^2$

Jeans mass $M_J = 3k_BTR/2G\mu m_{\text{H}}$; $\rho_J = (3/4\pi M^2)(3k_BT/2G\mu m_{\text{H}})^3$

Equilibrium Temperature $T_p = T_s(1-A)^{1/4}(R_s/2d)^{1/2}$

Diffraction Limit $\theta = \lambda/D$

Poisson noise uncertainty on n counts is $\sqrt{n}$