Phys 194–FYRE PS 3

Given: Sep 20, 2017. Due: Sep 27, 2017

Homework Policy: You can consult class notes and books. Always try to solve the problems yourself; if you cannot make progress after some effort, you can discuss with your classmates or ask the instructor. However, you cannot copy other's work: what you turn in must be your own. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Problem 1 Dispersion

The index of refraction of a radio wave with frequency ν going through an ionized medium (plasma) is:

$$\mu = \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2}$$

where $\nu_p = \sqrt{e^2 n_e/\pi m_e}$ is the "plasma frequency", e is the electron charge, m_e is the electron mass, and n_e is the number density of electrons, how many electrons per cm^3. In cgs units this is $\nu_p = 8.5\,\mathrm{kHz} n_e^{1/2}$. With this definition, the speed that a pulse moves at is $v = c\mu < c$.

a. Show that the time it takes a pulse to travel a distance d is:

$$t(f_1) = \int_0^d \frac{dl}{v(f_1)}$$

b. Compare this to $t(f_2)$. What is $\Delta t = t(f_1) - t(f_2)$? You might want to make the approximation that $\nu \gg \nu_p$.

c. Show that this gives you:

$$\Delta t = 4.15 \times 10^6\,\mathrm{ms} \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right) \times \mathrm{DM}$$

where $\mathrm{DM} = \int_0^d dl\, n_e$ and ν_1 and ν_2 are both in MHz. Even if you can't get the numerical factor out in front, can you show that this general relation is correct?

Problem 2 Using Dispersion

For a pulsar observed at a frequency of 1400 MHz, with a channel width of $\delta\nu = 1\,\mathrm{MHz}$, and a pulse period of 1 ms, what is the maximum DM such that the delay across the channel is less than one pulse period?

Problem 3 Dispersion in water

Take the speed of light in water to be:

$$\left(\frac{v(\lambda)}{c}\right)^{-1} = 1.7527 - 2.55\times10^{-3}\lambda^4 - 5.0\times10^{-3}\lambda^2 + 9.9\times10^{-3}\lambda^{-2} - 4.0\times10^{-4}\lambda^{-4} + 2.9\times10^{-5}\lambda^{-6} - 5.0\times10^{-7}\lambda^{-8} \quad (1)$$

with λ the wavelength in microns (Danilchenko 2003, arXiv:physics/0306020). If I had a 1 ns broadband pulse across the full optical range from blue to red, how far could it travel before it spread out to 2 ns?