Introduction

Astronomers now know that planets around other stars are commonplace in the Milky Way Galaxy. But with only a few exceptions, we cannot see these planets directly because any light they reflect or emit is overwhelmed by the luminosity of the stars they orbit. In this lab, you will investigate the indirect detection of exoplanets using the observed reflex motion of stars.

Learning Objectives

At the completion of this lab, you should be able to:

1. Explain why a star wobbles in response to motion of its planet
2. Explain what is being measured in a radial velocity exoplanet detection
3. Interpret a radial velocity curve in terms of orbital motion of star and planet
4. Identify the period of a planet from a star’s radial velocity (RV) curve
5. Describe how we use Kepler’s third law to estimate the planet’s semimajor axis
6. Describe the effects of multiple planets on the observed radial velocity

Reflex Motion [5 pts each; 30 points total]

Open the PhET *My Solar System* program[^1]. Set the number of bodies to 2, and check the boxes for “System Centered” and “Show Traces.” Set up a very simple solar system with the following parameters for the star and planet:

Press *Start* and observe the motions of the star and planet. In the diagram below, sketch the orbits and use arrows to indicate the direction of motion.

1. How does the **orbital period** (the time required to complete one full orbit) of the planet compare to the orbital period of the star?

2. In the diagram below, indicate the directions of the force exerted by the star on the planet ($F_{sp}$) and the force exerted by the planet on the star ($F_{ps}$) with arrows. Let the length of the arrows indicate the strength of the force (longer means stronger, etc.)
3. Two students discuss the simulation and the diagram:

**Student One:** We know that the star’s gravity pulls on the planet. Newton’s third law says that the planet must also pull on the star. So both the star and the planet should be orbiting.

**Student Two:** But the star is much bigger, so it should have more gravitational force.

Do you agree or disagree with each of these students? Explain briefly.

4. The students above discuss Newton’s third law of equal and opposite forces. Should they also be discussing Newton’s second law $F = ma$? Explain how this relates to the motion of the star and the planet.

5. Based on the gravitational forces you have identified, explain why the star follows a small orbit and the planet follows a large orbit.

6. The motion of the star caused by the planet is called **reflex motion**. What would change about the star’s reflex motion if the mass of the planet were reduced to 1? Make a prediction, and then test your prediction with the simulation. What happens to the star’s reflex motion when the mass of the planet is increased to 20? Explain.
Measuring Motion [5 pts each; 60 points total]

1. The drawing below shows a distant star that is being observed from the Earth. Use arrows to indicate motions of the star that would cause an observer on Earth to detect a Doppler shift in the star’s spectrum.

![Diagram of a star with arrows indicating directions]

2. The diagram below shows the orbit of the star. At each labeled position, draw an arrow to indicate the direction of the star’s motion at that position in its orbit (each arrow should be a straight line!).

![Diagram of a star orbit with arrows indicating directions]

3. The motion of a star toward and away from the observed is called the radial velocity. The radial velocity is negative when the object is moving toward the observer, and positive when the object is moving away.

   At which of the four labeled positions will the star have the largest positive radial velocity?

4. At which of the four labeled positions will the star have the largest negative radial velocity?
5. Are there any positions along the star’s orbit where its radial velocity is 0? Explain.

6. Based on your answers on the previous questions, sketch the radial velocity curve of the star as observed from the Earth for two complete orbits. Label the point on the curve with the corresponding letter each time the star passes through positions A, B, C, and D.

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<table>
<thead>
<tr>
<th>Time</th>
<th>Radial Velocity</th>
</tr>
</thead>
</table>
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7. How can you tell from the radial velocity curve you have drawn when the star has made one complete orbit?

8. How can you use the radial velocity curve of the star to determine the period of the planet?

9. The **amplitude** (difference between largest negative and largest positive) of the star’s radial velocity depends on the strength of the gravitational pull of the planet on the star. The plot below shows the radial velocity curve for a star of mass $M_S$ orbited by a planet of mass $M_p$. For simplicity, we’ll stick to circular orbits.
Say we somehow increased the mass of the planet, $M_p$. How would this change the gravitational pull between the star and planet? How would this change the motion of the star? Use *My Solar System* to test your prediction.

10. How would increasing $M_p$ change the observed radial velocity of the star? Sketch the new curve on the plot above and explain your reasoning.

11. Keep $M_p$ the same, but now move the planet closer to the star. (You will need to adjust the planet’s speed to keep a circular orbit.) How does this change the gravitational pull between the star and planet?
12. How does decreasing the distance between the star and planet change the observed radial velocity curve of the star? Hint: there are two ways this will change the curve! Sketch the new curve on the plot above and explain your reasoning.

Interpreting Radial Velocity Plots [5 pts each; 55 points total]

1. The plot below shows the radial velocity of star X. This star is known to host an exoplanet, X b (the second planet discovered around this star would be X c, etc.).

![Radial Velocity Plot](image)

Use the radial velocity plot above to find the orbital period of the planet X b in days, and explain your reasoning.

2. Kepler’s third law of planetary motion states that the period $P$ of the planet is related to the semimajor axis of the orbit $a$ by:

$$P^2 \propto a^3$$
The Earth-Sun distance is 1 AU. What is the orbital period of the Earth’s orbit (in days)? You can assume the star has a mass of $1 \, M_{\odot}$.

3. Use a ratio with the values of $P$ and $a$ to find the semimajor axis of the orbit of $X\, b$ in AU. i.e.,

$$\left( \frac{P_X}{P_{\text{Earth}}} \right)^2 = \left( \frac{a_X}{a_{\text{Earth}}} \right)^3$$

and solve for $a_X$.

4. In our solar system, the innermost planets from the Sun are Mercury (0.38 AU), Venus (0.72 AU), and Earth (1.00 AU). How does the size of the orbit of $X\, b$ compare to the orbits of these planets?
5. Our own solar system has eight planets, and many other planetary systems also have multiple planets. How does that change the observed radial velocity plot? Let’s try it in *My Solar System*. Set the number of bodies to 3 and check the boxes for “System Centered” and “Show Traces.” Set up a planetary system with the following parameters for the star and now two planets:

<table>
<thead>
<tr>
<th>Position</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>X</td>
</tr>
<tr>
<td>Star</td>
<td>60</td>
</tr>
<tr>
<td>Planet b</td>
<td>6</td>
</tr>
<tr>
<td>Planet c</td>
<td>2</td>
</tr>
</tbody>
</table>

Press *Start* and sketch the motion of the star below.

6. Why does the star now have this very different motion than what we saw in the single planet system?

7. Based on their masses and distances, which planet will cause the largest amplitude reflex motion of the star? Explain your reasoning.

8. What are the orbital periods of each planet?
9. On the axes below, sketch the radial velocity curve for this star as seen from Earth. (We’ll assume Earth is viewing this planet system edge on, unlike your face-on view in My Solar System.) Be sure your plot matches your reasoning from the previous question.

10. Look at the orbit of Planet b (the inner planet). What causes Planet b’s motion to change slightly over several orbits? (Hint: think of all the gravitational forces in this system.)

11. The plot below shows the radial velocity of a hypothetical star with two exoplanets. Points denote the observations taken over a time span of roughly 2 years, and the line shows the motion of the star determined by astronomers.
Based on this plot, describe the planetary system around this star. Estimate the orbital periods of the planets, and compare the masses of the planets. Explain your reasoning.