Clocking Dead Stars with Radio Telescopes

Lecture I Preliminaries

Course Description: What is FYRE?

- First Year Research Experience
- Experience for *you* and *us*
- You will:
  - Learn background material on pulsars and radio astronomy
  - Learn introductory Unix usage and python programming
  - Learn to conduct observations using the Green Bank Telescope, one of the world's premier radio telescopes
  - Participate in small groups to adopt and completely "solve" a pulsar, determining all of the relevant physical information and placing it into the context of the larger population
  - Work as a class to "solve" a binary pulsar, which has the potential to be significantly more interesting but which is much rarer and requires new concepts and techniques
  - Write proposals for new telescope time in small groups, which will be judged by an external panel of collaborators. The best proposals may be awarded time on the Green Bank Telescope

Evaluation will be:

- Weekly problem sets (10%), with the best 10 of 11 counting.
- Weekly in-class assignments (50%)
- Participation (20%)
- Final proposals and presentation (20%)

I.2 Physics Synopsis

The level of physics and math that you are expected to be familiar with (but not necessarily know in detail) is:

Newton's Laws : most importantly, $F = ma$

Kinetic Energy : $K = \frac{1}{2}mv^2$

Gravitation : $F = GM_1M_2/r^2$ (on the surface of the Earth, $F = gm$)

Potential Energy : $U = -GM_1M_2/r$ (from gravity; on the surface of the Earth, $U = gmh$)

Centripetal Acceleration : $a = v^2/r$

Ideal Gas Law : $PV = NRT$

Circumference of a Circle : $2\pi r$

Area of Circle : πr^2

Surface Area of a Sphere : $4\pi r^2$

Volume of a Sphere : $\frac{4}{3}\pi r^3$

Radians : $180° = \pi$ radians, $\sin(\pi/2) = 1$, $\sin\pi = 0$, etc.

Small Angles : $\sin x \approx x$ for x very small and measured in radians. Also, $\tan x \approx x$, and $\cos x \approx 1$ (**draw these**)

Scientific notation : $A \times 10^a \cdot B \times 10^b = (AB) \times 10^{a+b}$

I.2.1 Greek

If we use a symbol you don't recognize or can't read, **ask**!

I.3 Precision

We often do not know things very precisely. So we use $\sim$ and $\approx$ and related symbols. $\sim$ is for when we know something to *an order of magnitude*. So we if we know that $x \sim 5$, we know that x is between $5/3$ and $5 * 3$, where 3 is roughly $\sqrt{10}$. This means that the possible range for x is in total a factor of 10. We will also sometimes use $\sim$ to mean *scales as*. For example, if you were to estimate the height of a person as a function of their weight (for a wide range of people), you might expect that as you double the weight, the height changes by $2^{1/3}$. We could write height$\sim$weight$^{1/3}$. There will be a lot of variation, but this is roughly correct.

$\approx$ means more precision. It doesn't necessarily have an exact definition. But generally, if we say $x \approx 5$, that means that 4 is probably OK but 2 is probably not.

Finally, we have $\propto$, which means *proportional to*. This is more precise that the *scales as* use of $\sim$. So while for a person height$\sim$weight$^{1/3}$ is OK, for a sphere (where we know that volume is $4\pi/3r^3$) we could write volume$\propto r^3$: we take this as correct, but leave off the constants ($4\pi/3$ in this case).

I.3.1 Small Angles

For small angles θ, $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$. We need θ to be in radians. But we also often deal with fractions of a circle. A circle has $360°$. We break each degree into 60 minute (or *arcminutes*): $1° = 60'$. And each arcminute into 60 seconds (or *arcseconds*): $1' = 60''$, so $1° = 3600''$. But we also know that 2π radians is $360°$, so we can convert between radians and arcsec. This will come up frequently: $1'' = 360 \times 3600/2\pi \approx 1/206265$ radians.

I.4 Celestial Sizes, Distances, and Coordinates

I.4.1 Units

Astronomy emphasizes *natural* units ($\odot$ is for the Sun, $\oplus$ is for the Earth):

- $M_\odot = 2 \times 10^{30}\,\mathrm{kg}$ (solar mass)
- $R_\odot = 7 \times 10^{8}\,\mathrm{m}$ (solar radius)
- $M_\oplus = 6 \times 10^{24}\,\mathrm{kg} \approx 3 \times 10^{-6}\,M_\odot$ (earth mass)
- $M_\mathrm{J} = 2 \times 10^{27}\,\mathrm{kg} \approx 10^{-3}\,M_\odot$ (Jupiter)
- $L_\odot = 4 \times 10^{26}\,\mathrm{W}$ (solar luminosity or power)
- light year $= 10^{16}\,\mathrm{m}$: the *distance* light travels in one year (moving at $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$)
- Astronomical Unit = AU = $1.5 \times 10^{11}\,\mathrm{m}$ (distance between earth and sun)
- parsec = parallax second = pc = $3 \times 10^{16}\,\mathrm{m} = 206,265\,\mathrm{AU}$

And then we use usual metric-style prefixes to get things like kpc, Mpc, etc.

BUT most astronomy work doesn't use kg, or m (i.e., SI). And so we also don't use derived units like J or N. Instead we use cgs units: cm, g, s. Which means that units are:

mass g ($10^{-3}\,\mathrm{kg}$)

length cm ($10^{-2}\,\mathrm{m}$)

energy erg ($10^{-7}\,\mathrm{J}$)

force dyne ($10^{-5}\,\mathrm{N}$)

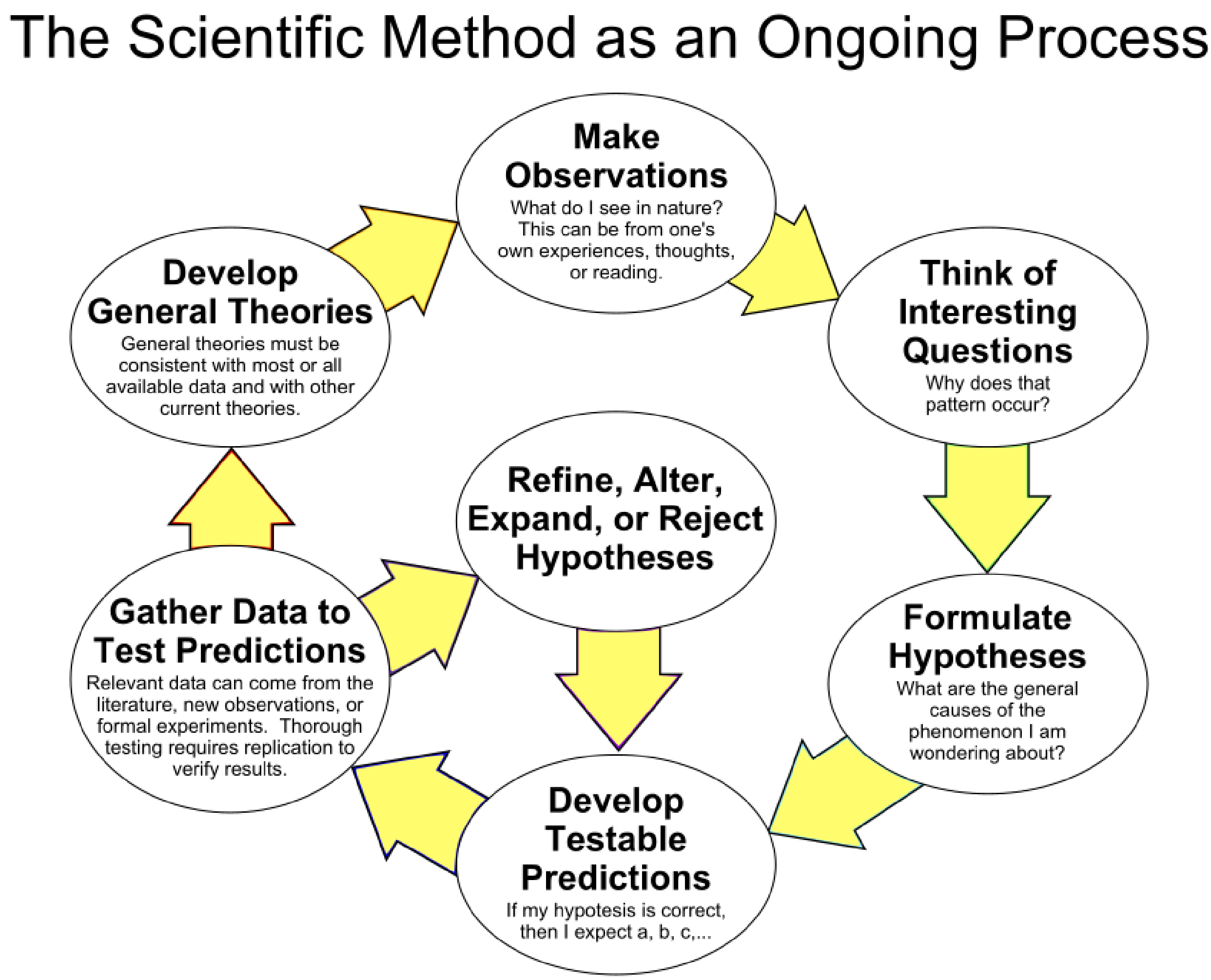
Almost all equations are the same just with different constants, e.g., $G = 6.7 \times 10^{-8}\,\mathrm{cm^3\,g^{-1}\,s^{-2}}$ instead of $6.7 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$. Except E&M, but we won't do a lot of that here.

- $M_\odot = 2 \times 10^{33}\,\mathrm{g}$ (solar mass)
- $R_\odot = 7 \times 10^{10}\,\mathrm{cm}$ (solar radius)
- $M_\oplus = 6 \times 10^{27}\,\mathrm{g} \approx 3 \times 10^{-6}\,M_\odot$ (earth mass)
- $M_\mathrm{J} = 2 \times 10^{30}\,\mathrm{g} \approx 10^{-3}\,M_\odot$ (Jupiter)
- $L_\odot = 4 \times 10^{33}\,\mathrm{erg/s}$ (solar luminosity or power)

- light year $= 10^{18}$ cm: the *distance* light travels in one year (moving at $c = 3 \times 10^{10}\,\mathrm{cm\,s^{-1}}$)
- Astronomical Unit = AU = 1.5×10^{13} cm (distance between earth and sun)
- parsec = parallax second = pc = $3 \times 10^{18}\,\mathrm{cm} = 206,265\,\mathrm{AU}$

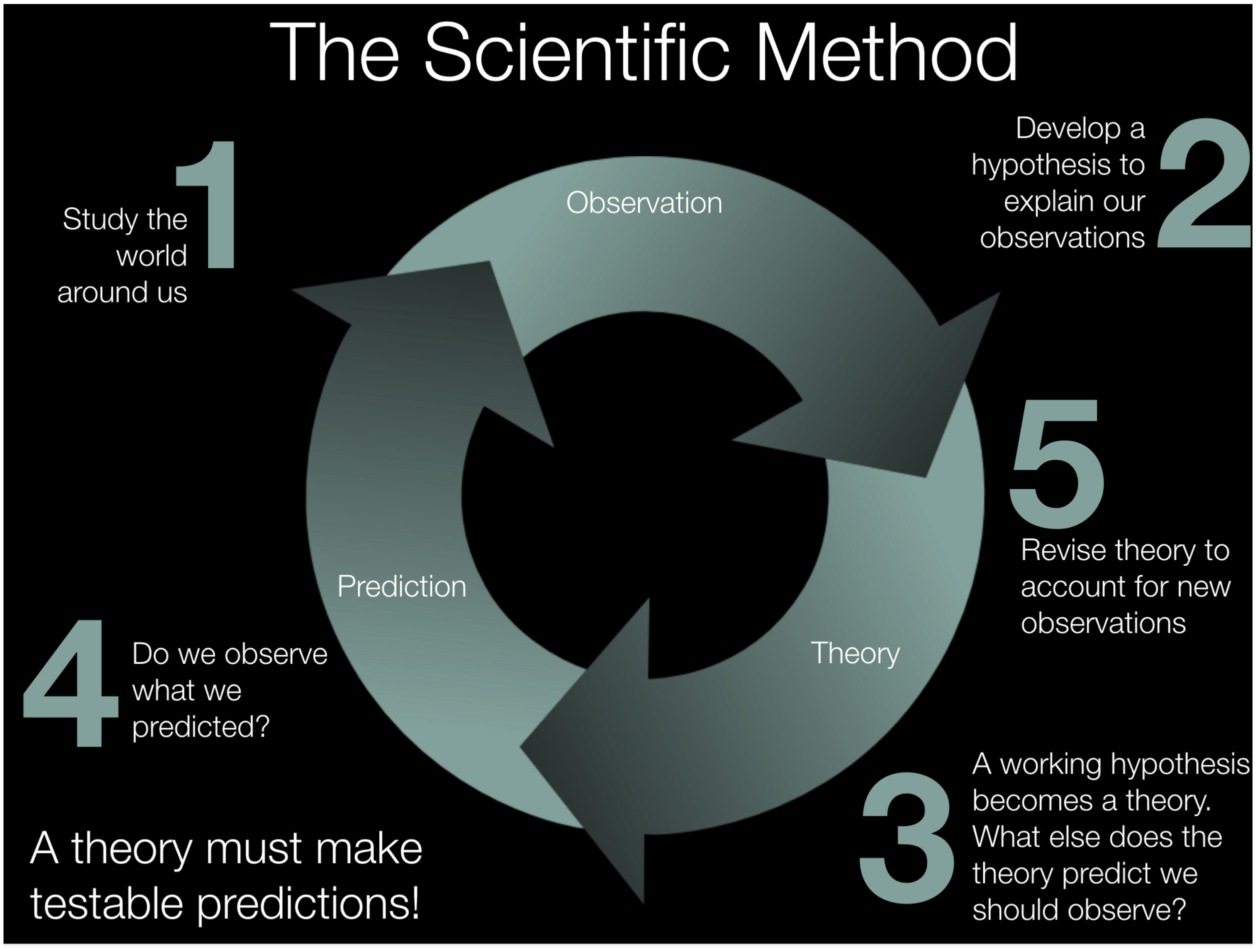
Google/Wolfram Alpha/astropy can be very helpful when checking unit conversions.

I.5 Scientific Method



Can we do this? Why or why not?

Now the astronomy version:



Science must be testable. A theory is not a "hunch" or "opinion" in science. Instead it is an extremely strong statement that provides an explanation of a natural phenomenon based on a wealth of well-documented evidence:

- A theory must be testable and based on observation
- A theory must be falsifiable
- A theory is always subject to revision and change

Let us come up with a hypothesis that can tested *observationally* but not *experimentally*.

Now let us come up with a hypothesis that cannot be tested at all.

Lecture II Neutron Stars

II.2 Higher Masses: What Happens?

Once we get to Fe in the core, we cannot get energy out. The star is still shining (so it's losing energy) but no longer creating it. So the core starts to cool. In order to support the rest of the star, the pressure needs to be the same, so the density goes up and up to keep the pressure constant.

If the mass of the Fe core is less than the Chandrasekhar mass, then electron degeneracy pressure can support the star. But once it gets past there that is not enough. At this point, it is roughly 5×10^9 K, 5000 km in radius.

As it gets to the Chandrasekhar mass, electrons cannot support the star. It starts to collapse. The collapse liberates some gravitational energy, but the Fe *absorbs* that energy, liberating protons. Protons take up extra space, and they are increasingly squeezed by the rest of the star.

Remember β decay (nuclear decay):

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

This happens spontaneously for various nuclei. The inverse can also happen, although it isn't spontaneous:

$$p^+ + e^- \rightarrow n + \nu_e$$

When things get too dense, the inverse reaction is energetically favorable. This makes a bunch of neutrons, removing electron support. Neutrinos also leak out, removing energy.

So the core collapses down in a free-fall timescale of $\sim 1/\sqrt{G\rho} \sim 10\,\mathrm{s}$ or less. This reaction happens at a density of $\sim 10^9\,\mathrm{kg\,m^{-3}}$. Much of the star is blown off in a *supernova explosion*: the gravitational energy of 10^{46} J is released mostly as neutrinos, with a small amount creating a blast wave of material moving at 10,000 km/s.

II.3 Core Collapse

It gets squeezed down to $\sim 10\,\mathrm{km}$. This is a core-collapse SN (there are other kinds), and happens for $M < 25M_\odot$ or so. A lot of the gravitational energy is releasd:

- 10^{46} J total
- 10^{44} J is in the KE of the ejected material
- 10^{43} J is in photons ($10^{10}L_\odot \sim L_{\rm galaxy}$ for 10 days)

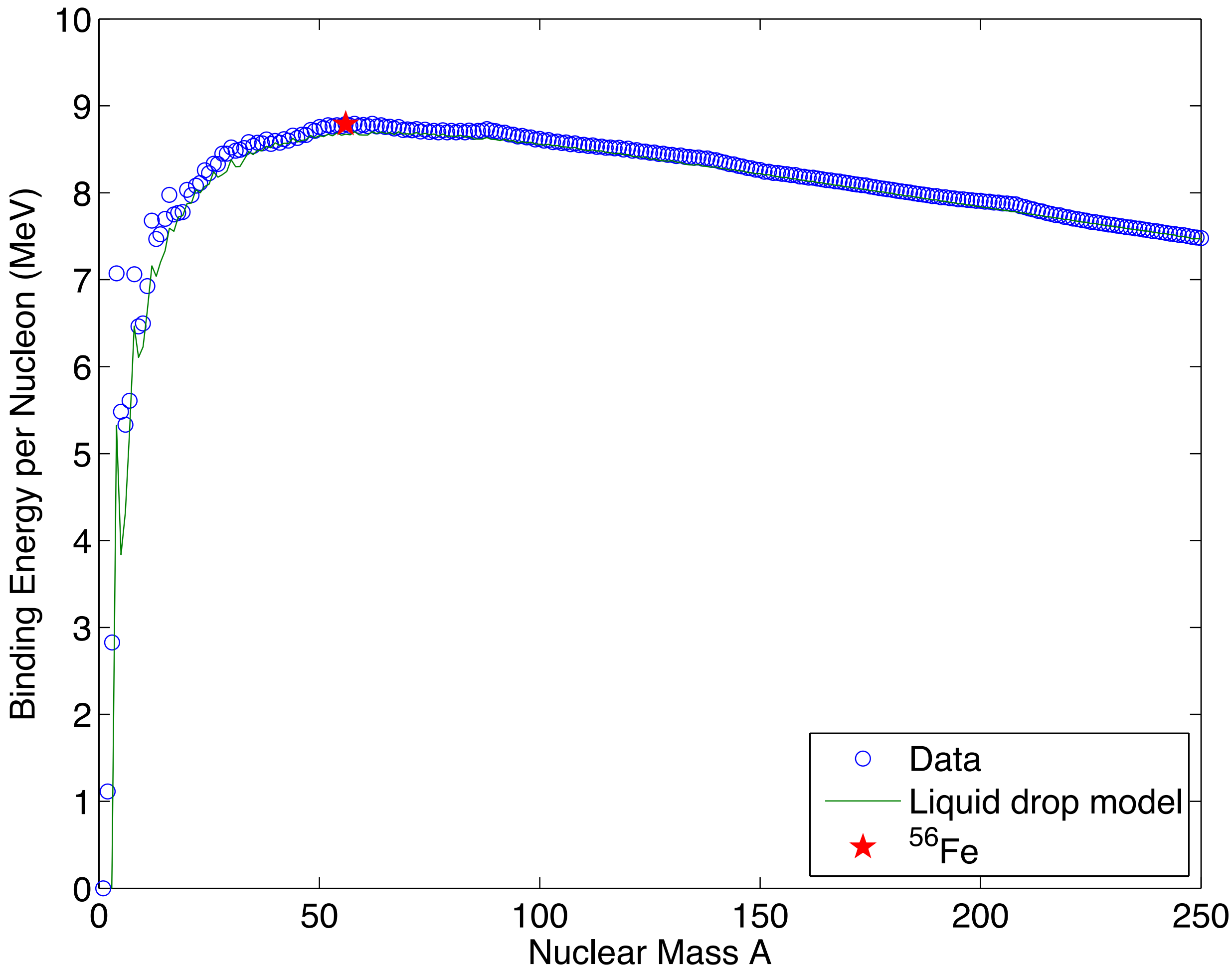
For example, SN1054 which is 2 kpc away was seen during the day quite easily.

99% of the energy (or more) goes out as neutrinos ($10^{19}L_\odot$). About 20 were detected from a SN 50 kpc away.

A lot of the blasted away material goes into the nearby interstellar space, enriching it with metals that were made in the star. This is how elements heavier than C/O get into the universe.

II.4 What Remains

Stability of nuclei: peaks near ^{56}Fe. For smaller nuclei, too many nucleons are near the surface, and for more massive nuclei the Coulomb repulsion matters more (remember the liquid drop model):



But this only works in isolation. With relativistic electrons present the equilibrium is different. They have enough energy for inverse beta decay. This turns protons into neutrons, and we end up with nuclei with many many neutrons (heavier than Fe). For example, with $\rho \sim 10^{14}\,\mathrm{kg\,m^{-3}}$, we might have ^{76}Fe and ^{78}Ni.

This keeps going as the material gets cooler and denser. Eventually, around $4 \times 10^{14}\,\mathrm{kg\,m^{-3}}$, we get *neutron drip*. Neutrons leak out of nuclei, and we get a new equilibrium with heavy nuclei, neutrons, and electrons. We can calculate this OK for $\rho < \rho_{\mathrm{nuc}} = 2.3 \times 10^{17}\,\mathrm{kg\,m^{-3}}$. Beyond this density we do not understand the many-body interactions that occur, and it gets even more uncertain beyond $10^{18}\,\mathrm{kg\,m^{-3}}$ (pions, muons, hyperons, ...).

Why are free neutrons OK? For most neutrons, cannot have beta decay since electrons are blocked from forming by Pauli exclusion principle: $n \to p + e^- + \bar{\nu}_e$ cannot happen freely.

A neutron star is like a white dwarf, except instead of electron degeneracy, the reaction $p + e \to n$ makes neutrons, and neutron degeneracy pressure supports it. For $1.4 M_\odot$ the size is about 10 km. This gives $\rho \sim 10^{17-18}\,\mathrm{kg\,m^{-3}}$, compare to a nucleus is 2×10^{17}.

$$\rho_{\mathrm{WD}} \sim \frac{m_H}{(h/m_e c)^3} \sim 10^8\,\mathrm{kg\,m^{-3}}$$

$$\rho_{\mathrm{NS}} \sim \frac{m_H}{(h/m_p c)^3} \sim 6 \times 10^{17}\,\mathrm{kg\,m^{-3}}$$

We are confining one nucleon (proton or neutron) in a box. For the WD the size of the box is the de Broglie wavelength of the electron. For the NS it's the de Broglie wavelength of the neutron. Since the wavelength is $\sim 1/m$, the neutron's box is much smaller.

Consider a nucleus. The distance between neutrons is $r_0 \sim 10^{-15}$ m. If you take a Solar mass in neutrons, that means $A \sim M_\odot/m_{\mathrm{H}} \sim 10^{57}$ neutrons, so the size is $R_{\mathrm{NS}} \sim r_0 A^{1/3} \sim 10\,\mathrm{km}$.

The properties of this object: surface gravity $g \sim 10^{12}\,\mathrm{m\,s^{-2}}$, escape speed $\approx 0.6c$.

The interior is very complicated: still under investigation. Likely superconducting (no electrical resistance) superfluid (no friction).

We have an upper limit to neutron star mass: keep $v_n < c$ gives a limit of $2 - 3M_\odot$ (details are hard).

II.4.1 Spin

Angular momentum $J = I\omega$, $\omega = 2\pi/P$. Conserved. What made the NS?

$$\frac{R_{\mathrm{core}}}{R_{\mathrm{NS}}} \sim \frac{m_e}{m_n} \left(\frac{Z}{A}\right)^{5/3} \sim 500$$

going from the core (supported by electron degeneracy) to the NS. Conserving J:

$$I_{\mathrm{core}}\omega_{\mathrm{core}} = I_{\mathrm{NS}}\omega_{\mathrm{NS}}$$

and $I \approx MR^2$. So

$$\omega_{\mathrm{NS}} \sim \omega_{\mathrm{core}} \left(\frac{R_{\mathrm{core}}}{R_{\mathrm{NS}}}\right)^2$$

since the mass is the same. Or, going to period:

$$P_{\mathrm{NS}} \sim 4 \times 10^{-6} P_{\mathrm{core}}$$

Much faster! Core can rotate as fast as 30 min, so P_{NS} can be as little as 5 ms.

II.4.2 Magnetic Field

Instead of angular momentum, conserve magnetic flux $\Phi = BR^2$. Same arguments give $B_{\rm NS} \sim 250,000 B_{\rm core}$. What might the initial field be? We measure fields of $10\,\mathrm{T}$ in white dwarfs, so the field in a NS could be up to 10^7 T in NS. In fact we see NSs with fields up to $1000\times$ this.

II.4.3 A Limit To Rotation?

How fast can a NS rotate? 2 limits:

- Keep the equator $< c$
- centripetal force $<$ force of gravity

First one:

$$\frac{2\pi R}{P} = v_{\rm equator} < c$$

would give a limit of $2\pi R/c = 0.2\,\mathrm{ms}$.

The second:

$$\frac{v^2}{R} = \frac{GM}{R^2} = \frac{4\pi^2 R}{P^2}$$

would give a limit $P < \sqrt{4\pi^2 R^3/GM} = 0.4\,\mathrm{ms}$. (this is actually Kepler's third law, $P^2 \propto R^3$).

Lecture III Pulsars

Neutron stars were predicted in 1934 (neutrons discovered in 1932) and thought to be associated with SNe, but not seen for > 30 yr.

Jocelyn Bell was looking at radio emission over time. Studying scintillation, or flickering, which is irregular changes. Saw that there were little blips that appeared regularly, every 1.337 s. This came from the same part of the sky, which rose at a different time every night (*not* from Earth). These are *pulsars*.

Nature of pulsars was deduced by Tommy Gold. Based on facts:

- Periods of ~ 1 s common, although the Crab pulsar has $P = 33$ ms
- Periods are very stable, change by $\dot{P} \sim 10^{-15}$ s/s

Models:

Binary Star Would need P from orbital period. Can use Kepler's third law to show that for $M = 1\,M_\odot$, $a = 1.6 \times 10^6$ m. Compare to $R = 7 \times 10^8$ m for the Sun, or 5×10^6 m for Sirius B.

Could be orbiting NSs. However, GR says that orbit should gradually grow closer, which period is observed to grow longer.

Pulsating Star WDs oscillate with $P = 100 - 1000$ s. Can show that $P \sim 1/\sqrt{G\rho}$. If were a pulsating NS, then would have period of $\sim 10^{-4}$ times that of WD (ρ is 10^8 times higher), so this is OK, but maybe too short for slow pulsars.

Rotating Star As we saw, can rotate at ~ 1 ms and be OK for a NS. If were WD, would not periods of ~ 10 s at minimum, so can't work.

Result is that pulsars are rotating NSs.

III.1.4 Crab Pulsar

Center of Crab SNR, from 1054 AD. See $P = 33$ ms, with $\dot{P} = 4.2 \times 10^{-13}$ s/s. What does this imply about change in rotational energy? $I = \frac{2}{5}MR^2$ for uniform sphere. For NS,

$$I \approx 0.24 \left(\frac{M}{M_*}\right)^{1/3} \alpha_G^{-5/2} m_n \left(\frac{h}{m_n c}\right)^2 = 2.5 \times 10^{38}\,\mathrm{kg\,m^2} \left(\frac{M}{M_*}\right)^{1/3}$$

See $\dot{P}$. $\omega = 2\pi/P$, so $\dot{\omega} = -2\pi\dot{P}/P^2 = -2.4 \times 10^{-9}\,\mathrm{s^{-2}}$ (increase by a ms every 90 yr). Use $E_{\mathrm{rot}} = \frac{1}{2}I\omega^2$, so identify:

$$\frac{dE_{\mathrm{rot}}}{dt} = I\omega\frac{d\omega}{dt} = -4\pi^2 I\frac{\dot{P}}{P^3}$$

If we take $I = 10^{38} I_{38}\,\mathrm{kg\,m^2}$, have 4.6×10^{31} W. But we can also measure the amount of energy radiated away by the Crab Nebula and find 5×10^{31} W. These balance, and the energy of the Crab Nebula is supplied by the slowing rotation.

Overall, pulsars were found to be rotating neutron stars. We see blips when the "lighthouse" beam crosses the Earth **show animation**. The majority of the energy from the spin-down is invisible: the radio blips are a tiny fraction of the energy.

It is the strong magnetic field that makes this happen.

III.1.5 Magnetic Dipole Model

Light cylinder: where v to go around is c. We take the magnetic field to be a dipole, $B(r) = B_0(r/R)^{-3}$. A changing magnet releases electromagnetic power per unit area S (*Poynting flux*) $\sim cB^2/\mu_0$. We can roughly relate the spin-down energy loss $I\omega\dot{\omega}$ to the Poynting flux through the light cylinder:

$$4\pi R_{\rm LC}^2 S_{\rm LC} \approx I\omega\dot{\omega}$$

with $\omega = 2\pi/P$, $\dot{\omega} = -2\pi\dot{P}/P^2$. $R_{\rm LC} = cP/2\pi$, so $S_{\rm LC} = (c/\mu_0)B_{\rm LC}^2 = (c/\mu_0)B_0^2 R^6/R_{\rm LC}^2$. So we have:

$$4\pi R_{\rm LC}^2 \frac{c}{\mu_0} B_0^2 \frac{R^6}{R_{\rm LC}^6} = 4\pi R^6 \frac{c}{\mu_0} B_0^2 \left(\frac{cP}{2\pi}\right)^{-4} \sim \frac{R^6}{\mu_0}\frac{B_0^2}{c^3} P^{-4} \sim I\frac{\dot{P}}{P^3}$$

This gives:

$$B_0^2 \sim \frac{c^3 \mu_0 I}{R^6} P\dot{P}$$

So from the spin period and the rate at which it is slowing down, we can determine what the magnetic field is!

We can then use this (assuming B =constant) to get $P(t)$. The equation above is a simple ODE. We assume that the spin-down is constant, and have $P(0) = P_0$. Solve:

$$\frac{dP}{dt} = \frac{A}{P(t)}$$

With the solution

$$P(t) = \sqrt{2At + P_0^2}$$

with

$$A = \frac{R^6 B_0^2}{\mu_0 c^3 I} = \dot{P}_{\rm now} P_{\rm now}$$

If we assume that $P_{\rm now} \gg P_0$, then $P(t) \approx \sqrt{2At}$. Alternatively,

$$t = \frac{1}{2A}\left(P(t)^2 - P_0^2\right)$$

We find that the age is $\tau \approx P/2/\dot{P}$, so we also get the age of the system from P and $\dot{P}$. Do this for the Crab pulsar get 1250 years, which is very close to the true age of about 950 years (since people saw the supernova).

P-$\dot{P}$ diagram: HR diagram for pulsars. **draw**. Move through the diagram from upper left to lower right until you die from low voltage (don't actually die, just shut off). This takes 10^{7-8} yrs to get to $P = 10\,\mathrm{s}$ from a typical starting point of 10 ms. Usually born with 10^8 T, but there is a range.

Millisecond pulsars: $P = 1.56\,\mathrm{ms}$, $\dot{P} = 1.1 \times 10^{-19}\,\mathrm{s/s}$. This gives $B = 9 \times 10^4\,\mathrm{T}$, so much smaller than normal pulsar. And age of 2.3×10^8 yr. Cannot get there via normal evolution. Note that many of these are in binaries. Scenario is that MSPs live/die in in binary as normal PSRs. The transfer mass, angular momentum. Reborn (recycled) as MSPs.

III.1.6 Rotation

Conserve angular momentum. Everything has some. Sun rotates $\sim$ 1/month.

Go from core (WD) to NS. We have:

$$\frac{R_{\rm core}}{R_{\rm NS}} \approx \frac{m_n}{m_e}\left(\frac{Z}{A}\right)^{5/3} = 512$$

for $Z/A = 26/56$ for iron. This gives us the ratio of sizes. We can then conserve angular momentum (assuming mass is the same), $L = I\omega$ with $I = CMR^2$ (C depends on internal structure).

$$I_{\rm core}\omega_{\rm core} = I_{\rm NS}\omega_{\rm NS}$$

which gives

$$\omega_{\rm NS} = \omega_{\rm core}\left(\frac{R_{\rm core}}{R_{\rm NS}}\right)^2$$

Or, in terms of period P,

$$P_{\rm core} = P_{\rm wd}\left(\frac{R_{\rm NS}}{R_{\rm core}}\right)^2 \approx 4 \times 10^{-6} P_{\rm core}$$

If we use $P_{\rm core} = 1350\,\mathrm{s}$ for a known WD, we find periods of a few ms for a NS.

How does this compare with the minimum period? Use what we derived before, which can also be expressed as:

$$\frac{GM}{R^2} = \omega_{\max}^2 R$$

which gives

$$P_{\min} = \frac{2\pi}{\omega_{\max}} = 2\pi\left(\frac{R^3}{GM}\right)^{1/2} = 0.6\,\mathrm{ms}\left(\frac{M_*}{M}\right)$$

So as long as the period is $> 1\,\mathrm{ms}$, we don't have to worry about the NS breaking up.

III.1.7 Magnetic Field

Just as the angular momentum is conserved, so will be magnetic flux. This is because the material is a very good conductor. Roughly, $\pi R^2 B$ is a constant, so:

$$B_{\rm NS} \approx B_{\rm core}\left(\frac{R_{\rm core}}{R_{\rm NS}}\right)^2 = 1.3 \times 10^{10}\,\mathrm{T}$$

based on the most extreme WD field of $5 \times 10^4\,\mathrm{T}$ (more typical is $10\,\mathrm{T}$, and $2 \times 10^{-4}\,\mathrm{T}$ for the Sun). So the B could be really high, although it is usually a little more modest ($10^8\,\mathrm{T}$).

III.1.8 Braking Index

Spin-down could come from magnetic dipole or other sources. Parameterize:

$$\dot{\Omega} \propto -\Omega^n \tag{1}$$

with n the braking index. For magnetic dipole model get $n = 3$. Can show that for gravitational waves get $n = 5$, but otherwise a similar spin-down law. Measure it through:

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} \tag{2}$$

but can only measure this for young pulsars when higher derivatives are measureable. We find a generic age:

$$\tau = \frac{P}{(n-1)\dot{P}}\left(1 - \frac{P_0}{P}\right)^{n-1} \tag{3}$$

So the actual age depends on n (which may not necessarily be constant) and on the ratio between the initial and current periods.

III.1.9 Dispersion

Radio waves propagating through an ionized medium. Plasma frequency is finite:

$$\omega^2 = \omega_p^2 + k^2c^2 \tag{4}$$

with $\omega_p^2 = 4\pi n_e e^2/m_e$. So we get a group velocity:

$$v_g \approx c\left(1 - \frac{\omega_p^2}{2\omega^2}\right) \tag{5}$$

As ω decreases v_g decreases. So lower frequencies arrive later. Can parameterize in terms of total delay:

$$t(\omega) = \int_0^L \frac{dl}{v_g} = \frac{L}{c} + \frac{2\pi e^2}{m_e c\omega^2} \times DM \tag{6}$$

with DM the dispersion measure, the integral of the electron density:

$$DM \equiv \int_0^L n_e\,dl = \langle n_e\rangle L \tag{7}$$

Can use this to estimate how far away pulsars are based on DM, since it is proportional to L.

III.1.10 Glitches

Spin-down is mostly steady, based on change in Ω with I constant. But this doesn't have to be the case. The crust+charged particles rotate at one frequency, while the superfluid n's rotate at another. What keeps them together? There isn't a lot of friction between them. So the spin-down (which torques B, and hence the charged particles) will mean that after some time the crust will be slower than the superfluid interior. Some strain will build up, and all of a sudden (vortex unpinning) it will come undone. The two components will come back into equilibrium. This results in a deposit of angular momentum into the crust (and B), so the pulsar appears to spin-up a little bit in a very short amount of time.

We see $\Delta\Omega/\Omega \sim 10^{-6}$ (or lower), as well as a change in $\dot{\Omega}$. These come from recoupling between the two pieces, and then there is a relaxation time after that as the system comes into equilibrium.

Assume I_s is moment of superfluid, and I_n is moment of the rest. After a glitch, assume that ν_n changes discontinuously. Assume coupling between components has timescale τ_c. So:

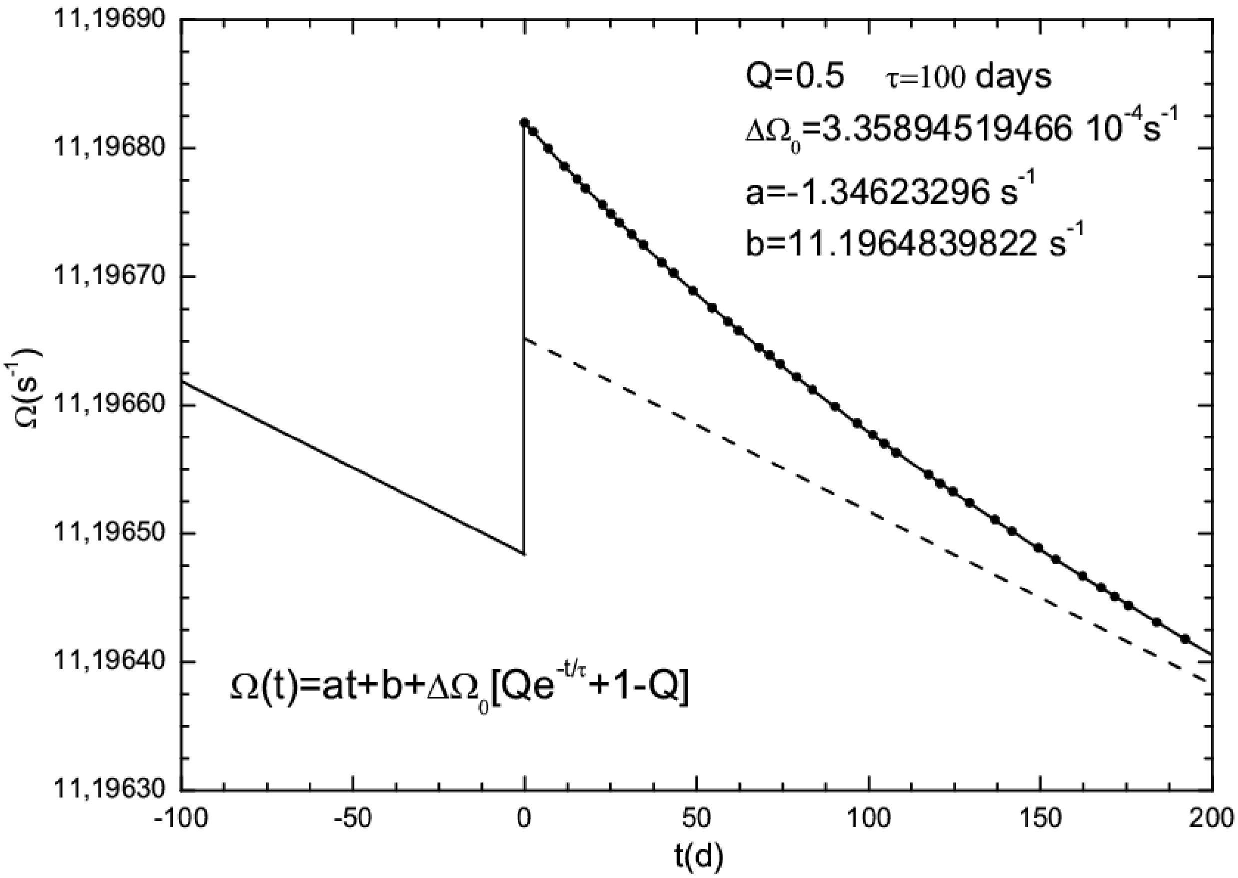
$$I_n\dot{\Omega} = -\alpha - \frac{I_n(\Omega - \Omega_s)}{\tau_c} \tag{8}$$

$$I_s\dot{\Omega}_s = \frac{I_n(\Omega - \Omega_s)}{\tau_c} \tag{9}$$

α is for external torques (spin-down). This gives:

$$\Omega(t) = \Omega_0(t) + \Delta\Omega_0(Qe^{-t/\tau_c} + 1 - Q) \tag{10}$$

where Q describes the healing — how close to the original frequency does it come back. τ_c is weeks to months, so a lot of the interior should be superfluid.



III.1.11 Magnetosphere

A vacuum dipole is not a good approximation. If B is strong, will be induced $E = v \times B$ which can be very high. But this will be cancelled by plasma in the magnetosphere.

Consider a conducting sphere rotating at Ω. Charges would orient on surface to cancel induced E such that:

$$E + (v \times B) = E + ((\Omega \times r) \times B) = 0 \tag{11}$$

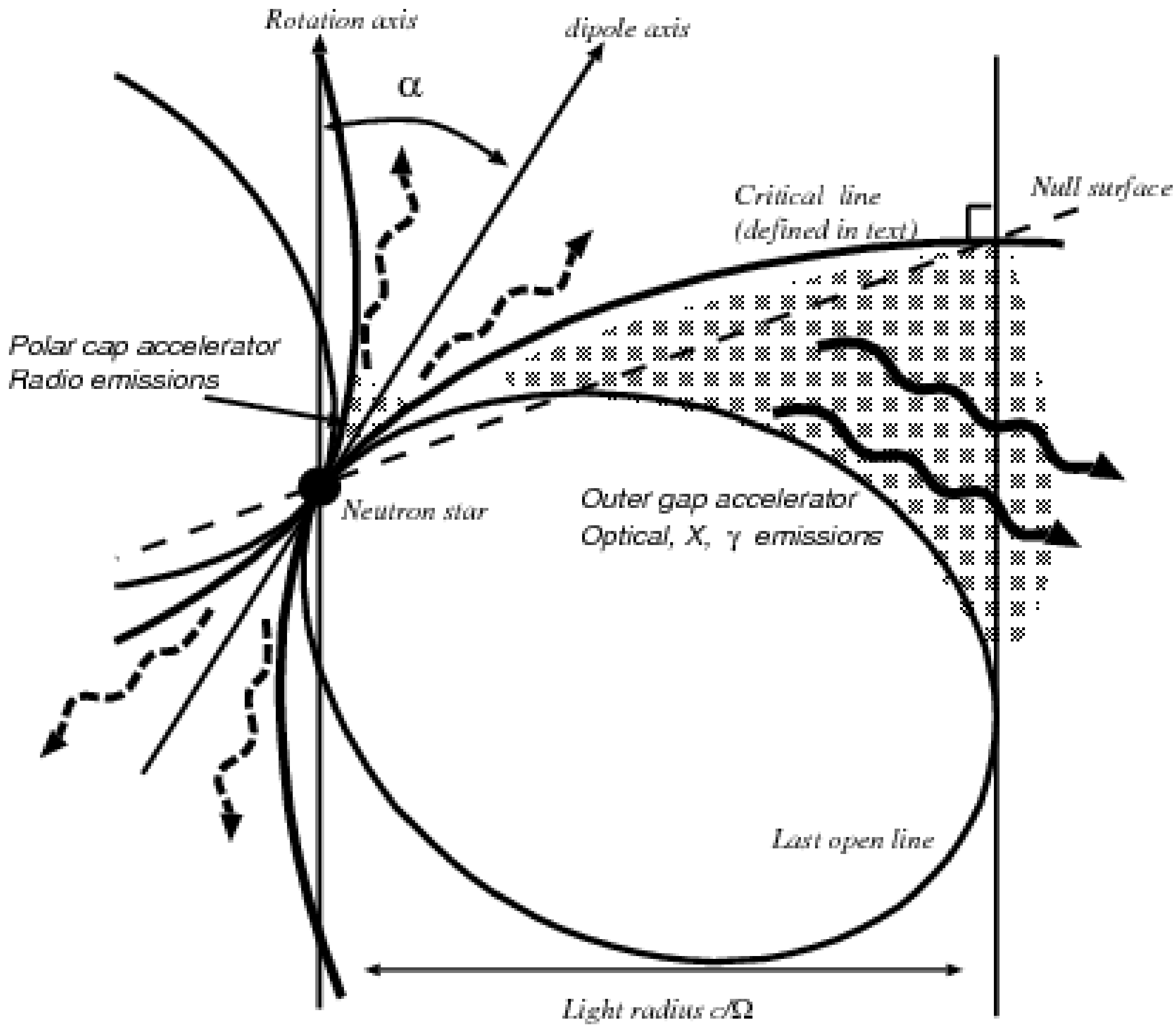
These charges make E through Maxwell's equations. At the surface, we have $\nabla^2 E = 0$, which has an electrostatic potential:

$$\phi = -\frac{B_0 \Omega R^5}{6r^3}(3\cos^2\theta - 1) \tag{12}$$

If we then look at $\partial\phi/\partial r$, there is a strong E_r which will pull charges off the surface. $E_r \sim \Omega R B \sim 6 \times 10^{12}\,\mathrm{V/m}$ for a period of 1 s. Compare Lorentz force to gravity:

$$\frac{e(v \times B)}{GMm_e/R^2} \sim \frac{e\Omega R^3 B}{GMm_e} \approx 10^{12} \tag{13}$$

The magnetosphere will be filled by charges pulled from the surface.



There are various regions in the magnetosphere with “gaps”, where charges can be accelerated, are potential locations for the high-energy emission.

Lecture IV Basic Statistics & Fitting a Model to Data

Given a collection of data, it is often useful to be able to describe its distribution with simple statistical measures like mean, median, mode, variance, and standard deviation. Each of these statistics gives us a glimpse of how the entire dataset is distributed. First, we will define each of these quantities, then we will extend our discussion to how they apply to normal distributions and the central limit theorem; finally, we will begin to describe fitting a model to data with least squares fitting.

IV.1.12 Basic statistics

To begin, we will compute statistics for discrete, independent, and identically distributed values. This means that each value is separate (not smooth/continuous), is not affected by the values of other data, and individual trials are replaceable. In class, we created our own dataset by rolling 6-sided dice (10 times per person). For calculating the following statistics, imagine we have rolled the following numbers: 1, 1, 4, 5, 3, 4, 4, 6, 1, 6.

Mean: *the average value of a dataset.* Sometimes we will refer to the mean value as an "expectation value," and often, we will use μ to represent the mean. To calculate the mean of N values,

$$\bar{x} = \frac{1}{N}\,\Sigma_{i=1}^{N} x_i = \mu_x. \tag{14}$$

Using this equation with the values above gives a mean of $\mu_x = 3.5$. Try this for yourself!

Median: *the middle value of a dataset when values are sorted in numerical order.* To find the median of our sample dataset, sort the values in numerical order (1, 1, 1, 3, 4, 4, 4, 5, 6, 6), then find the median "index": $i_{\mathrm{med}} = (N+1)/2$. In this case, $i_{\mathrm{med}} = 11/2 = 5.5$, so we calculate the mean by averaging the 5th and 6th x-values. In other words, $\mathrm{Med}(x) = (x_5 + x_6)/2 = 4$.

Mode: *the most common value in a dataset.* In our example dataset, three trials result in $x = 1$ and three result in $x = 4$. Therefore this distribution is "bimodal."

Variance: *the spread of values about the mean.*

$$\mathrm{Var}(x) = \sigma_x^2 = \frac{1}{N}\,\Sigma_{i=1}^{N}(x_i - \mu)^2 \tag{15}$$

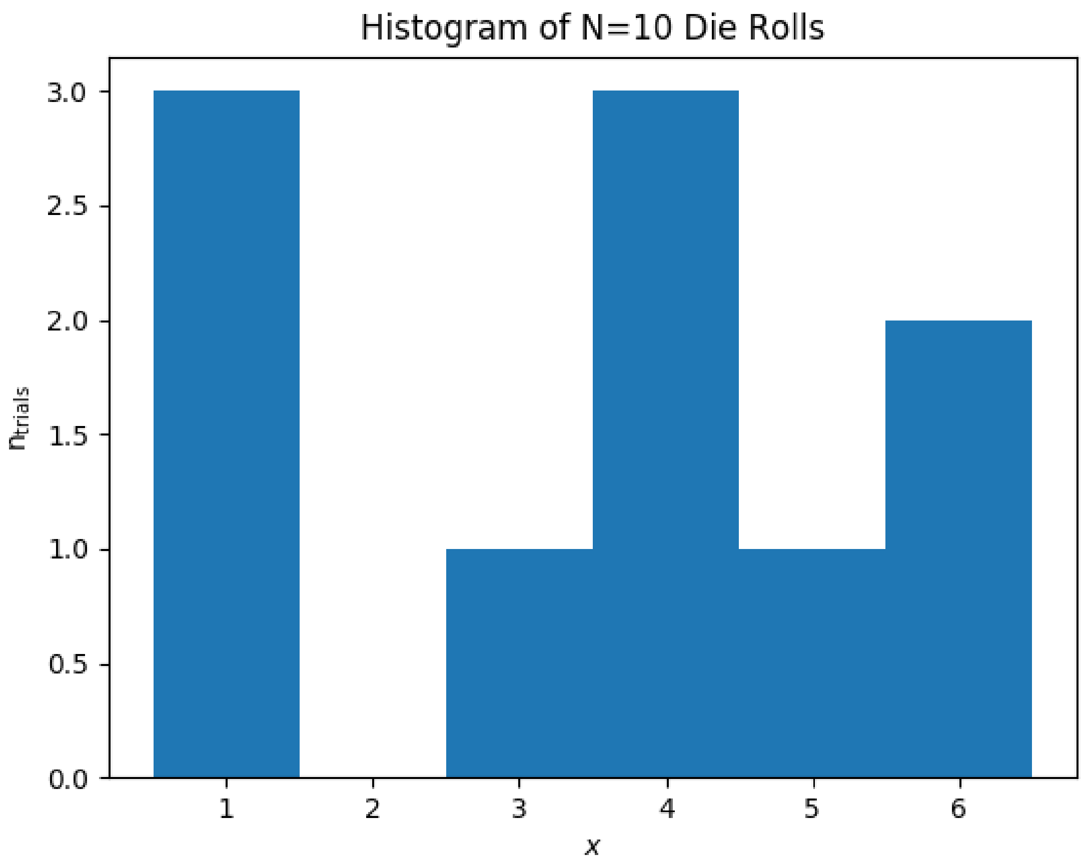
Our sample dataset has a variance, $\sigma_x^2 = 3.45$

Standard Deviation: *square root of the variance.*

$$\mathrm{Std}(x) = \sigma_x = \sqrt{\frac{1}{N}\,\Sigma_{i=1}^{N}(x_i - \mu)^2} \tag{16}$$

Our sample dataset has a standard deviation, $\sigma_x = 1.86$. Looking ahead, standard deviation (σ) is commonly used to express the width of a normal distribution (see below).

All of these statistics also have mathematical definitions for distributions of continuous (smooth, non-discrete) functions, but it would be helpful to get a better idea of what these distributions/functions look like first. By binning our sample results, one way to visualize the data is by plotting a histogram (shown below). A histograms has a discrete number of bins, and therefore is well-suited to displaying the results of multiple trials of rolling a 6-sided die.



Other distributions are continuous, meaning that they can take on *any* values between $-\infty$ and ∞. These distributions are represented with functions, the most common of which is called the normal distribution (Gaussian function, shown below – soon). The normal distribution has the functional form:

$$f(x) = A\, e^{-(x-\mu)^2/2\sigma^2} \tag{17}$$

where $A = (2\pi\sigma^2)^{-1/2}$ and μ and σ represent the function's mean and standard deviation. The normal distribution is one of the most commonly-used distribution in statistics, physics and many other fields. Stated very loosely, the *central limit theorem* says that for most kinds of things, if you do them enough and collect enough data, you will get a Gaussian.

What does this mean exactly? Think about the distribution of peoples' heights, a distribution well-modeled by a normal distribution. Most men's/women's heights are near the average value, μ, while extreme heights – both short and tall – are *far* less prevalent. For a Gaussian distribution, 68% of the area under the curve is within 1σ of the mean, 95% is within 2σ, 99.7% within 3σ, and so forth. In fact, if you were to add up all of the area under the normal distribution represented by $f(x)$ for all values of x,

$$\int_{-\infty}^{\infty} dx\, f(x) = 1. \tag{18}$$

The fact that the integral (area under the curve) for $f(x)$ is equal to one is no coincidence; the normalization coefficient (A) is chosen such that this is the case. In fact, all *probability density functions* (PDFs) work this way. They provide the full range of possibilities of a given variable (e.g. x) and give the probability of x being between some range of values x_{low} and x_{high}. Therefore, it makes sense that the probability of x being between $-\infty$ and ∞ would be one! A second requirement for PDFs is that they always have positive values.

To explore the normal distribution a bit further, let us try to calculate its full width at half maximum (FWHM) when $\mu = 0$. To do so, set $f(x) = A/2$ and solve for x. You should find that $\text{FWHM} = 2\sigma\sqrt{2\ln 2}$, showing that a Gaussian's width depends entirely on its standard deviation, σ. Furthermore, a Gaussian's mean, median, and mode are all equal. With the equations provided below for continuous distributions, you can try calculating some of these quantities.

Mean:

$$\mu_x = \int_{-\infty}^{\infty} dx\, x\, f(x) \tag{19}$$

Median: given by m in the following expression.

$$\int_{-\infty}^{m} dx\, f(x) = \int_{m}^{\infty} dx\, f(x) = 0.5 \tag{20}$$

Mode: the maximum value of $f(x)$; the peak.

Variance:

$$\ldots \tag{21}$$

Standard Deviation:

$$\ldots \tag{22}$$

IV.1.13 Fitting a Model to Data (Linear Least Squares)

In the previous section, we briefly mentioned the idea of fitting a Gaussian to histograms of men's/women's heights to construct PDFs describing their distribution. To begin, let's take a detailed look at exactly the steps and assumptions that go into modeling a set of data with a simple, linear curve – generating a *linear least squares fit*.

Imagine we have a dataset consisting of country's areas versus their population sizes and say we would like to explore fitting those data with a linear model. Visualize a scatter plot with a point for each country's area (x_i) on the horizontal axis, and its population size (y_i) on the vertical axis. You might imagine the larger the country, the larger the population, therefore we will attempt to model the aggregate behavior of these data with a line of the form,

$$\hat{y}_i = mx_i + b \tag{23}$$

where x_i values are the same as those of the data points, $\hat{y}_i$ give population sizes *predicted* by our model, m and b represent the model's slope and y-intercept respectively. For the sake of simplicity (and since the concept of population is hard to fathom in a country with zero area), we will assume $b = 0$ for the following discussion, but it would be instructive to see if you can write the equations without making this simplifying assumption.

In order to find the best model to fit the data, we take the square of the difference between individual data points (y_i) and corresponding values predicted by our model ($\hat{y}_i$), computed using x_i. This is effectively one form of the chi squared function, $\chi^2 = \Sigma_i \, (O_i - E_i)^2/E_i^2$ using observed (O_i; data) and expected (E_i; model) values respectively. Here,

$$\chi^2 = \Sigma_{i=1}^{N} (y_i - \hat{y}_i)^2 \tag{24}$$

and we would like to minimize the squared differences between data and model values to achieve the best fit. This is where the name *least squares* comes from. In calculus, you have performed minimization by taking the derivative and setting it equal to zero (e.g. think about how you would find the minimum value of an upward-facing parabola). In our case, we want to find the slope of our linear fit that minimizes the χ^2 function, thus we differentiate the function with respect to m and solve for m:

$$\frac{d}{dm}\chi^2 = 0 \tag{25}$$

$$\frac{d}{dm}\left[\Sigma_{i=1}^{N} (y_i - mx_i)^2\right] = 0 \tag{26}$$

$$2\,\Sigma_{i=1}^{N} (mx_i - y_i)\, x_i = 0 \tag{27}$$

$$m\,\Sigma_{i=1}^{N} x_i^2 = \Sigma_{i=1}^{N} x_i\, y_i \tag{28}$$

$$m = \Sigma_{i=1}^{N}\, x_i\, y_i / \Sigma_{i=1}^{N}\, x_i^2 \tag{29}$$

This is a powerful expression; by simply plugging in data values x_i and y_i respectively, we can now find the slope m for a line that best fits the data! Think about how we would proceed if we had not set $b = 0$. What additional steps would need to be taken to find the best-fit m and b values simultaneously for a given dataset?

Lecture V Binary Systems

Since momentum is conserved, the two bodies will orbit their common center of mass. The COM can move, but it will move at a constant velocity. This is defined such that:

$$m_1 r_1 = m_2 r_2$$

Both stars must go around in the same time such that the line between them always goes through the COM:

$$P = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$

(assuming circular motion). Or $r_1/v_1 = r_2/v_2$. Combining these we get:

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

We also have the force:

$$F = G\frac{m_1 m_2}{(r_1 + r_2)^2}$$

As before, this must be the force needed to keep an object in a circular orbit. So on object 1, $F = m_1 v_1^2/r_1$:

$$\frac{m_1 v_1^2}{r_1} = G\frac{m_1 m_2}{(r_1 + r_2)^2}$$

Divide both sides by m_1, and use $P = 2\pi r_1/v_1$:

$$\frac{4\pi^2 r_1}{P^2} = \frac{G m_2}{(r_1 + r_2)^2}$$

But we also define $R = r_1 + r_2 = r_1(1 + r_2/r_1)$. With the ratio of the masses intead of radii:

$$R = r_1(1 + \frac{m_1}{m_2}) = \frac{r_1}{m_1}(m_1 + m_2)$$

Put this in:

$$\frac{4\pi^2 R^3}{G} = (m_1 + m_2)P^2$$

This is Kepler's third law! It's a bit more general than for just a system like the Earth and the Sun. We often write a instead of R.

This works well for a star, where we measure the radial velocity through a Doppler shift. A velocity would change the frequency of all periodic things. This would apply to the radio emission from the pulsar, but since we don't know that the emission should be restricted to just a single emission line then it doesn't help us. It would change the frequency of the pulses themselves, But for pulsars we can be more precise by using phase (i.e., pulse number of time-of-arrival) instead.

As an example, consider a rotating source which evolves as:

$$P = P_0 + \dot{P}(t - t_0)$$

So if we want to meaure $\dot{P}$, it comes down to measuring the slope of the line fit to $P(t)$. But what we are doing here is losing track of all information between observations. i.e., if we measure $P(t)$ on Monday and $P(t)$ on Friday, the only piece of information used between them is a rough starting point for P. You don't use the fact that the rotation *must* be continous between them. So you should know not only what $P(t)$ is but what the pulse number is. More generally, we can talk about measuring times-of-arrival (TOAs) as a proxy for measuring a phase, $\phi(t)$:

$$\phi(t) = \phi_0 + f(t - t_0) + \frac{1}{2}\dot{f}(t - t_0)^2$$

[Note that we use frequency f instead of period P since the math is a bit easier. But we can use $P(t) = 1/f(t)$ and take the time derivative:

$$\frac{dP(t)}{dt} = \dot{P} = -\frac{1}{f(t)^2}\frac{df(t)}{dt} = -\dot{f}/f^2$$

So we requiring that the pulsar rotate in a known way between the observations. This is more demanding on us because we can easily lose track of the pulse number and get the wrong answer. But if we do it right then we end up measuring $\dot{f}$ based on a measurement that gets bigger as $\propto t^2$, which will mean we can do it faster and more precisely.

Going back to binaries, we want to make sure we look not at a velocity but at a phase. Changes in time-of-arrival look like delays to the pulse phase. We call it a_1. This is the projected size of the *pulsar*'s orbit, not the whole size of the orbit. We also call this the Römer delay.

Another problem is that we do not know the total size of the orbit, only the size along the line-of-sight. If the orbit were inclined the equations would look the same. We should just have the wrong mass for the companion.

If we restrict to circular orbits, we have:

$$v_1 = \Omega_b a_p \sin i \cos \Omega_b t$$

and

$$a_1 = -Omega_b^2 a_p \sin i \sin \Omega_b t$$

with a_p the pulsar's semi-major axis, i the inclination ($0°$ is face-on), and $\Omega_b = 2\pi/P_b$ the orbital frequency. The second is not an acceleration but a delay: it is the integral of the velocity. There is also an arbitrary time zero-point (or orbital phase zero-point) that we don't necessarily care about but which we need to know, T_0. But let's derive the mass function. We have Kepler's third law:

$$\frac{4\pi^2 a^3}{G} = (M_p + M_c)P_b^2$$

with a instead of R. We don't measure a, we measure $a_p = aM_c/(M_p + M_c)$, which is just how much the pulsar moves. Or actually just $a_p \sin i$. And we also have $a_c = aM_p/(M_p + M_c)$ for the companion. Solve for a:

$$a = a_p \frac{M_c + M_p}{M_c}$$

or re-write:

$$a = (a_p \sin i)\frac{M_c + M_p}{M_c \sin i}$$

and substitute:

$$4\pi^2 (a_p \sin i)^3 \left(\frac{M_c + M_p}{M_c \sin i}\right)^3 = G(M_p + M_c)P_b^2$$

We can move a few things around and get:

$$\frac{4\pi^2}{G}\frac{(a_p \sin i)^3}{P_b^2} = \frac{(M_c \sin i)^3}{(M_p + M_c)^2}$$

But we usually write it in terms of $m_p = M_p/M_\odot$ and $m_c = M_c/M_\odot$, masses in units of solar masses. So we get:

$$\frac{4\pi^2}{G}\frac{(a_p \sin i)^3}{P_b^2} = M_\odot \frac{(m_c \sin i)^3}{(m_p + m_c)^2}$$

or the mass function:

$$f(m_p, m_c) = \frac{(m_c \sin i)^3}{(m_p + m_c)^2} = \frac{4\pi^2}{G}\frac{(a_p \sin i)^3}{M_\odot P_b^2} = \frac{4\pi^2 x_1^3}{T_\odot P_b^2}$$

with $T_\odot = (GM_\odot/c^3) = 4.925 \times 10\,\mu$s which enables us to write $x_1 = a_p/c$ as a delay in seconds.

When we measure systems we measure x_1 and can usually get P_b. But the equation still has 3 more unknowns, m_c, m_p, and $\sin i$, with only a single equation to connect them. So 1 equation and 3 unknowns. This cannot have a single solution. Instead we must use other information.

We do know that $0^\circ < i \leq 90^\circ$, but this usually isn't good enough. If we have some idea what the companion is (say from optical observations) we could limit m_c. Similarly we might have some idea what m_p is since it's a pulsar, so $m_p \approx 1.4$, but we don't know exactly. What can we do?

What about a second pulsar. So far we only know of one of these, the Double Pulsar. In that case we also get an x_2 measurement:

$$\frac{4\pi^2 x_1^3}{T_\odot P_b^2} = \frac{(m_2 \sin i)^3}{(m_1 + m_2)^2}$$

and:

$$\frac{4\pi^2 x_2^3}{T_\odot P_b^2} = \frac{(m_1 \sin i)^3}{(m_1 + m_2)^2}$$

But this still isn't enough. Divide one by the other:

$$\frac{x_1^3}{x_2^3} = \frac{m_2^3}{m_1^3}$$

so $m_2/m_1 = x_1/x_2$, which is just what we had worked out before. So if we measure two pulsars we can get the *mass ratio*, but we don't know the individual masses or the inclination.

How about if we measure a Doppler shift, say from a normal star in the system. Then we effectively measure $v_c \sin i = 2\pi a_c \sin i/P_b$. Or we are in the same situation as before where we have determined the mass ratio but still need another variable to determine the individual masses or inclination.

The main way out of this is General Relativity. Instead of Newton's gravity, we modify gravity a little (at least in this regime). So orbits lose a little energy over time and shrink, and an orbit won't exactly go back to where it started from. If that orbit is an ellipse, the ellipse will precess. Rather than use full GR we usually look at a Post-Newtonian or Post-Keplerian solution, which looks at just the initial corrections to Newton needed to do things quickly. They work for (almost?) all systems so far. We have three PK terms:

- Einstein delay γ (only for elliptical orbits), coming from moving the pulsar in and out of the potential well of the companion and changing its speed. This is because in GR a moving clock changes its time, as does a clock moving in gravitational fields (has been verified on Earth).
- Precession $\dot{\omega}$: change the arbitrary phase.
- Orbital shrinkage $\dot{P}_b$: change the orbital period (like $\dot{P}$, usually best measured from a fit of orbital phase vs. time).

These are all functions of the same m_p and m_c. So if we measure 2 of them we have a system with as many unknowns (3) as equations (3) – if we know more because of another pulsar or a companion we have *over-determined* the system. If we measure a 3rd PK parameter we can test GR itself. There are also another couple of PK parameters related to the Shapiro delay: the delay a pulse takes going into and out of the potential well of the companion. Only happens for orbits near edge-on.

[Note: we said that phases were better than periods. But when you are first solving a system sometimes knowledge of the phase is very hard to use, so you will use periods to get a rough $P(t)$ or orbit.]

Lecture VI Noise

Why do we need big telescopes? How long do we have to observe in order to see something useful? How big a telescope do you need?

photons come at some rate: source has a flux density F_ν in J/s/m^2/Hz. To get rate of photons:

$$F_\nu \times \frac{\text{telescope area}}{\text{photon energy}} \times \text{bandpass} \times \text{efficiency}$$

gives N (photons/s). Number detected is $N \times$time:

$$n = F_\nu \frac{A}{h\nu} \Delta\nu \Delta t \eta$$

A bright radio source has a flux density of 1 Jy or $10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$. Take a telescope diameter of 100 m and a bandpass of 100 MHz, at a frequency of 1.4 GHz, so we get:

$$10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}} \times \pi 50^2\,\mathrm{m}^2 \times 100\,\mathrm{MHz} \times 0.1 \times (1.4\,\mathrm{GHz}h)^{-1} = 8 \times 10^8\,\mathrm{photons/s}$$

Or, in 1 min we get:

$$8 \times 10^8\,\mathrm{photons/s} \times 60\,\mathrm{s} = 5 \times 10^{10}\,\mathrm{photons}$$

It the optical/IR we are in the limit of small numbers of photons, and it actually makes sense to count them. But here in the radio we are flooded with huge numbers of tiny (in terms of energy) photons, and it makes more sense to think about this in terms of total energy received. We treat photons more as waves than as particles in this limit.

But still, how long do we need to observe for? How many seconds are enough?

Consider a source with flux density S. Telescope has real area A, and effective area $A_e = \eta A$ (what might account for $\eta < 1$?)?

We can think about the radio telescope as a device that measures energy per unit bandwidth per unit time (or power per unit bandwidth) equivalently as a circuit attached to a resistor at a temperature T. The resistor is at a finite temperatuer. Random motions of electrons will lead to a varying voltage across the circuit which leads to a power per bandwidth $P = k_B T$ (you'll learn to derive this later). We define an antenna temperature as the equivalent temperature of a circuit to give the same power as what we see on-sky. So we get:

$$S = \frac{2k_B T_A}{A_e} = \frac{T_A}{G}.$$

The factor of 2 accounts for two polarizations. So we have defined a *gain* $G = A_e/2k_B$, which comes in the weird units of $\mathrm{K/Jy}$. It converts a source flux density into an equivalent temperature. Larger numbers are better (proportional to area). So Parkes is 0.64 K/Jy (64 m), while Arecibo is 20 K/Jy (300 m).

We can then think about everything in terms of temperature. So a source will have an equivalent temperature $T_{\text{source}} = GS_{\text{souce}}$. While the brightest radio sources can be ~ 1 Jy, the brightest pulsars are ~ 10 mJy (note that this is very dependent on frequency) so in most cases the temperature of the telescope would change by $< 1\,\mathrm{K}$. And there are other sources of noise. We talk about a total system temperature:

$$T_{\rm sys} = T_{\rm receiver} + T_{\rm sky} + T_{\rm other}$$

So the receiver has a finite temperature, which typically at GBT will be $\sim 20\,\mathrm{K}$. We also have to worry about the temperature of the sky. At GHz frequencies that will be very small (minimum is $3\,\mathrm{K}$ from the CMB), but at low frequencies like with the MWA that can be hundreds or thousands of degrees. And there are other sources of noise too. Usually we try to figure out which source of noise is worst, and don't try too hard to eliminate the other sources if they are smaller. i.e., we can get cooled receivers at the GBT with $T_{\rm rec} = 20\,\mathrm{K}$. But at MWA frequencies we have $T_{\rm sky} = 1000\,\mathrm{K}$, so that doesn't make sense. Instead we can use room-temperature receivers with $T_{\rm rec} = 300\,\mathrm{K}$ and we still don't notice.

We often parameterize a system in terms of the System-Equivalent Flux Density (SEFD): this is the strength of a source (in flux density units, Jy) which gives the same temperature as $T_{\rm sys}$. So $\mathrm{SEFD} = T_{\rm sys}/G = 2k_B T_{\rm sys}/A_e$. This has the two pieces we most need to control when designing an experiment: make A_e big and $T_{\rm sys}$ small.

What does this mean? We still don't know how long we have to observe. We have said that a source works like a circuit producing an amount of power per bandwidth. But the receiver will have its own power per bandwidth. And all of that power will make fluctuations in the voltage we get out of our circuit. We find that for a system with temperature $T_{\rm sys}$, we get fluctuations that have amplitude:

$$\Delta T_{\rm sys} = \frac{T_{\rm sys}}{\sqrt{2\Delta t \Delta f}}$$

where again the 2 is for polarizations, Δt is the integration time (how long we point the telescope) and Δf is the bandwidth. So what we are seeing here is that for a unit time ($1\,\mathrm{s}$) and a unit bandwidth ($1\,\mathrm{Hz}$), we get $\Delta T_{\rm sys} = T_{\rm sys}/\sqrt{2}$. But we can average N independent samples together using the central limit theorem, with $N = \Delta t \Delta f$. Then the noise goes down as $1/\sqrt{N}$.

We then compare the change in temperature from our source $T_{\rm source} = GS$ to the fluctuations from the system. We need the change from the source to be $> 5 \times \Delta T_{\rm sys}$ or so, such that we can be really certain we are seeing something real. So we can look at a signal-to-noise ratio:

$$\mathrm{S/N} = \sqrt{2\Delta t \Delta f}\frac{GS}{T_{\rm sys}}$$

So we wnt Δt to be high, Δf to be high, S, to be high, G to be high, and $T_{\rm sys}$ to be low.

We can correct this slightly because pulsars only pulse for a short time, and if we know that we can use the off-pulse and compare with the on-pulse.

If we have an interferometer this doesn't really change. Roughly we can still use the total area of the telescope in computing G, and nothing else is different.