Name: ____________________

Date: ____________________

Phys 194–FYRE Assignment #8
Fourier Transforms

Assignment Policy: You can consult class notes, books, and online resources. You can work in small groups (2 or 3), but you must turn in your own work. Make sure you are clear about the process you use to solve the problems: partial credit will be awarded.

Today we will look at **time-series** data and find hidden periodicities through the Fourier Transform. By **time-series** we mean data that have $Y_i = Y(t_i)$ for a time samples t_i. First, copy the file `assignment8_1.dat` from `shared/UWM_FYRE/Assignment-8` on the `nanograv` server into your directory. Then fire up `python`. We will want to start our python with:

```
%matplotlib inline
import math
import numpy as np
import matplotlib.pyplot as plt

import scipy.signal
```

The last command will give us access to Fourier Transforms.

Getting Started

1. Load in the data with
   `t,Y=np.loadtxt('assignment8_1.dat',unpack=True)`
   Try plotting the data. What is the period of the signal? What is the frequency (make sure you use units for both; the time axis is in seconds)? What is the amplitude?

2. Now make a power spectrum. To do this we need to first know how often the axis was sampled. So we want to calculate:
   `dt=t[1]-t[0]`
   And then we can say:

```
f,P=scipy.signal.periodogram(Y,fs=1/dt,scaling='spectrum')
```

Specifying `fs` (the sampling frequency) means that the frequency axis will have proper units of Hz. Specifying `scaling='spectrum'` changes the units of the output power spectrum. Note that this is not a raw Fourier Transform: Fourier Transforms return complex numbers with both amplitude and phase information. Here we have instead only a power, so all phase information is lost, and the amplitude is squared. Plot the power spectrum. Can you identify a spike that shows us the periodicity of this signal? We want to have `python` tell us what the position of the spike is. The best way to do that is to find where the power array `P` is at its maximum value: `P==P.max()`. That actually returns an array of *boolean* values (True or False) – try it. The boolean array can then be used to tell us which element of the frequency array it corresponds to: `f[P==P.max()]` will be the frequency of the spike. What is that value? Can you determine the amplitude of that signal from the power spectrum?

3. Now look at `assignment8_2.dat`, `assignment8_3.dat`, `assignment8_4.dat`. For each of these try to determine the frequency, amplitude, and phase from plotting $Y(t)$, and the repeat it with the power spectrum. How do those numbers compare? Does the amplitude you read off the power spectrum relate directly to the amplitude you see on the $Y(t)$ plot?

Now With Noise

Those were easy because we could identify the signals by eye. Now we will add noise and see how easy we can recover the signals.

1. Load in `assignment8_5.dat`. If you plot $Y(t)$ you can hopefully still see the periodic signal. Can you estimate the signal-to-noise ratio (SNR): the amplitude of the signal divided by the "amplitude" of the noise (formally we would use the standard deviation or something similar, but in this case you can't do that easily because the signal is so strong).

2. Now create and plot the power spectrum. Can you still see the periodicity? If so determine the frequency and amplitude of the signal. Then you can use the power spectrum to measure the standard deviation (remember `P.std()`) to get the SNR. What do you find?

3. Repeat this for `assignment8_6.dat` and `assignment8_7.dat`. Can you still identify the periodicity by eye in those cases? What do you get for the SNR from the power spectrum?

Two Signals

1. Look at `assignment8_8.dat`. The signal $Y(t)$ there contains two sine waves added to each other. Can you figure out their amplitudes and periods just from the $Y(t)$ plot?

2. Now try doing it with the power spectrum.

3. Repeat on `assignment8_9.dat`.

Not Sines

1. Look at `assignment8_10.dat`. Plot $Y(t)$. It's no longer a sine wave. This shape is more like a pulse. What is the period? Amplitude?

2. Now look at the power spectrum. What do you see?

3. How can you estimate the properties of the signal from the power spectrum? Contrast this with the sine case. What do you see about the amplitude of the signal?

4. Look at `assignment8_11.dat`, `assignment8_12.dat`, `assignment8_13.dat`. Repeat the examination in both the time-domain ($Y(t)$) and the frequency domain (power-spectrum). Describe your results both qualitatively (in words) and quantitatively (in numbers).